A MANUAL OF HARMONY FOR USE IN MUSIC-SCHOOLS AND SEMINARIES...

Oscar Paul

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A
MANUAL OF HARMONY

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FOR USE IN

MUSIC-SCHOOLS AND SEMINARIES AND FOR SELF-INSTRUCTION

BY

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TRANSLATED FROM THE LATEST GERMAN EDITION

BY

THEODORE BAKER
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TRANSLATOR'S PREFACE.

The teacher who, by doing away with a mass of cumbrous and antiquated Rules, and substituting therefor a simple and immutable Principle, lightens and abridges the student's task without any sacrifice of thoroughness, may justly be regarded as a promoter of the best interests of art.

This fact has long been recognized by those studying under Prof. PAUL's personal supervision. Through the publication of his Manual of Harmony the study of the first principles of musical art was greatly simplified for all familiar with the German language. It has been the aim of the translator, himself a pupil of the author, to present, as nearly as might be, the German *spirit* of this work in a good English *body*; to adhere as closely as possible to the original text, without, however, hesitating to make such slight deviations as appeared needful for securing directness and clearness of expression.

The elementary law underlying all forms of harmonic combination, the "Interconnection of the Chords", is here set forth and illustrated in the most thorough and perspicuous manner, thus sweeping away at a blow the arbitrary "rules" and "exceptions" of former systems. The science of thorough-bass notation is exhibited with a precision and fulness of detail such as may be sought for in vain even in books exclusively devoted to this subject. (This chapter is given in greatly abridged form in the present edition.) The appendix contains, in brief, such information as is necessary for a clear understanding of the mathematical department of musical science, together with references to many of the highest authorities in musical literature. The controversial tone pervading the whole work, which may be considered by some critics as a blemish, affords a refreshing contrast to the dogmatical laying down of the law too common among books of this class, and will hardly fail to stimulate bright minds to *think for themselves*, an advantage which cannot well be overestimated. The popularization of many of HAUPTMANN's theories, through their practical application, forms a prominent and highly valuable feature of this work.

The (perhaps) somewhat labored explanation of progressions between "indirectly connected" triads (pp. 17—19) is to be ascribed to HAUPTMANN's influence, and may, like the paragraph on the resolution of the chord of the seventh on the second degree in major, be thought pedantic; in the latter case at least this censure would be unjust, as the author's own exposition sufficiently proves.

In the assurance that the work will recommend itself far better than any extended eulogy could do, this translation is now laid before the American musical public.

LEIPZIG, December, 1883.

THE TRANSLATOR.

AUTHOR'S PREFACE.

The excellent results attending the teachings of the two masters, Moritz Hauptmann and E. Fr. Richter, have been gratefully acknowledged by all those who were so fortunate as to enjoy their instruction in harmony and counterpoint. To the practical discernment of the experienced teacher, E. Fr. Richter, the student owed his rapid comprehension of the theory of music, and a speedy familiarity with the science of chords in its entirety; but for the correction of many errors, — for directing the musical contemplation to the nature of harmony, — for obtaining a clear perception of that which may be accomplished in this art, — and for the thorough mastery of harmonic combinations, which latter, based upon the indispensable principle of the interconnection of chords, were examined and illuminated from every point of view, his thanks were due the master Moritz Hauptmann. As a preparation for the study of that grand monument of genius, the work entitled "Die Natur der Harmonik und der Metrik" [The Nature of Harmonic and Metrical Science] by Hauptmann, an extended course of scientific and musical training is required; Richter's Manual of Harmony, even in the latest editions, ignores — doubtless intentionally — the light which the researches of science and pedagogics during the last twenty years have thrown upon the harmony of Sebastian Bach, the nature of the affinity of chords, the subject of acoustics, and on the diversity in the perception of tones. Death has unhappily put an end [to the oral teaching of both these eminent scholars.

For these reasons one of their former pupils now ventures to publish a Manual of Harmony, which, while strictly adhering to scientific principles, is nevertheless written in popular form, and easily intelligible even to the inexperienced. Its object is, to make the student quickly acquainted with the science of chords, to teach him, through the working out of numerous exercises, the treatment of figured basses in all the keys, and, by means of the accompanying explanatory observations, to enable him to comprehend the masterworks of earlier as well as of later date, and to attain the degree of insight and independence requisite for supplying, in the compositions of SEB. BACH and his contemporaries, such parts as are indicated merely by figures. In this Manual, which is intended for music-schools and seminaries, and also for self-instruction, the author has endeavored to unite brevity and clearness of style with a positive and well-grounded exposition of the formation of the chords, an independent arrangement of suitable examples, and a systematic explanation of the harmonic progressions employed in strict composition, showing plainly that upon these latter are based the regular forms found in the free style; he has attempted to write a manual of harmony which should meet the practical and scientific requirements of the present, and prove really useful to the student of music. The observant critic will perceive that the author sought especially, in his disposition of the triads, to make all constituent elements of the key available, and, in the exercises on the treatment of these chords, to introduce the augmented triad; to arrange the diatonic and chromatic progressions of the chords of the seventh methodically; to explain the structure of the chords of the ninth and of suspended chords; to further an understanding of the nature of augmented chords, and of the harmonies belonging to the "extended" minor modes; and, finally, to present a complete analysis of the system of thorough-bass notation employed in the BACH period, in order to ensure a proper comprehension of the same.

Although claiming the independent authorship of this work, the writer is far from considering it perfect; regarding it as a duty to adhere to the principle, that everyone should, until the close of his career, seek to *learn*, and to combat *his own infirmities*.

While ready to defend the principles laid down in this book, the author will be thankful for any corrections proffered by competent critics.

May this Manual of Harmony meet with a friendly and favorable reception.

LEIPZIG, May, 1880.

OSCAR PAUL.

TABLE OF CONTENTS.

Chapter I. pp. 1—6.

The Scales. — Structure of the same. Difference between the major and minor scales. Difference between the harmonic and melodic minor scales. View of the scales.

Chapter II. pp. 6—12.

Intervals. — Formation of the same above and below a given basis. Consonances. Diatonic and chromatic dissonances. The intervals in pure and tempered harmony.

Chapter III. pp. 12—28.

Keys and Chords. — Keys in general. The major mode and its structure. Triads of the major mode. Classification of triads into those connected through the third or fifth, or indirectly. Triads on the second and seventh degrees. Progressions between triads. Contrary motion. View of the triads. Exercises in progressions between the fundamental triads in major. Special reference to the progressions between triads connected through the fifth in major. The tritone; not to be used in melodic progression. Closes, authentic, plagal, and perfect. Exercises in progressions between fundamental triads, with special reference to chords connected through the fifth. Sequences. Review of all intervals, taking one tone as basis for the formation of the same.

Chapter IV. pp. 28—31.

The Minor Mode and its Triads. — Interconnection of the triads in minor. Augmented and diminished triads.

Chapter V. pp. 31—37.

Progressions between the Triads in Minor. — Avoidance of the augmented second. View of the triads in minor. Exercises in progressions between the same.

Chapter VI. pp. 37—38.

On the Formation of Closes with the Triads of the Minor Mode. — Authentic, plagal, perfect and deceptive closes.

Chapter VII. pp. 38 —51.

The Inversions of the Triads. — Fundamental position of the triad. "Close" and "open" position of the chord. First and second inversion (= second and third position) of the triad, caused by setting the third and

fifth respectively as bass, *i. e.* as basis of the chord. The doubling of any tone in the triad in four-part harmony. Chord of the fourth and sixth, its employment and preparation. Its relation to the other chords. Examples in major. Exercises in progressions between the triads and their inversions in major. The same in minor.

Chapter VIII. pp. 51—66.

Chords of the Seventh. — General conception of the term. Formation. Chords of the seventh in major. Dominant chord of the seventh. Its preparation. Its resolutions in major. Employment of the ligature in harmonic examples. Exercises in progressions between the chords in major, using the dominant chord of the seventh in its fundamental position. Chords of the seventh in minor. Resolutions of the dominant chord of the seventh in minor. Exercises including the same. Chord of the seventh on the seventh degree in major and minor. Comparison of the chord of the seventh on the seventh degree in major with that on the second degree of the relative minor key. Resolution of the diminished chord of the seventh in minor. The *combined* (major and minor) mode. Resolutions of the various chords of the seventh in major. Exercises in progressions between the fundamental chords of the seventh and the triads in major.

Chapter IX. pp. 66—73.

Chords of the Seventh in Minor. — Resolutions of the various chords of the seventh in minor. Progressions between chords of the seventh connected through the fifth. Exercises in progressions between the fundamental chords of the seventh and the triads in minor.

Chapter X. pp. 73—86.

Inversions of the Chords of the Seventh. — Their progressions to triads. Progressions between chords of the seventh connected through the third and fifth. Comparison of the chord of the seventh on the fourth degree in minor with that on the second degree in the relative major key. Exposition of the chords of the seventh by the aid of the *combined* mode. Their chromatic progression. The same as based upon the principle of interconnection, illustrated by stationary tones. Passing sevenths. Arsis and thesis. Observance of these rhythmical relations in connection with passing sevenths. Exercises on the chords of the seventh and their inversions, in various harmonic progressions in major and minor. Exercises in the employment of passing sevenths, using the melodic minor scale.

Chapter XI. pp. 86—93.

Chords of the Ninth. — Explanation of the same. Their positions. Organ-point. Exercises.

Chapter XII. pp. 93—100.

Suspensions. — Suspensions before triads, chords of the sixth, and chords of the fourth and sixth. Suspensions before fundamental chords of the seventh, chords of the fifth and sixth, chords of the third, fourth and sixth, and chords of the second. Incorrectly formed suspensions. Suspensions with an upward resolution. Exercises in the employment of suspensions.

Chapter XIII. pp. 100—107.

The Augmented Chords. — Augmented chord of the sixth, of the fifth and sixth, of the third, fourth and sixth, and of the second, fourth and sixth. Altered chords non-existent. Chromatic passing chords not to be confounded with these latter. Derivation of the augmented chords from those of the "extended" minor mode. Explanation of the latter. Exercises in employing the chords of the "extended" minor mode.

Chapter XIV. pp. 108—119.

Modulation. — The relation of the various keys to one another. Modulation with triads. With chords having four tones [tetrads]. Examples in modulation. The four different clefs: soprano, alto, tenor and bass. Exercises. Short pieces serving as an introduction to independent four-part composition.

Chapter XV. pp. 119—122.

Fifths and Octaves. — Reference to the Bach period. Views of C. Ph. E. Bach on parallel fifths and octaves, taking into consideration the middle (inner) parts in relation to each other and to the bass, and also the outer (highest and lowest) parts. Upward and downward progression of the bass. Progression between the triads on the fifth and sixth degrees in minor. Chase after parallel fifths and octaves.

Chapter XVI. pp. 123—139.

Thorough-bass Notation. — Explanation of the system of notation used by Bach and his contemporaries. Strokes through the figures. The chromatic signs (♯, ♭, ♮). Peculiar manner of employing the same in the Bach period. Method of indicating the third. The *Be-quadratum* and *Be-rotundum*. *Basso continuo.* Where several like chords occur in succession, the first only is figured. Figuring of dotted notes and rests. Difference in the treatment of short and long pauses. Division of the figures according to value of the notes. Manner of indicating three-part harmony in the "gallant" (free) style. Telemann's curve. Manner of indicating the chord of the fifth and sixth. The cross-relation. Difference between the two species of cross-relation. Enharmonic changes. To familiarize the pupil with Bach's figuring, exercises are given from C. Ph. E. Bach's "Essay" which were, perhaps, written by Seb. Bach himself.

Chapter XVII. pp. 140—142.

Passing and Changing notes. — Their division into two classes. Their various employment in the strict and free styles.

Chapter XVIII. pp. 142—144.

Organ-point. — Its entrance. Organ-point in the soprano and middle parts. Organ-point as employed by Rameau and C. Ph. E. Bach.

Chapter XIX. pp. 144—146.

Syncope, Retardation, Anticipation, and After-striking. — Difference in meaning of these terms. Various explanatory examples in notes.

Chapter XX. pp. 147—151.

On harmonizing a Melody, or Cantus firmus. — Choral Composition. — *Cantus firmus* in the soprano. Exercises. Choral-books by Sämann and A. Dörffel, Fischer, Rinck, and Schicht. Seb. Bach's Choral-songs.

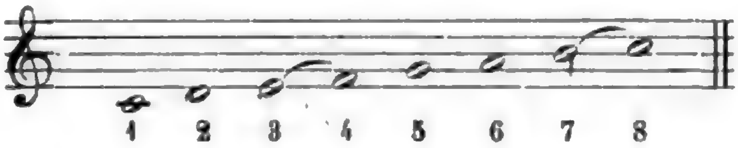
Appendix. pp. 152—168.

Range of musical tones. Acoustic relations as determined by HELMHOLTZ. Intensity, pitch, and quality of tone. Designation of the several octaves according to absolute pitch. Diatonic and chromatic. Period of vibration. Comparison of the tones with regard to the number of vibrations, according to SCHEIBLER. The pure and tempered systems of harmony. Arrangement of the consonances according to euphony. Overtones. Tones of combination: (1) differential tones; (2) tones of summation. The triad on the second degree in major a diminished triad. Plagal and authentic. Church-modes. Breitkopf and Härtel's Palestrina edition. BÖHME's Old German Song-book. The systems of alphabetical notation devised by HAUPTMANN and HELMHOLTZ, for use in acoustic researches. HAUPTMANN's system most available in practical study. M. W. DROBISCH's calculations for the systems of pure harmony and of equal temperament. Consonant and dissonant intervals in pure harmony, and their relations in respect to vibration. RISCHBIETER's treatise on the chord of the fourth and sixth. Thesis and arsis. WESTPHAL's "Elements of Musical Rhythm". Remarks on the treatment of the accompaniment, in J. D. HEINCHEN's Manual of Thorough-bass, 1711 and 1728; also in PETRI's Introduction to practical Music, 1782. Dr. RUST's prefaces to the edition of the Bach gesellschaft. CHRYSANDER's biography of HÄNDEL.

CHAPTER I.

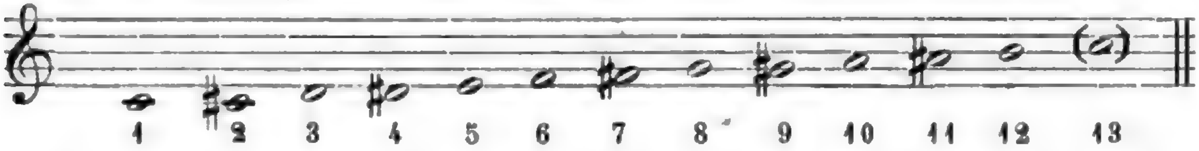
The Scales.

In the scale a definite position is assigned to each tone. The harmonic relations of the various tones can, however, be comprehended only after gaining a knowledge of the chords. The beginner must therefore first of all learn the structure of the several major, minor and chromatic scales mechanically. All scales progress, both upwards and downwards, in a succession of seconds. A second is the progression from any tone in the scale to the next one. Those seconds which occur in the major and minor scales are called diatonic seconds, to distinguish them from the chromatic seconds, which are found in the chromatic scale. [See Notes I and II]. In the major and minor scales we find [with the single exception noticed below] major and minor diatonic seconds; in the chromatic scale only minor diatonic seconds and chromatic seconds. [The progression of a second is called a *step*, to distinguish it from progression by a wider interval, which is called a *skip*.] The scale of *C*-major is composed of the following tones:



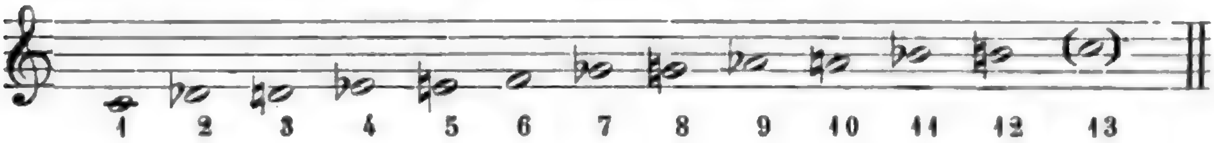
On examining this scale we perceive, that between the first degree and the second, the second and third, the fourth and fifth, the fifth and sixth, and the sixth and seventh, major seconds are to be found; between the third and fourth, and seventh and eighth degrees, are minor seconds.

On the other hand, the chromatic scale beginning on *C*, which is formed by using sharps (♯) (*i. e.* signs which raise the notes before which they are set by a chromatic semitone):



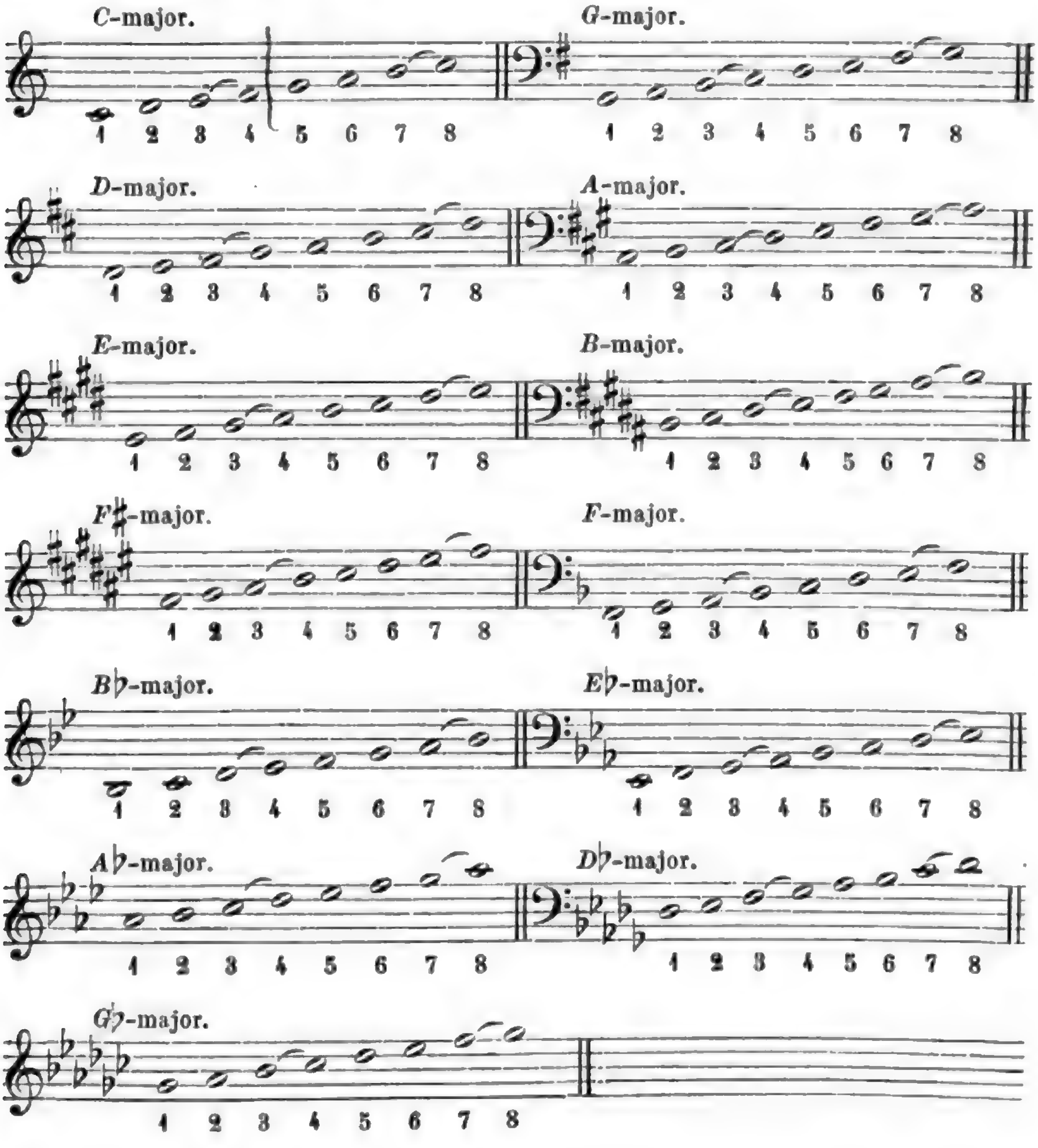
has chromatic seconds between 1 and 2, 3 and 4, 6 and 7, 8 and 9, 10 and 11; and diatonic minor seconds between 2 and 3, 4 and 5, 5 and 6, 7 and 8, 9 and 10, 11 and 12, 12 and 13.

If the chromatic scale be formed with the aid of flats (♭) (*i. e.* signs which lower the notes before which they are set by a chromatic semitone) and of naturals (♮) (*i. e.* signs which restore a note previously raised by a sharp or lowered by a flat to its original pitch):

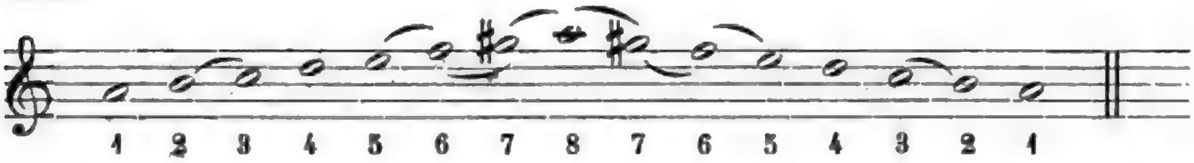


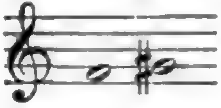
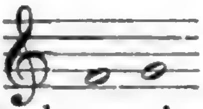
chromatic seconds occur between 2 and 3, 4 and 5, 7 and 8, 9 and 10, 11 and 12; diatonic minor seconds between 1 and 2, 3 and 4, 5 and 6, 6 and 7, 8 and 9, 10 and 11, and 12 and 13.

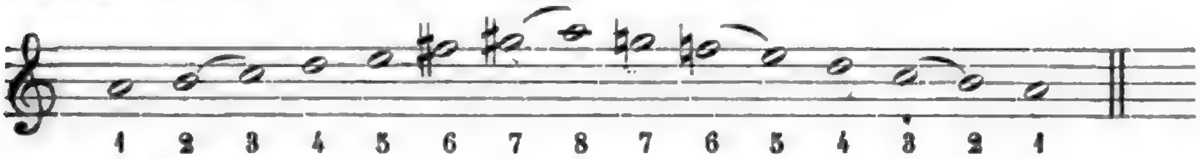
All major scales are formed alike. They may be exhibited, using the violin-clef and the bass-clef alternately, and marking the diatonic minor seconds by curves, as follows:



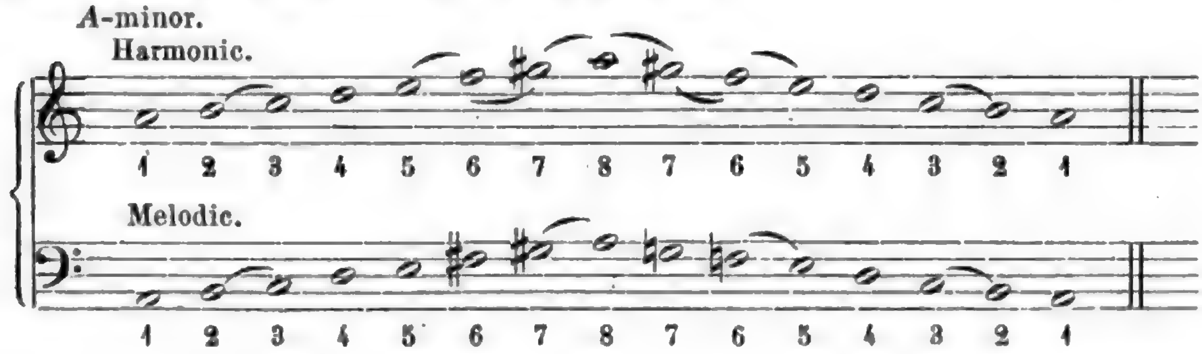
The minor scale has two forms: the *harmonic* and the *melodic*. Taking the scale of *A*-minor as model, the harmonic minor scale would appear thus:



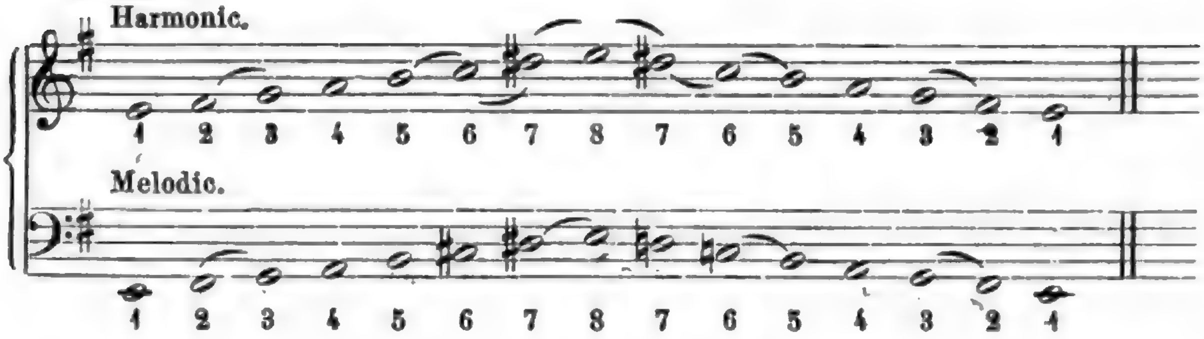
In this scale minor diatonic seconds are found between the second and third, fifth and sixth, and seventh and eighth degrees; major diatonic seconds between 1 and 2, 3 and 4, 4 and 5. Between the sixth and seventh degrees  an interval occurs which is greater by a chromatic second than the major second . This interval between the sixth and seventh degrees of the harmonic minor scale is called an *augmented* second, the interval of a major second being augmented by a chromatic semitone. This augmented second is considered an unmelodic interval in the strict style of composition, and therefore undergoes a change in the *melodic* minor scale. In upward progression this change is effected by raising the sixth degree of the harmonic scale by a chromatic semitone (= chromatic second); and in downward progression by lowering the seventh degree by a chromatic semitone, in which latter case the sixth retains the same position as in the harmonic minor scale. The *melodic* scale of *A*-minor is therefore formed thus:



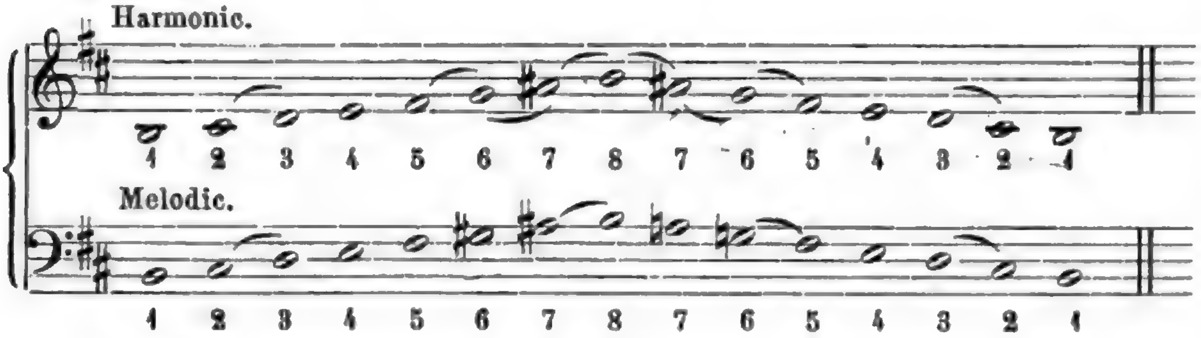
It is evident that the augmented second is removed; that between the third and fourth, and seventh and eighth degrees, moving upward, and between the sixth and fifth, and third and second degrees, moving downward, minor diatonic seconds are to be found; and that all the remaining steps are major diatonic seconds. The following harmonic and melodic minor scales, in which the minor diatonic seconds are marked by curves above, and the augmented seconds by curves below, should be thoroughly impressed upon the memory. [See Note III.]



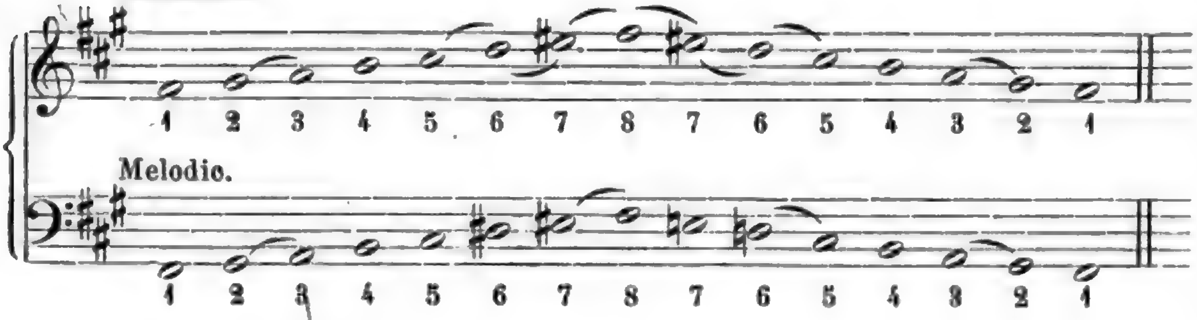
E-minor.



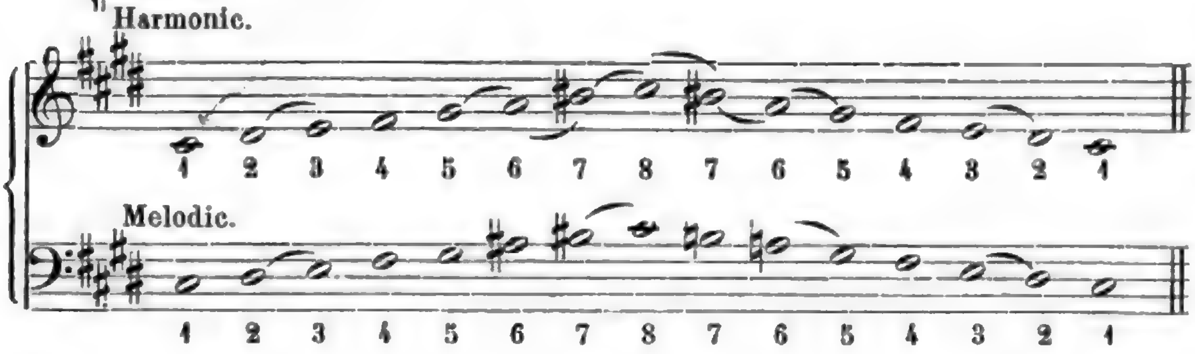
B-minor.



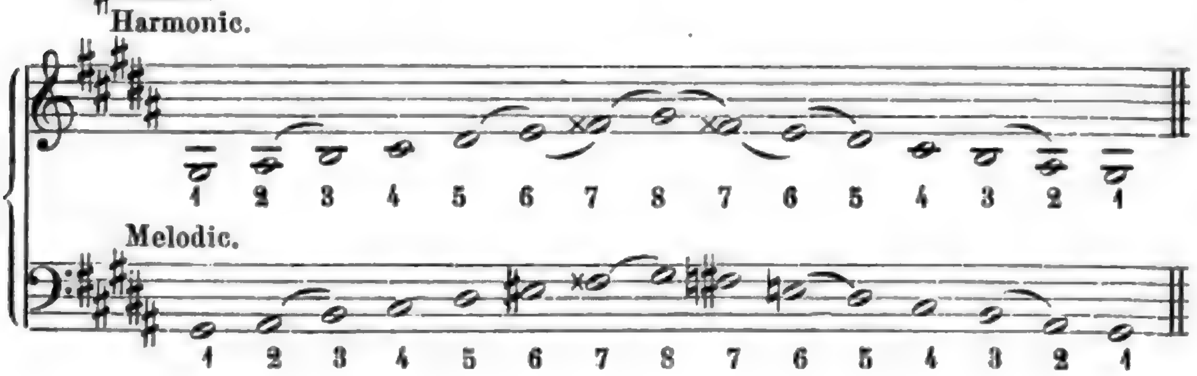
F♯-minor.



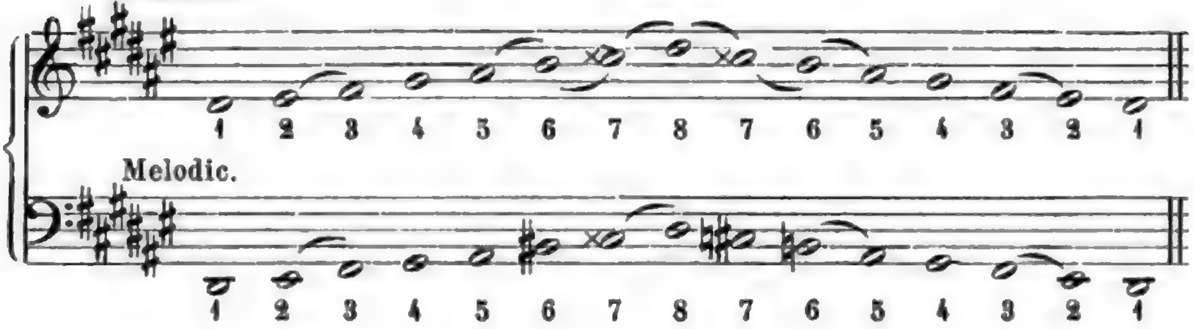
C♯-minor.



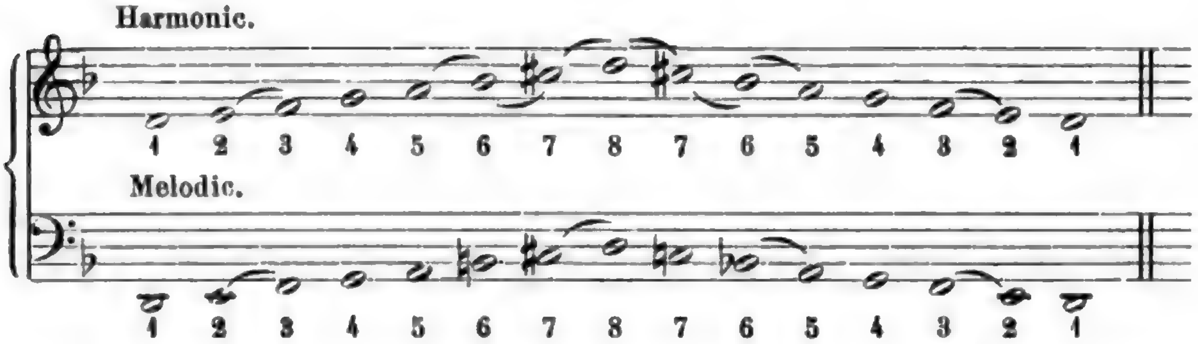
G♯-minor.



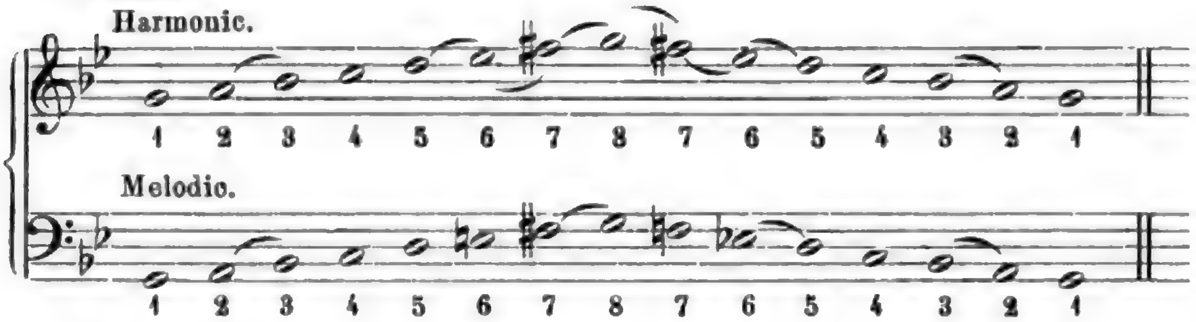
D♯-minor.



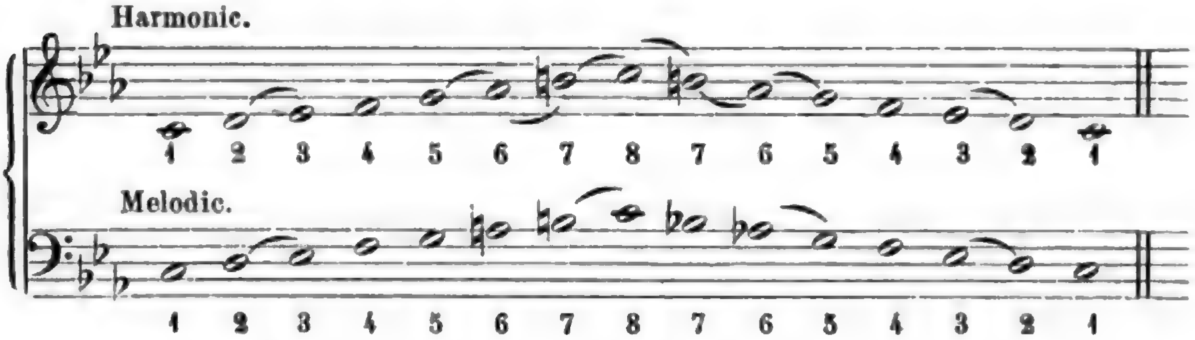
D-minor.



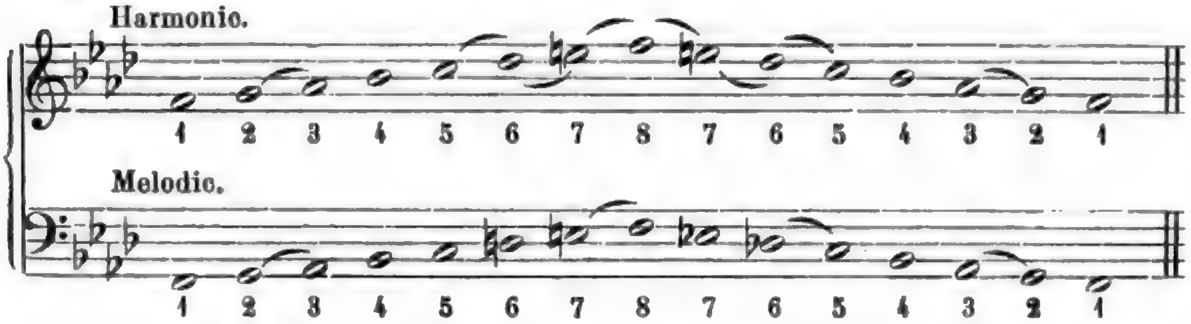
G-minor.



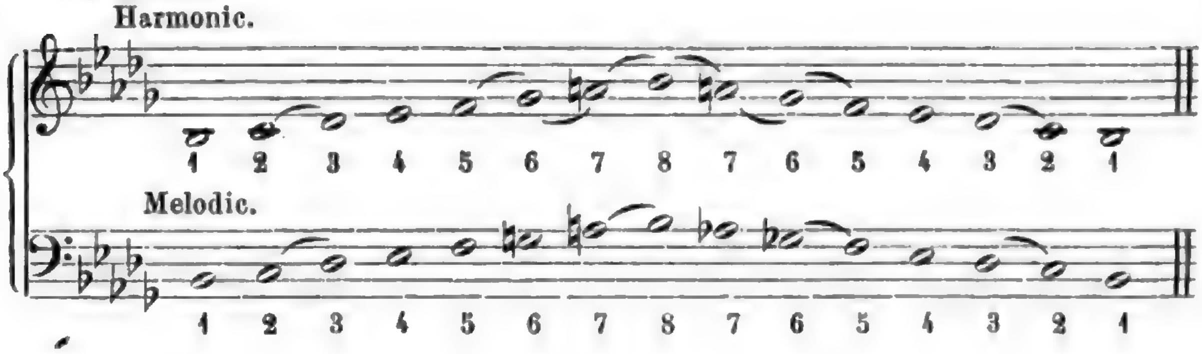
C-minor.



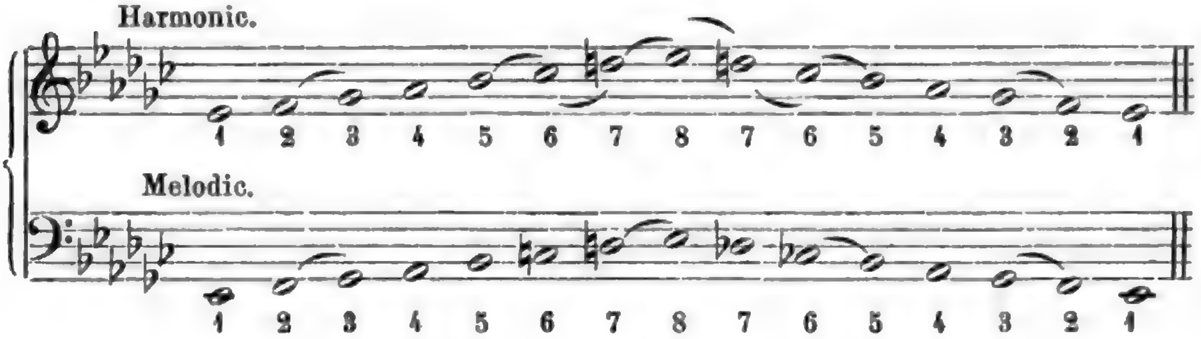
F-minor.



B♭-minor.



E♭-minor.



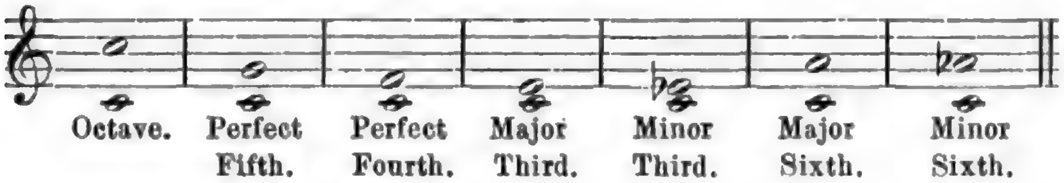
CHAPTER II.

The Intervals.

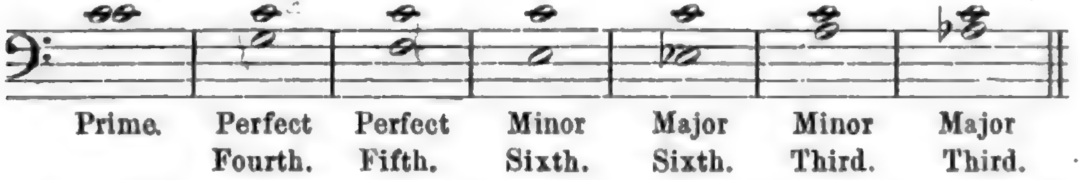
An interval is the difference in pitch (in point of gravity or acuteness) between two tones.

The musical intervals, taking the note *c′* as basis, are:

I. *a*) Consonances formed above the basis:



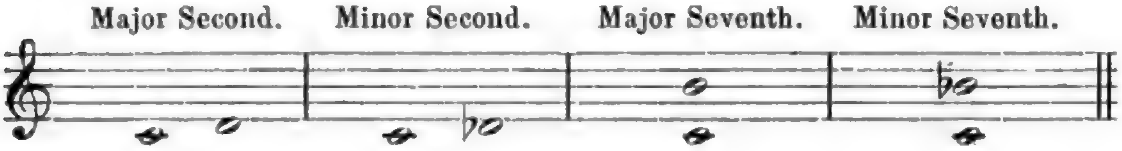
b) Consonances formed below:



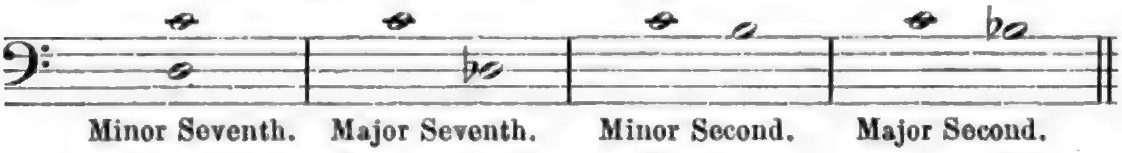
The octave, fifth and fourth are designated as *perfect* intervals because they are alike in major and minor. *C*-major and *C*-minor for instance have the same octave, fifth and fourth. The thirds and sixths, on the contrary, differ in major and minor; *C*-major has a major third and major sixth; *C*-minor (harmonic) has a minor third and a minor sixth.

From the above examples it is plain that the "Consonances formed below" are the inversions of the "Consonances formed above", and *vice versa*. The downward inversion of an interval consists in setting the upper note of said interval an octave lower, the lower note retaining its original position; the upward inversion consists in setting the lower note an octave higher, the upper note retaining its position.

II. (*a*) Diatonic dissonances formed above the basis:

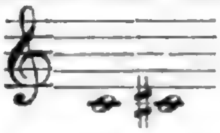


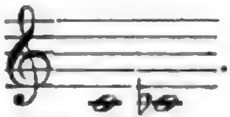
b) Diatonic Dissonances formed below:



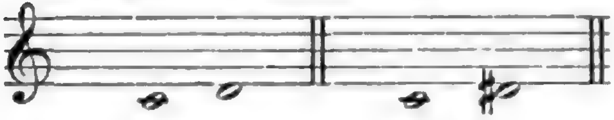
This table shows that the inversion of the major second gives the minor seventh; that of the minor second the major seventh; that of the major seventh the minor second; and that of the minor seventh the major second.

III. Chromatic dissonances and augmented and diminished intervals.

a) Chromatic second, formed above: ; this interval is formed by raising the tone taken as basis a chromatic semitone; in this case by a simple sharp; at other times by a ♮ or a ×. In various manuals of harmony this interval is designated as the "augmented prime". The prime and octave are, however, strictly speaking, invariable tones, which do not admit of expansion in either direction; in the formation of intervals the prime is to be regarded as the point of departure, and the octave as the extreme limit; it is therefore incorrect to speak of augmented and diminished primes or octaves.

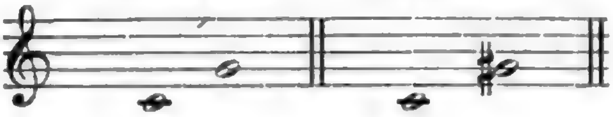
b) Chromatic second formed below: . Both of these chromatic seconds must be carefully distinguished from the minor diatonic seconds occurring in the diatonic scale, *e. g.* the interval *e-f* or *b-c'* in the scale of *C*-major, or in the harmonic *A*-minor scale; or in general the interval between the third and 4 ~~fifth~~, and seventh and eighth degrees in any major scale, and between the second and third, and fifth and sixth degrees in any harmonic minor scale.

c) Augmented intervals formed above. Every such interval is formed by raising the higher tone of a *major* or *perfect* interval a chromatic semitone. For example *c'-d'* is a major second, *c'-d'*♯ an augmented second formed above:



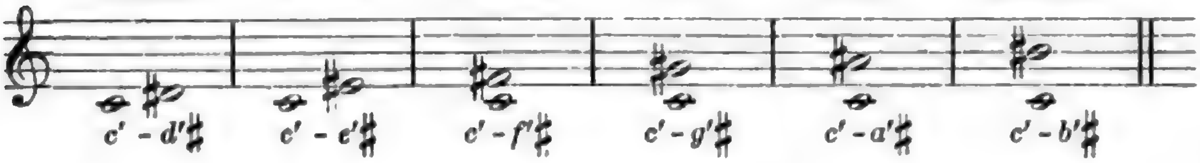
Major Second. Augm. Second.

c′-*g′* is a perfect fifth, *c′*-*g′*♯ an augmented fifth formed above:



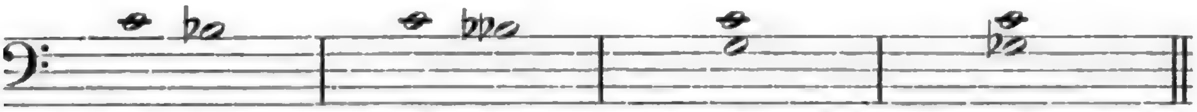
Perfect Fifth. Augm. Fifth.

The following intervals are therefore to be designated as augmented intervals formed above:



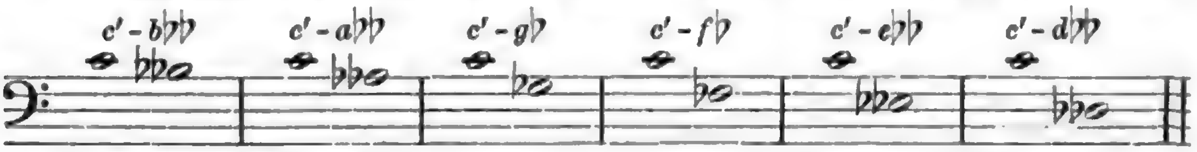
Augm. Second. Augm. Third. Augm. Fourth. Augm. Fifth. Augm. Sixth. Augm. Seventh.

d) Augmented intervals formed below. Each of these intervals is formed from either a *major* interval or a *perfect* interval by lowering the lower tone by a chromatic semitone. Taking the major second *c′*-b♭ for example, *c′*-*b*♭♭ would be the augmented second formed below; *c′*-*g* is a perfect fourth, *c′*-*g*♭ an augmented fourth formed below:



Major Second. Augm. Second. Perfect Fourth. Augm. Fourth.

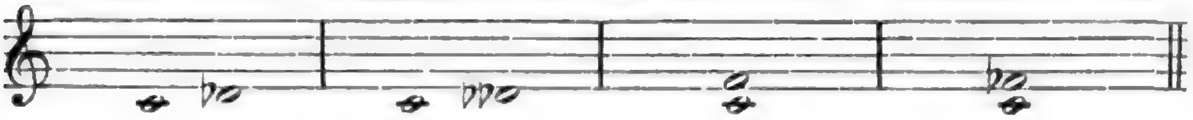
The augmented intervals formed below would therefore be as follows:



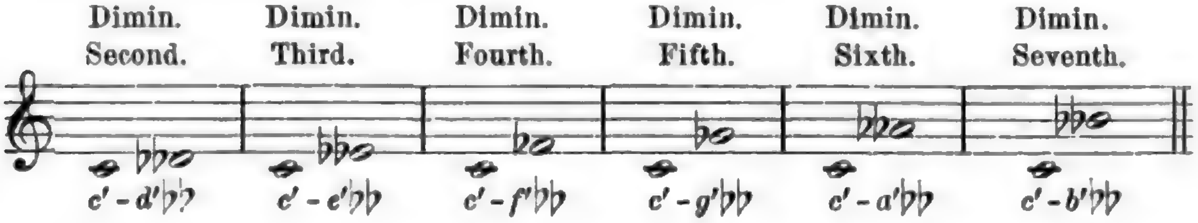
Augm. Second. Augm. Third. Augm. Fourth. Augm. Fifth. Augm. Sixth. Augm. Seventh.

e) Diminished intervals formed above. Each one of these intervals is formed from either a *minor* or *perfect* interval by lowering the higher tone a chromatic semitone. For instance, *c′*-*d′*♭ being a minor second, *c′*-*d′*♭♭ would be the diminished second formed above (in practice this interval is also called an "enharmonic step"); *c′*-*f′* is the perfect fourth formed above, and *c′*-*f′*♭ the diminished fourth:

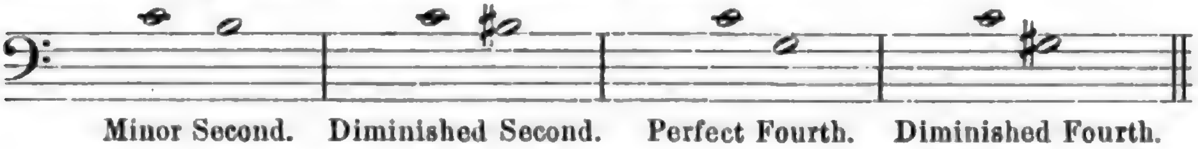
Minor Second. Diminished Second. Perfect Fourth. Diminished Fourth.



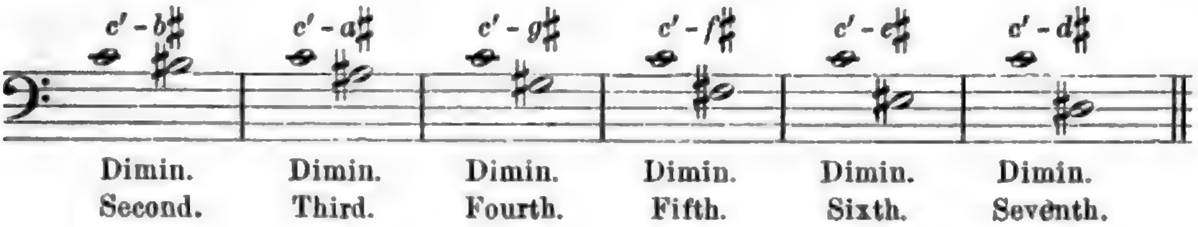
The following intervals are therefore to be considered as diminished intervals formed above:



f) Diminished intervals formed below. Each of these intervals is formed from either a *minor* or *perfect* interval formed below, by raising the lower tone by a chromatic semitone. For example, *c'–b* being a minor second formed below, *c'–b*♯ would be the diminished second (also called in practice, an "enharmonic step"); *c'–g* being the perfect fourth formed below, *c'–g*♯ would be the diminished fourth:

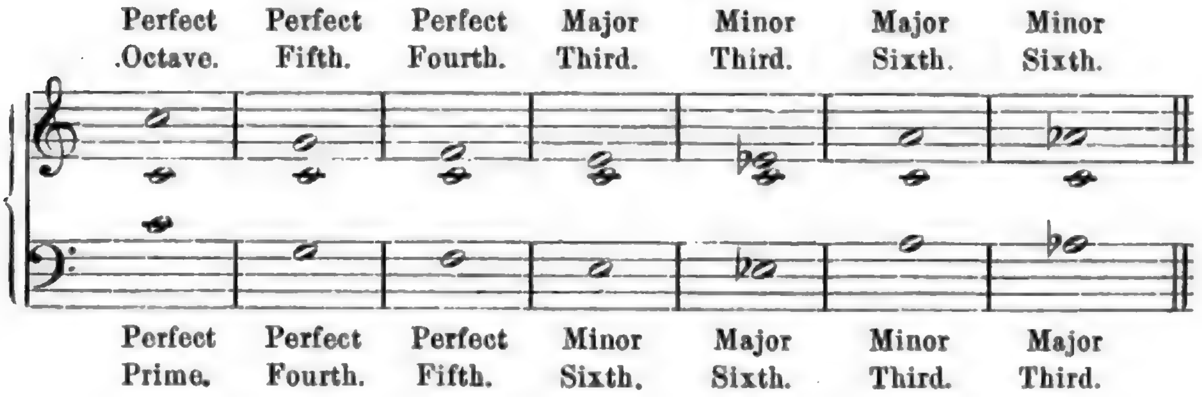


The following would consequently be the diminished intervals formed below:



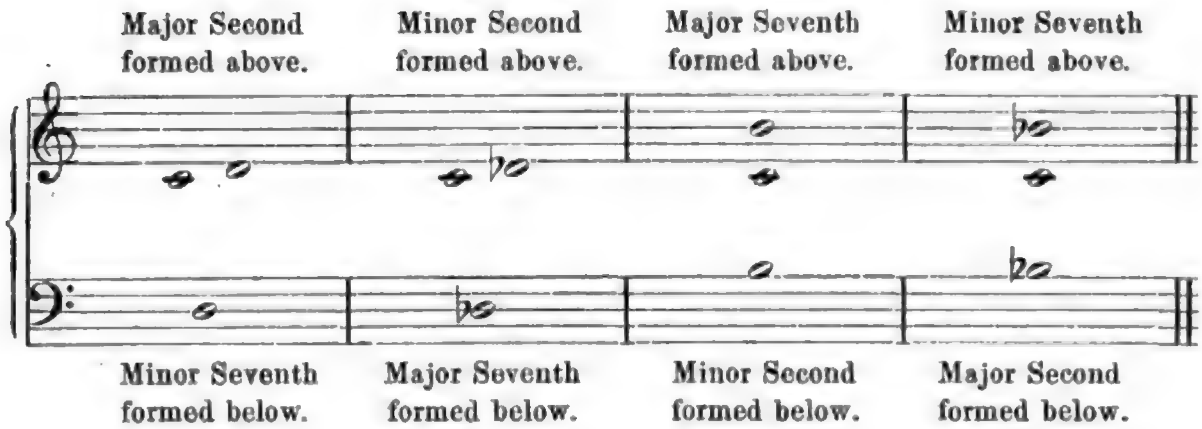
The pupil will notice, on examining these tables, that the diminished intervals formed above are the inversions of the augmented intervals formed below; and that the diminished intervals formed below are the inversions of the augmented intervals formed above. The following tables of all intervals and their inversions will facilitate the comprehension of the foregoing rules.

I. Consonances.

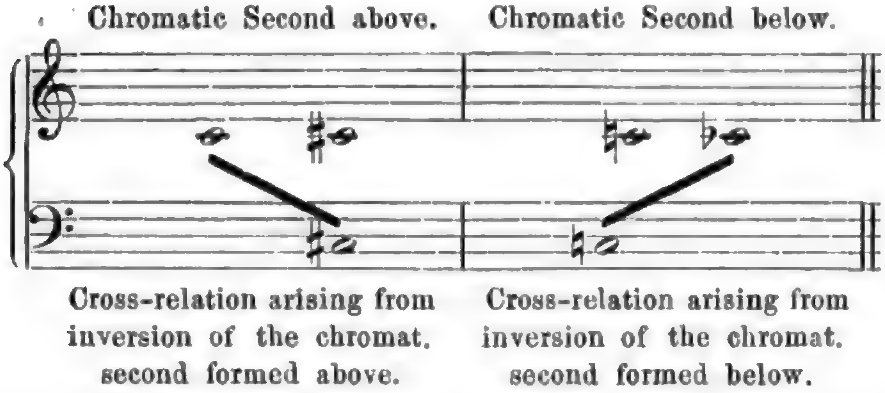


II. Dissonances.

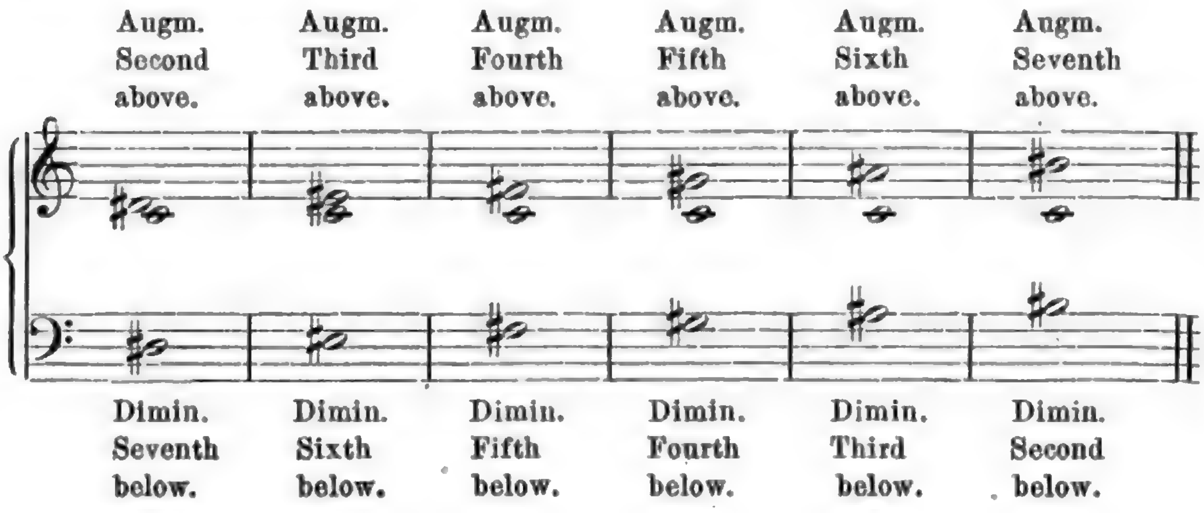
1. *Diatonic Dissonances.*

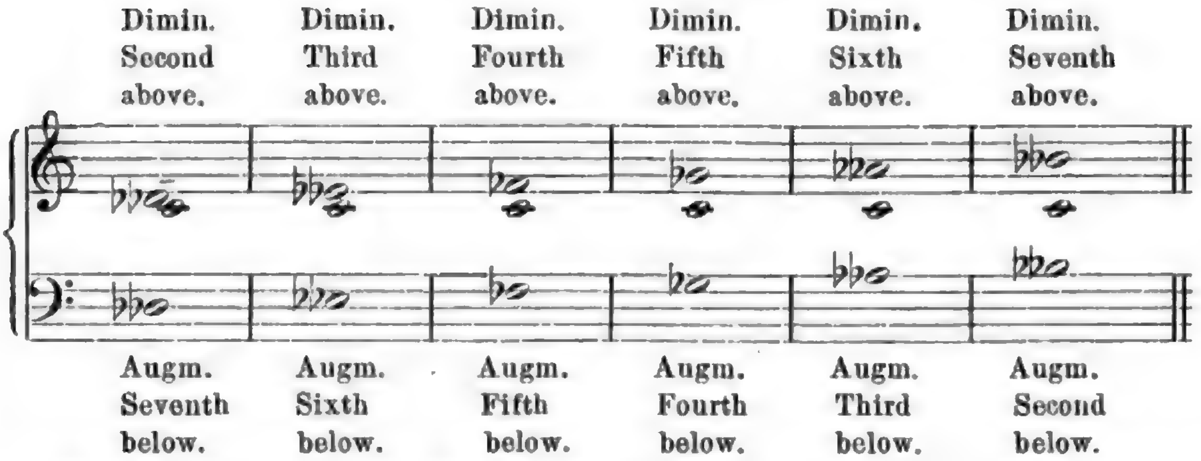

Major Second formed above.
Minor Second formed above.
Major Seventh formed above.
Minor Seventh formed above.
Minor Seventh formed below.
Major Seventh formed below.
Minor Second formed below.
Major Second formed below.

2. *Chromatic Dissonances.*


Chromatic Second above.
Chromatic Second below.
Cross-relation arising from inversion of the chromat. second formed above.
Cross-relation arising from inversion of the chromat. second formed below.

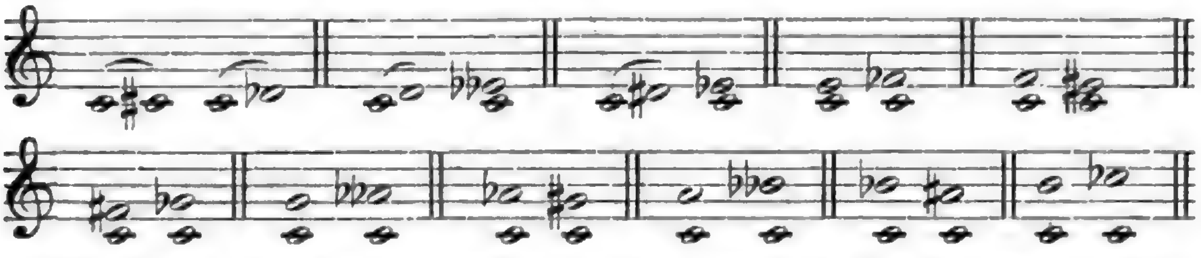
In examining the structure of these intervals, the relation subsisting between the tones constituting the chromatic seconds formed above and below, and their inversions, *i. e.* the cross-relation [also named the *inharmonic* or *false* relation], which latter is proscribed in strict composition, should be particularly observed.





The task now before the pupil consists in taking each note [in the chromatic scale] as basis, and naming the intervals to be formed upon it, without using music-paper or the piano. The employment of the latter would often only serve to confuse him, because of the "equal temperament" [see Note IV] of this instrument, in consequence of which such tones as *c′*♯ and *d′*♭, *c′* and *b*♯, *c′* and *d′*♭♭ etc. are sounded by one and the same key, and consequently have the same pitch; the ability to *think* in accordance with the system of pure harmony, which is, for musical orthography, the only correct and really significant one, will be seriously impaired, if the learner has not previously acquired practice in the formation and employment of the various intervals.

On the piano and organ, the tuning of which is "tempered" in consequence of their mechanism as keyed instruments, the exact pitch of the intervals cannot be ascertained, each key being used to produce tones dissimilar in their pitch and harmonic relations as well as in name. For instance, the tone *c′* is, on the piano and organ, the same as *b*♯ or *d*♭♭; the chromatic second *c′*-*c′*♯ the same as the diatonic minor second *c′*-*d′*♭; the augmented second *c′*-*d′*♯ the same as the minor third *c′*-*e′*♭ etc. The learner will therefore do well to conceive the musical intervals as sung rather than as played. In the "equal temperament" of the piano and organ the following intervals are, for example, alike:



which shows that musical orthography cannot be based upon the keyboard of the piano or organ. Correct musical thinking can be done only in accordance with the rules of pure harmony; the piano may be used occasionally when the learner finds it necessary to assist his thought by means of audible tones; in such cases the true harmonic relations should of course be carefully kept in view.

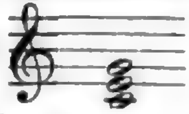
It may be mentioned here, that perfect octaves, fifths and fourths were formerly classed as perfect consonances, and thirds and sixths as imperfect consonances. This classification, which properly belongs under the head of "acoustics", is not applicable to musical practice, and stands in need of correction in any case [compare Notes IV and IX].

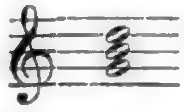
CHAPTER III.

Keys and Chords.

Every tone receives an intelligible designation and [harmonic] significance in the key. The key may be exhibited not only through the scale (*i. e.* the succession of notes which, arranged in a series of steps, are to be regarded as the separate elements of the key in their relation to the tonic), but also by means of letters, for the purpose of showing the relation of the chords one to another. As each individual tone obtains musical significance only through its connection with the key, in like manner every chord formed according to acoustic principles must be named with reference to its position in some key [see Note V]. Before proceeding to the system of alphabetical notation it is necessary to state, as an axiom, that each chord has an individual character, which, however, is made fully apparent only through its relation to other chords. This relation is based upon the

Interconnection of the Chords

an understanding of which is most readily obtained by using the system of alphabetical notation introduced by Moritz Hauptmann. According to this system, if the key of *C*-major be taken as illustration [of the *major mode* *], the tonic [or key-note] is represented by a capital *C*. The fifth, the opposite [see Note VI] of the tonic, is also represented by a capital letter, in this case *G*. The third, or mediant, is represented by a small *e*. These three notes constitute the major triad  [triad = common chord] which would be given, according to Hauptmann's formula, as *C-e-G*. The major triad is formed by a combination of two intervals, in the above example *C-e* and *e-G*; the lower, *C-e*, is a major third, and the higher, *e-G*, a minor third.

In the same way another major triad can be formed if *G* be taken as basis  or as expressed in letters *G-b-D*. As regards the relation

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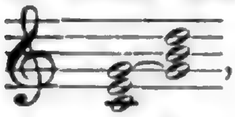
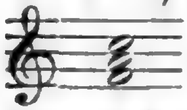
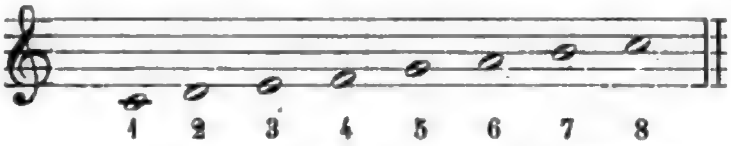
*) The term "mode" is synonymous with "model key", or key in the abstract. *Tr.*

in which these two chords *C-e-G* and *G-b-D* stand to each other , *C-e-G* is named the chord of the tonic, the foundation-chord of the key of *C*-major; *G-b-D* is the chord of the dominant, governing or determining the character of the same key above the chord of the tonic, and standing in opposition to that chord. The triad *F-a-C*  also stands in opposition to *C-e-G*, and is called the chord of the *sub*-dominant, determining the character of the key below the tonic, the note *F* being the fifth below the tonic *C*. These three chords *C-e-G*, *G-b-D* and *F-a-C* form the foundation of the key of *C*-major, which is represented in letters as follows: *F-a-C-e-G-b-D*, or as represented in notes:



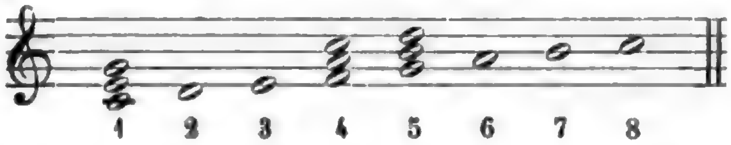
To ensure an understanding of the relations of the chords, the three

Fundamental Triads

C-e-G, *G-b-D* and *F-a-C* are marked by curves above the letters:

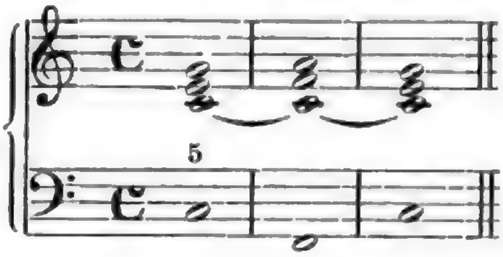
$$\overset{\frown}{F\text{-}a\text{-}C}\overset{\frown}{\text{-}e\text{-}G}\overset{\frown}{\text{-}b\text{-}D}.$$

These fundamental triads are based upon the first, fourth and fifth degrees of the scale, as shown below:



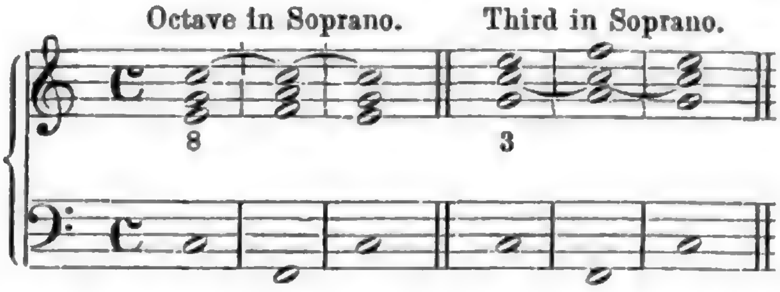
On examining the connecting links between the chords, *i. e.* such tones as are common to any two of them, it is evident that *C* is the connecting link between the chords *F-a-C* and *C-e-G*. The tone *C* being the fifth above *F* and the fifth below *G*, it is clear that the relation of the above-named chords to each other is dependent upon this position of *C*. These chords are consequently designated as "chords connected through the fifth". The two triads *C-e-G* and *G-b-D* are likewise chords connected through the fifth, the connecting note *G*, which is common to both, being the fifth above *C* and the fifth below *D*. The tonic triad *C-e-G* is therefore connected through the fifth with both the sub-dominant triad *F-a-C* and the dominant triad *G-b-D*. Taking an example in four-part harmony, which is chosen in order to obtain, through the bass, a firm foundation for the musical progressions, we perceive that the progression from one triad to another is accomplished by the aid of their connection through the fifth, the connecting tones giving harmonic stability to the progressions.

We first set the notes *C, F, C* as bass (or harmonic basis) for a progression of triads in which the fundamental chords formed on the above-named notes are to be employed, and write the example in close position:



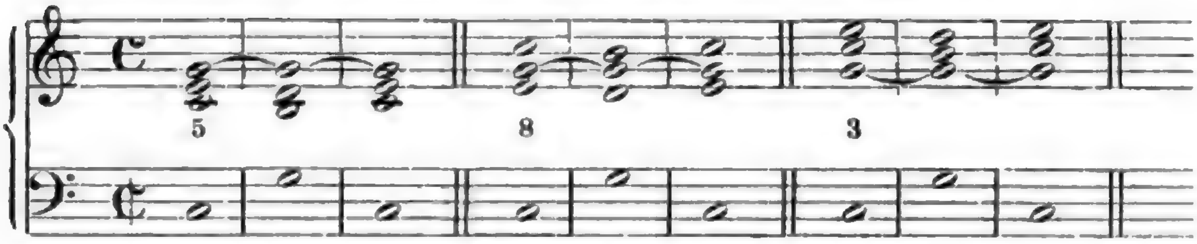
In this example the notes in close position are arranged in such a manner that *G*, the highest note of the first chord, which is the tonic triad *C–e–G*, stands to the bass-note *C* in the relation of the fifth. This relation is not altered by the fact that the note *C* (the so-called little *c* if named according to absolute pitch) lies an octave lower than the *C* (= *c′*) which appears in the third part. (The parts are here numbered according to the usual method, the highest being called the first, the next one below the second etc.) The position of the first triad, with the fifth in the first part [soprano] is marked at the beginning of an example in harmony, by a 5. The progression from this chord to the triad *F–a–C*, with which it is connected through the fifth, is accomplished by the movement of the notes *e* and *G* to *F* and *a* respectively, *C* as connecting link remaining stationary; in the retrogression from *F–a–C* to *C–e–G* the notes *F* and *a* move to *e* and *G*, *C* remaining as before stationary. The contrary motion observable in this example between the bass and the two highest parts is considered preferable, inasmuch as it brings the difference in character between the two forms of chords into sharper contrast.

The following examples show the progressions as they appear when the first triad takes the two remaining forms of close position:

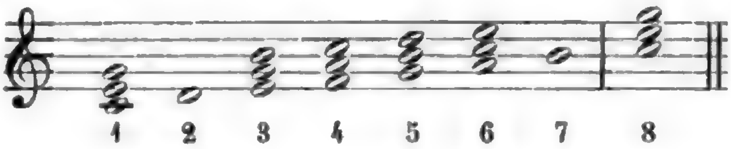


The figures 3, 5 and 8 indicate that the third, fifth or octave is to be taken in the soprano.

The progression from the tonic triad to the dominant triad, and *vice versa*, is to be effected in a similar manner, it being necessary to pay special attention to the connecting link *G* only. The examples here given in the three forms of close position will suffice to illustrate this connection:



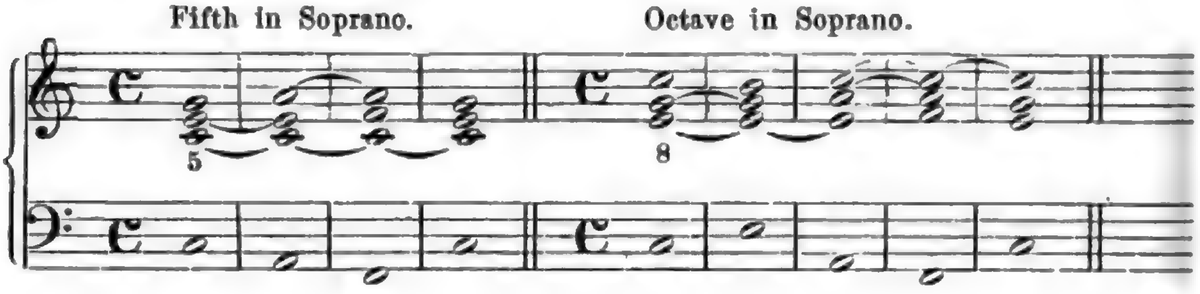
Proceeding with the analysis of the formula *F-a-C-e-G-b-D*, in which the fundamental triads are marked by curves, we find that another triad, of different character, lies between the triads of the tonic and sub-dominant, *i. e.* on the sub-dominant side of the formula; another triad of different character is also to be found between the triads of the tonic and dominant, *i. e.* on the dominant side of the formula. We mark these two triads *a-C-e* and *e-G-b* by curves under the letters; it must be borne in mind that these triads, while resembling each other, are quite different in character from the three fundamental triads. In respect to these latter it has already been shown that they have, through the position of the thirds, the major third being below and the minor third above, a major character; in the subordinate triads *a-C-e* and *e-G-b*, in which the position of the thirds is reversed, the minor third being below and the major third above, we find the opposite of the major character, which is the minor. These two minor triads are based upon the sixth and third degrees of the scale:

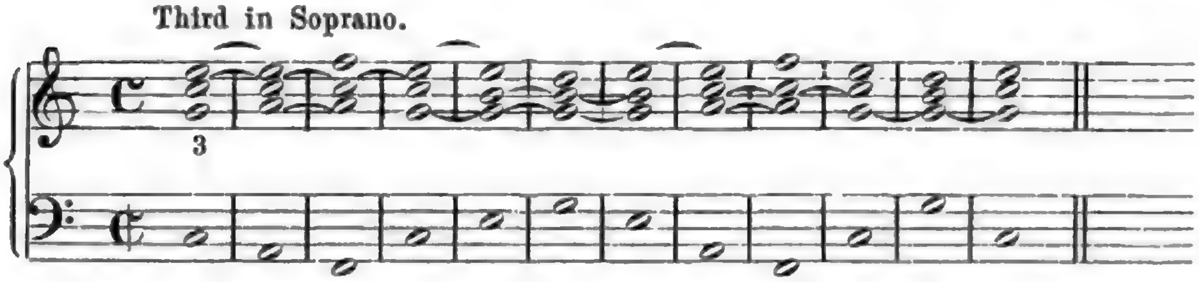


The triads of the second and seventh degrees still remain to be noticed. By means of our alphabetical formula the organic connection of these two triads with the fundamental triads can be shown, after pointing out the various connecting links to be found between the several fundamental and subordinate triads in the formula

F-a-C-e-G-b-D.

For instance, in *a-C-e* and *C-e-G* the tones *C-e* are common to both. These two tones form the interval of a major third; these triads are therefore to be regarded as "connected through the third". The triads *e-G-b* and *C-e-G* are also considered as "connected through the third", the tones *e-G*, which are common to both, constituting a minor third. After apprehending the principle of this relation it is easy to perceive that the triads *a-C-e* and *F-a-C* are connected through the minor third *a-C*, and also the triads *e-G-B* and *G-b-D* through the major third *G-b*. Progressions between triads thus connected are accomplished in the same manner as between triads connected through the fifth; except that in this latter case we had but one tone as connecting link, whereas between triads connected through the third two such tones are to be found, as is shown in the following examples:

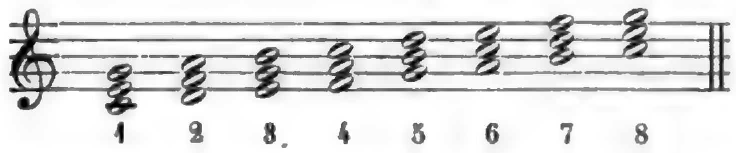




Triads having one tone in common, as marked by one ligature, are connected through the fifth; those having two tones in common, as marked by two ligatures, are connected through the third.

As remarked above, triads are still lacking on the second and seventh degrees of the scale; they may be added to the formula for *C*-major, *F-a-C-e-G-b-D*, in the following manner. The formula is enclosed by a line drawn at either limit */F-a-C-e-G-b-D/*: the fifth of the dominant triad, or *D*, is written next the fundamental note of the sub-dominant triad, which is *F*; and the fundamental note of the sub-dominant triad, *F*, is written beside the fifth of the dominant triad, *D*. The formula now appears thus:

D/F-a-C-e-G-b-D/F

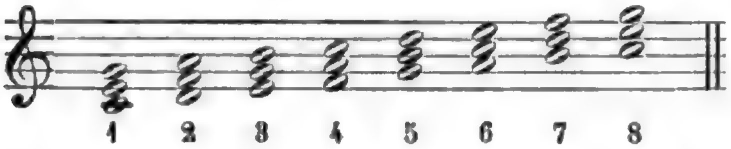


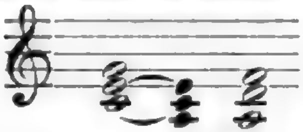
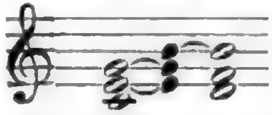
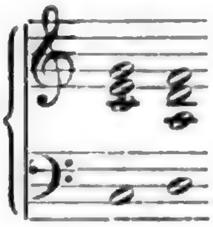
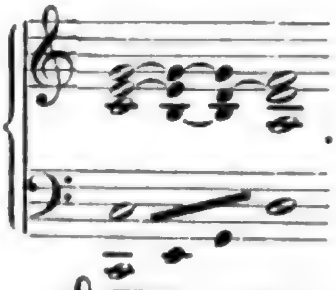
The two new triads obtained by this procedure, *D/F-a* and *b-D/F*, are based upon the second and seventh degrees of the scale. The chord upon the second degree sounds, on the piano or organ, *i. e.* in the tempered system of tuning, like a true minor triad. In pure harmony however this chord on the second degree of the major scale is by no means a true minor triad, for the reason that the interval *D-a* is not a perfect fifth, but is, as proved by mathematical analysis, a somewhat lesser interval. For this reason it would be well to designate this triad as a diminished triad of minor character; because, although in the system of "equal temperament" now generally employed it usually makes the same impression on the ear as a minor triad, its fifth appears in pure harmony, when compared with a perfect fifth, as diminished. The triad based upon the seventh degree of the scale is on the contrary immediately recognized as a diminished triad not only in pure but also in tempered harmony, its fifth being a decidedly lesser interval than the perfect fifth. [See Notes III and VII].

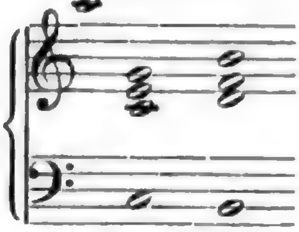
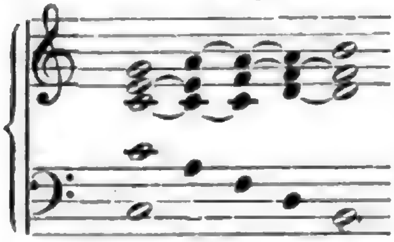
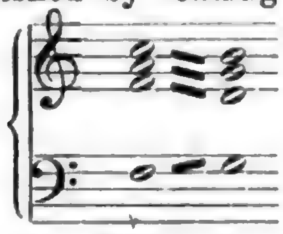
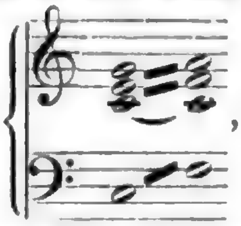
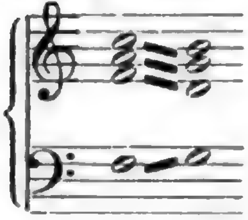
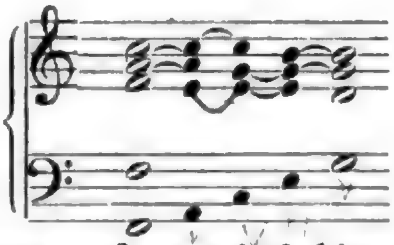
In *practice* the two intervals *b-D* and *D/F*, which, when united, constitute the fifth which forms the boundary of the key, as represented in the alphabetical formula, are treated as minor thirds.

The entire key has therefore the following triads:

D/F-a-C-e-G-b-D/F

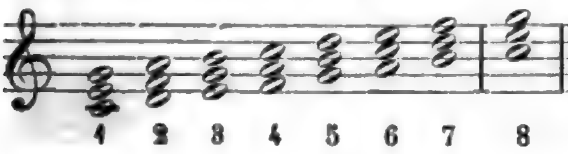


the fundamental triads are indicated by curves above the letters, and the subordinate triads by curves below. We observe, on inspecting the chords of the key of *C*-major, that neither the triad *D/F-a* nor the triad *b-D/F* is directly connected with the tonic triad *C-e-G*, they having no tone in common with the latter. A connection can be effected, in either case, through the aid of two other triads. These are, on the sub-dominant side: (1) *a-C-e*, which is connected with *C-e-G* through the third, and with *D/F-a* through the fifth; (2) *F-a-C*, which is connected with *C-e-G* through the fifth and with *D/F-a* through the third; on the dominant side: (1) *e-G-b*, which is connected with *C-e-G* through the third and with *b-D/F* through the fifth; (2) *G-b-D*, which is connected with *C-e-G* through the fifth and with *b-D/F* through the third. Hauptmann attempts, in view of these indirect connections, to explain the progression from *C-e-G* to *D/F-a* by interpolating, and to a certain extent using as a substitute for the former, the triad *a-C-e*: ; and by substituting in the progression form *C-e-G* to *b-D/F* the triad *e-G-b* for *C-e-G*: . The desired end is far more easily attained if we regard *D/F-a* and *b-D/F* as compound triads, and attempt to find the connecting harmonies between these compound triads and *C-e-G* (taking the latter as starting-point), by first gaining, within the boundaries of the enclosed formula */F-a-C-e-G-b-D/* a connection with *D* and *F*, and then finishing the progression. With the addition of a bass the correctness of the progression  would therefore be proved by interpolating following connecting harmonies: .

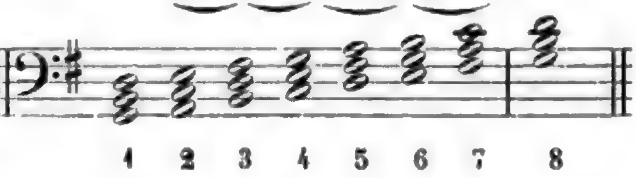
In the same way the correctness of the progression  can be shown by the employment of intermediate harmonies as follows: . It will be shown further on, in treating of the inversions of triads, for what reasons the triad *b-D/F* is not employed, as a rule, in the fundamental position in four-part harmony. In progressions between indirectly connected chords the beginner should observe the following practical rule, — that the bass is to be led in contrary motion to the other parts. For example, the sub-dominant triad *F-a-C* is not directly connected with the dominant triad *G-b-D*, these triads having no tone in common; the progression from one to the other is accomplished by leading the bass in contrary motion to the other parts, thus: . The meaning of the term "contrary motion" is made clear by the example itself, and is illustrated by dashes, the three in the upper staff having a different direction from the one in the lower staff. *Parallel* motion is shown by the following example: , in which the two dashes in the treble clef point in the same direction as the one in the bass clef. The direction in which the parts are led is clearly shown by the dashes. The correctness of the progression:  may also be proved by the interpolation of connecting harmonies, as here given:  .

Through these explanations and practical hints the learner will be enabled to write exercises having as their object harmonic progressions between the triads exhibited in the alphabetical formula. In order to become familiar with the structure of all the keys it is necessary that the pupil should learn them and write them out in letters and notes, and so impress them on the memory as to be able to answer correctly all questions on the position and interconnection of the triads in the major keys, as shown in the formulas. Following examples comprise all the major keys.

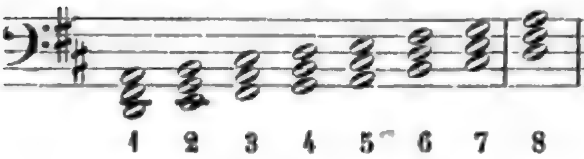
D/F-a-C-e-G-b-D/F



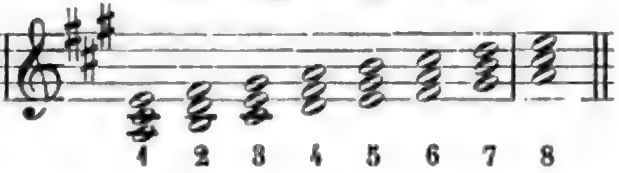
A/C-e-G-b-D-f♯-A/C



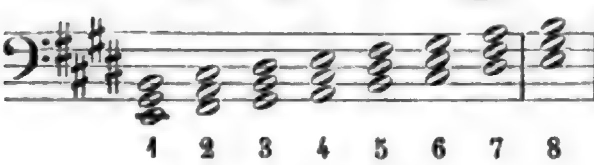
E/G-b-D-f♯-A-c♯-E/G



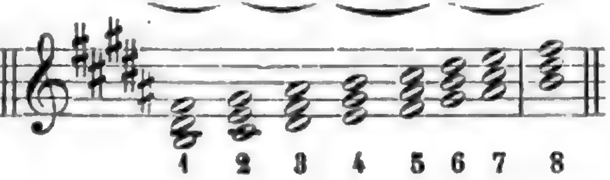
B/D-f♯-A-c♯-E-g♯-B/D



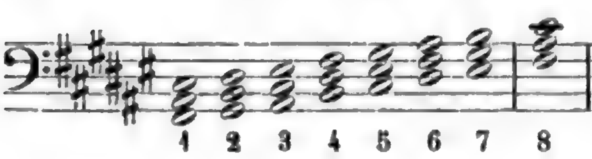
F♯/A-c♯-E-g♯-B-d♯-F♯/A



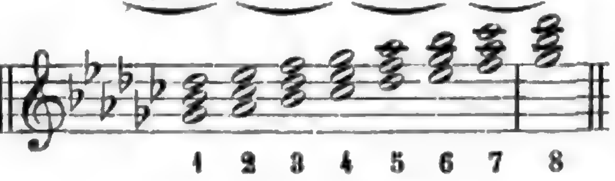
C♯/E-g♯-B-d♯-F♯-a♯-C♯/E

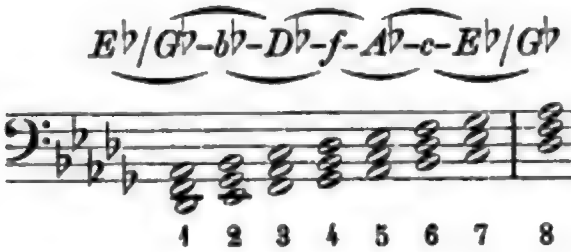


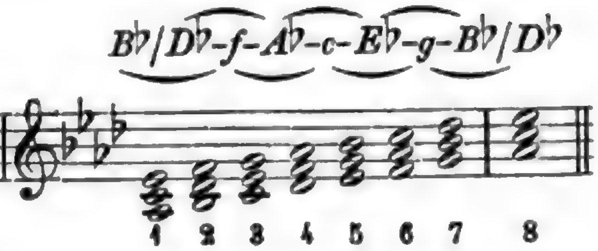
G♯/B-d♯-F♯-a♯-C♯-e♯-G♯/B

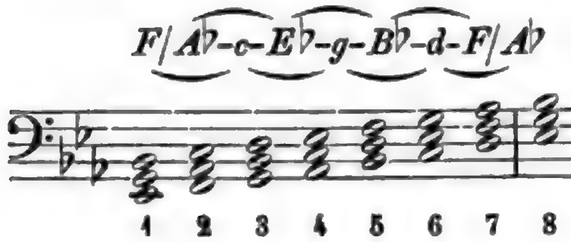


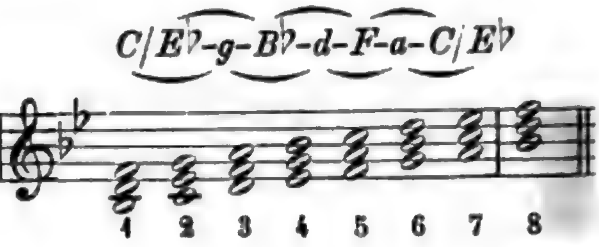
A♭/C♭-e♭-G♭-b♭-D♭-f-A♭/C♭

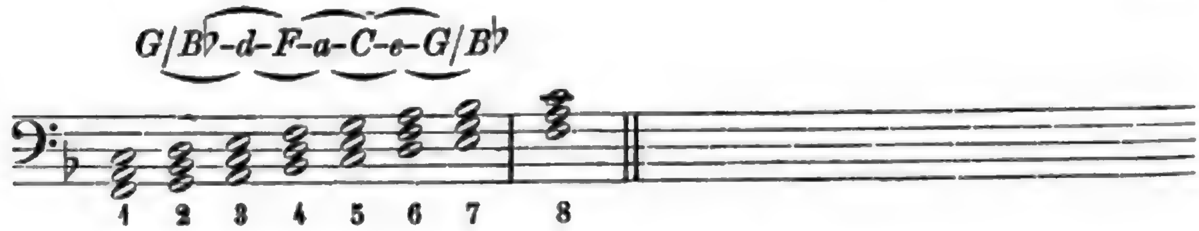




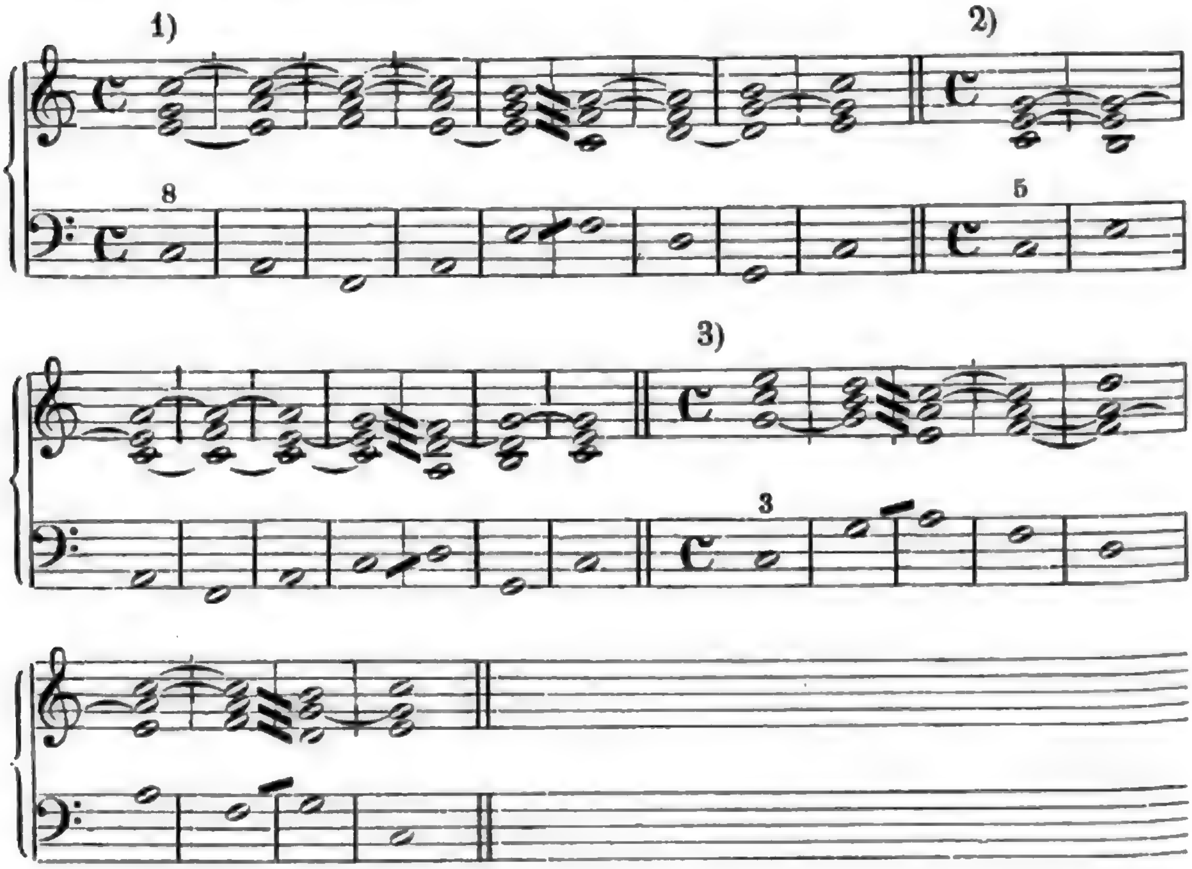








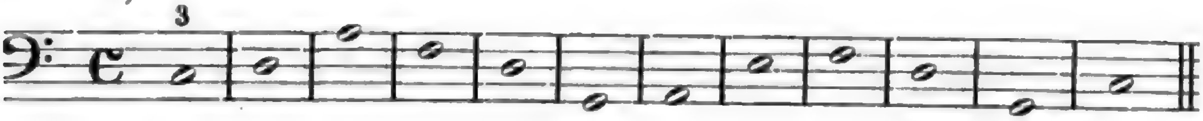
The three examples following, in which the opening chord is given in each of the positions mentioned above, show every species of connection between the triads.



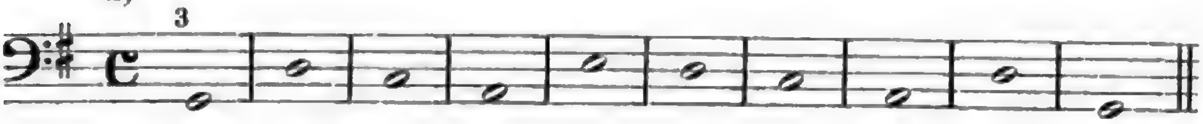
The learner should now write the three higher parts to the basses given below.

Exercises.

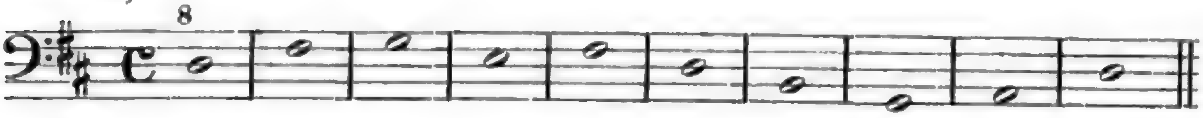
1)



2)



3)



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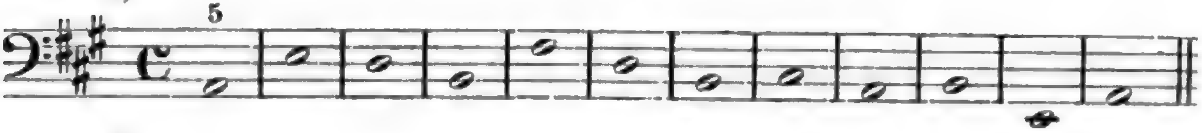
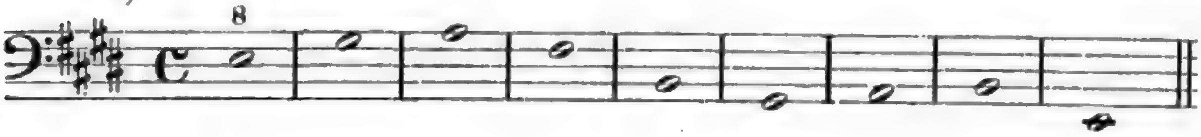
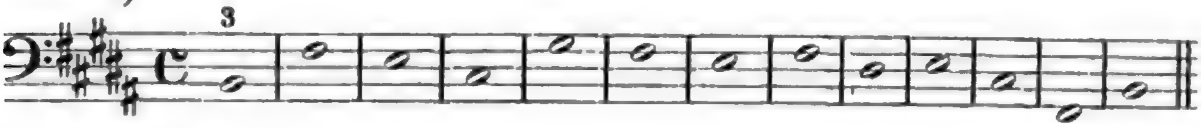


5)



6)



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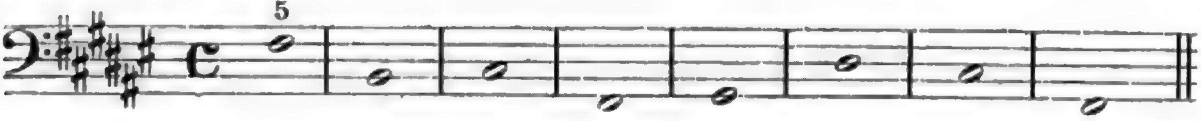

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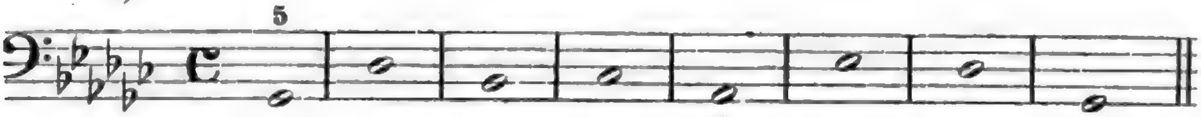

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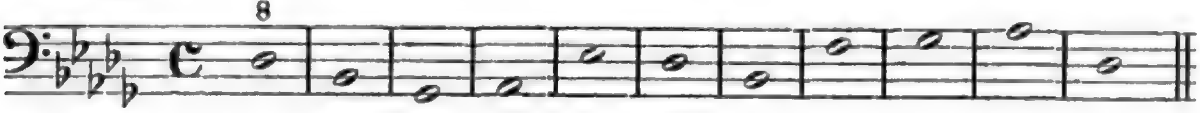

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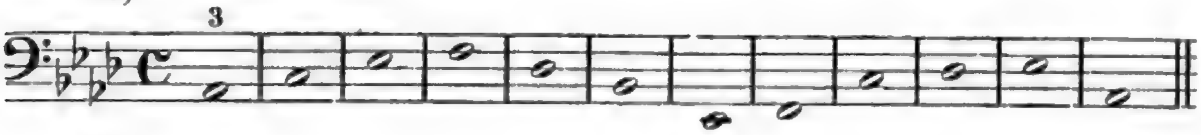
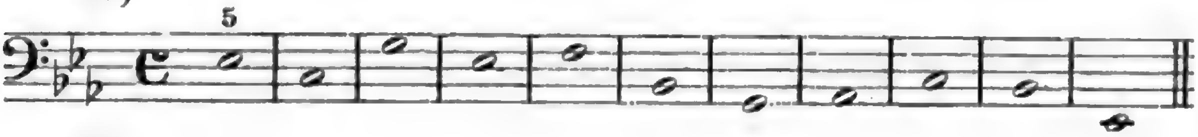
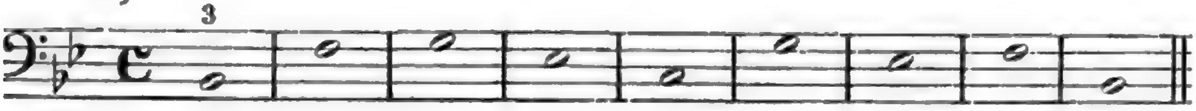


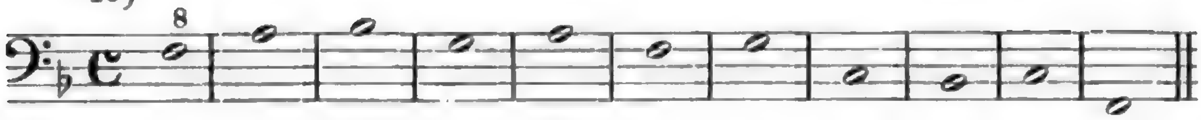
11)



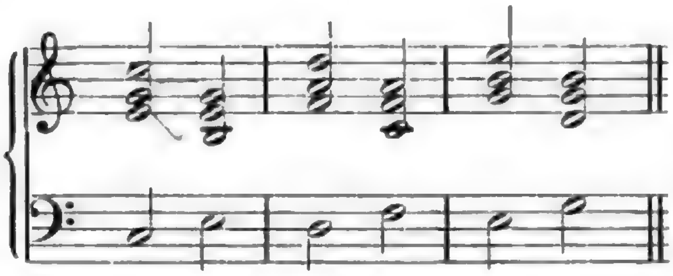
12)



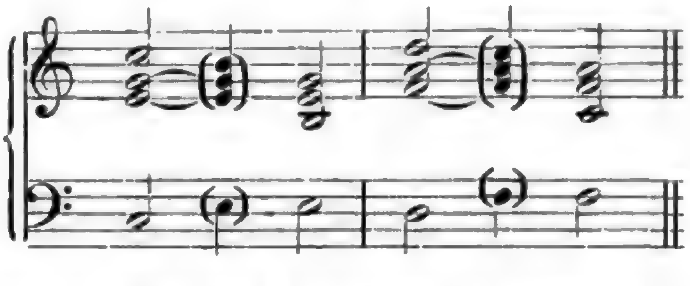
13)



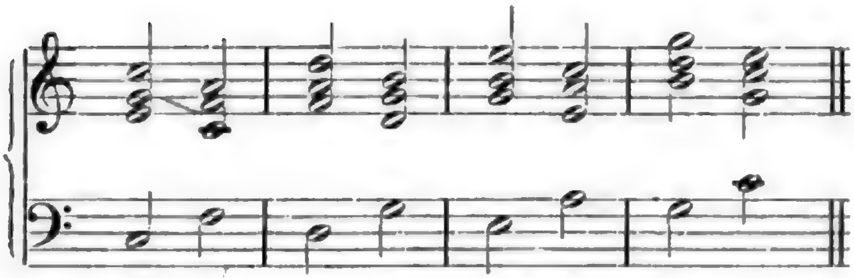
Chords connected through the fifth, between which, by reason of this characteristic relation, a certain antagonism is apparent, are allowed greater freedom of motion, even in strict composition, than chords connected through the third. Between these latter progressions in *contrary motion*, in the manner peculiar to the progressions between indirectly connected chords, are not permitted in strict composition, on account of their close affinity; such progressions in contrary motion are, on the other hand, allowed between chords connected through the fifth. Chords connected through the third can therefore not progress as in the following examples:



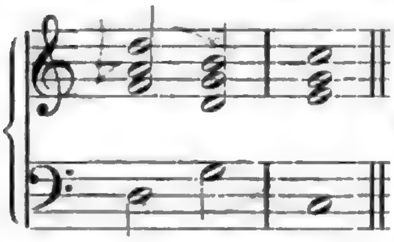
Such progressions would serve to show a lack of understanding for the affinity of the chords, although no mistake in the leading of the parts can be pointed out. These progressions are consequently permissible only in free composition, in which the intimate connection necessary in strict composition is not required. In this case the progressions may be explained by supposing that an intermediate or connecting chord (here given in quarter-notes) has been left out:



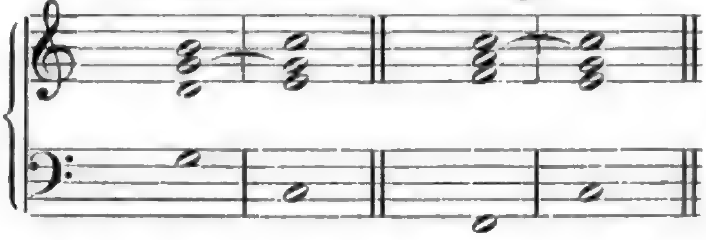
In these examples the affinity of the chords is shown to be felt, although, the intermediate chord being left out, it is not expressed. The progression from the intermediate chord to the one following consists simply in a change of the relative positions of the three parts above the bass; this progression being thus shown to be in a manner derived from a progression based upon the rules of strict composition, is consequently permissible in free composition. Between chords connected through the fifth, on the other hand, such progressions as are given below are allowed in strict composition:



Here we feel that these progressions are correct, and it can be proved, by the use of intermediate chords, that our internal consciousness corresponds to the nature of the case. Through these interpolations, as given further on, the melodic connection is shown to be formed by the soprano; we recognize the correctness of this progression from the fact, that the intermediate chords are connected with the first ones through the third, the progression being therefore accomplished under the rule for progressions between indirectly connected triads. For this reason the phrase:

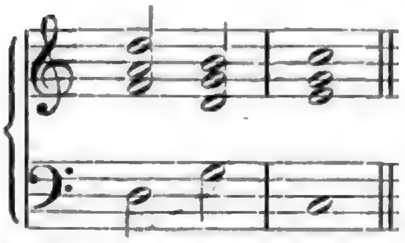


so often used as cadence or close, can be regarded only with approval. A close formed by triads is called an *authentic close* when the dominant triad precedes the tonic triad; and a *plagal close* [see Note VIII] when the sub-dominant triad precedes the tonic triad; *e. g.*

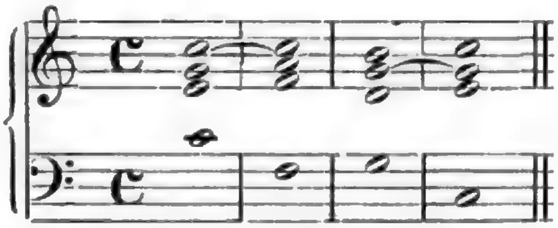


1. Authentic close. 2. Plagal close.

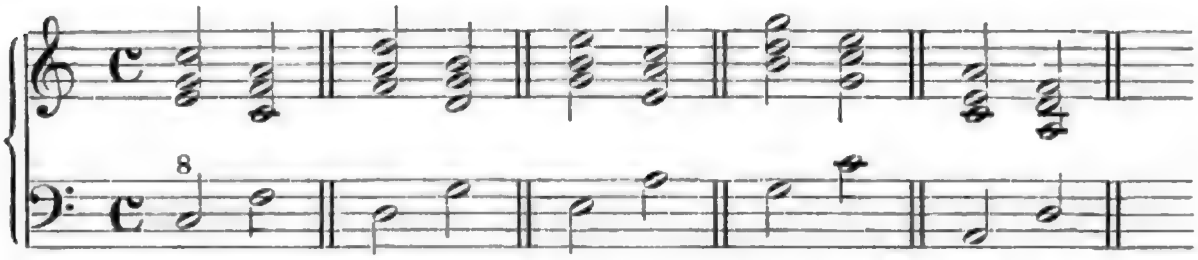
The phrase



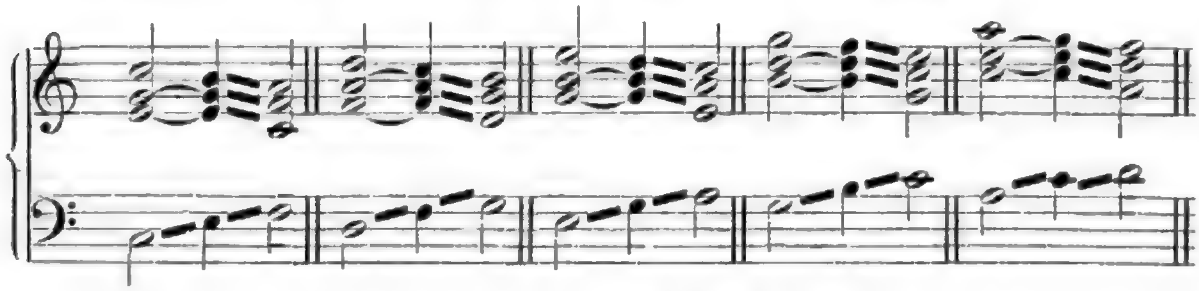
in which the [triads of the] second, fifth and first degrees occur in succession, is quite as correct as the two closes above-mentioned, and is, in fact, nearly as complete as the *perfect close,* which contains the chords of both dominant and sub-dominant:

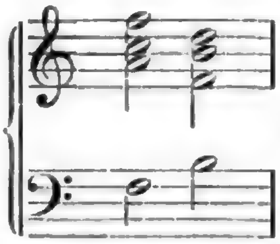


because two tones characteristic of the sub-dominant, namely the fundamental and the third, also appear in the triad *D/F-a*. Other chords connected through the fifth may progress in the same way, as the following examples show:

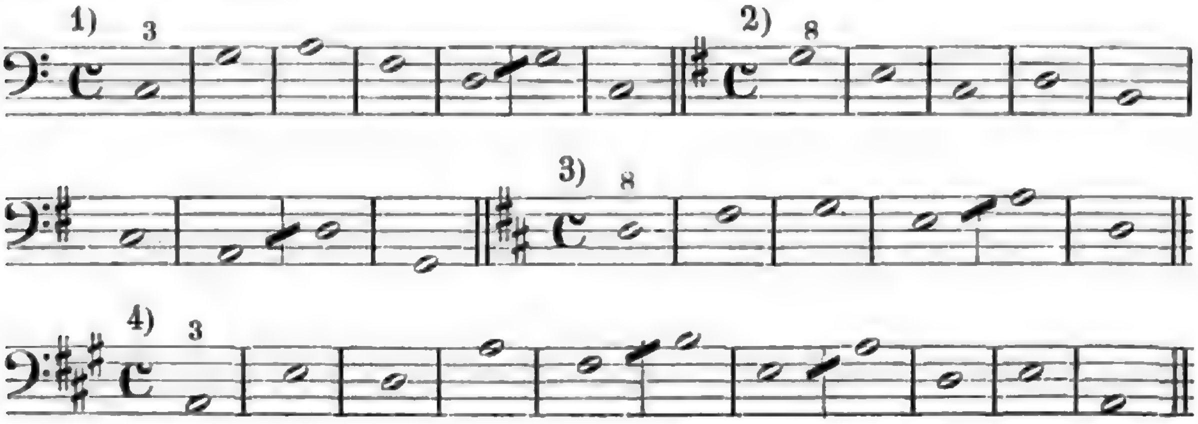


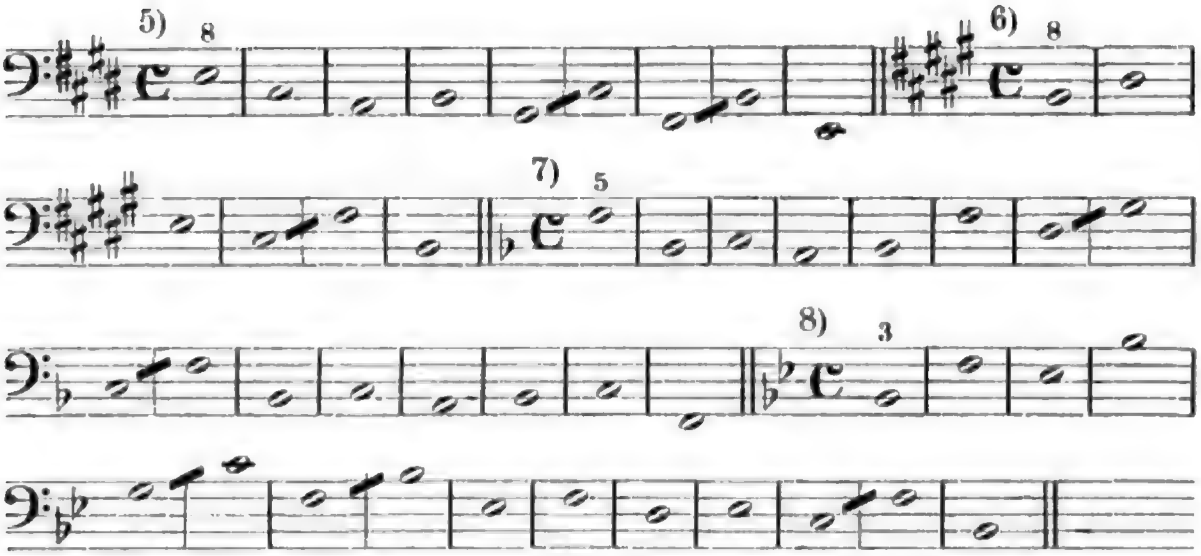
The proof of the correctness of such progressions is here given:



In the above examples the progression  is left out, the melodic progression *F-b* in the bass not being considered admissible in vocal music. This interval is called a *tritone*, being formed of three whole tones or major seconds; the two intermediate tones *G* and *a* should not be skipped in good vocal composition. By interpolating the chord *a-C-e* the tritone disappears. The above rule for the tritone, or augmented fourth, is applicable to all augmented intervals, the use of such intervals in melodic progression not being permitted in strict composition.

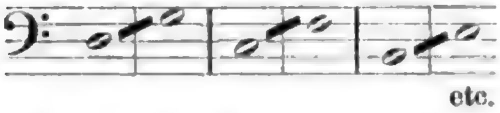
In the following exercises the pupil will have an opportunity of employing such progressions between chords connected through the fifth as were shown above.

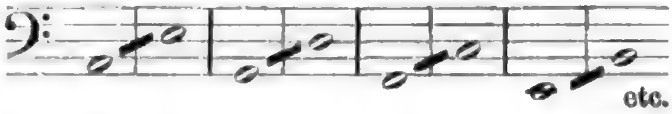




The bass-notes to such chords as should be connected by a progression in contrary motion are connected by dashes.

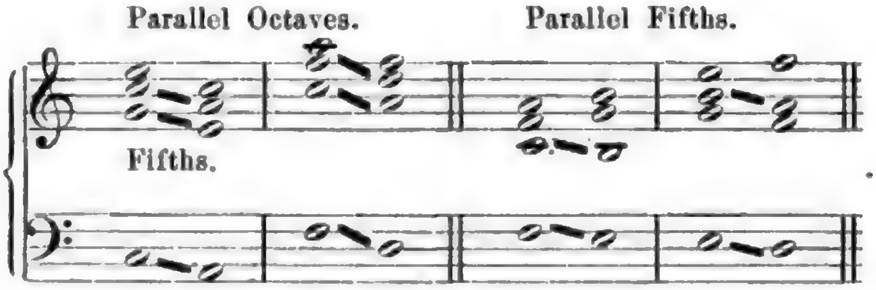
These exercises will give the learner sufficient practice in writing such progressions. His attention may here be called to the *sequences*, *i. e.* series of similar melodic progressions, as in fourths or fifths:





The sequences of fourths are well adapted to form a basis for harmonic progressions in contrary motion; where sequences of fifths occur the principle of connecting the harmonies through the fifth should be adhered to. The principle of indirect connection is in no case applicable to chords connected through the third; such progressions would make the composition appear clumsy, as was remarked above.

In all progressions already exhibited it is evident, that no two parts can move in *parallel octaves* or *parallel fifths*.

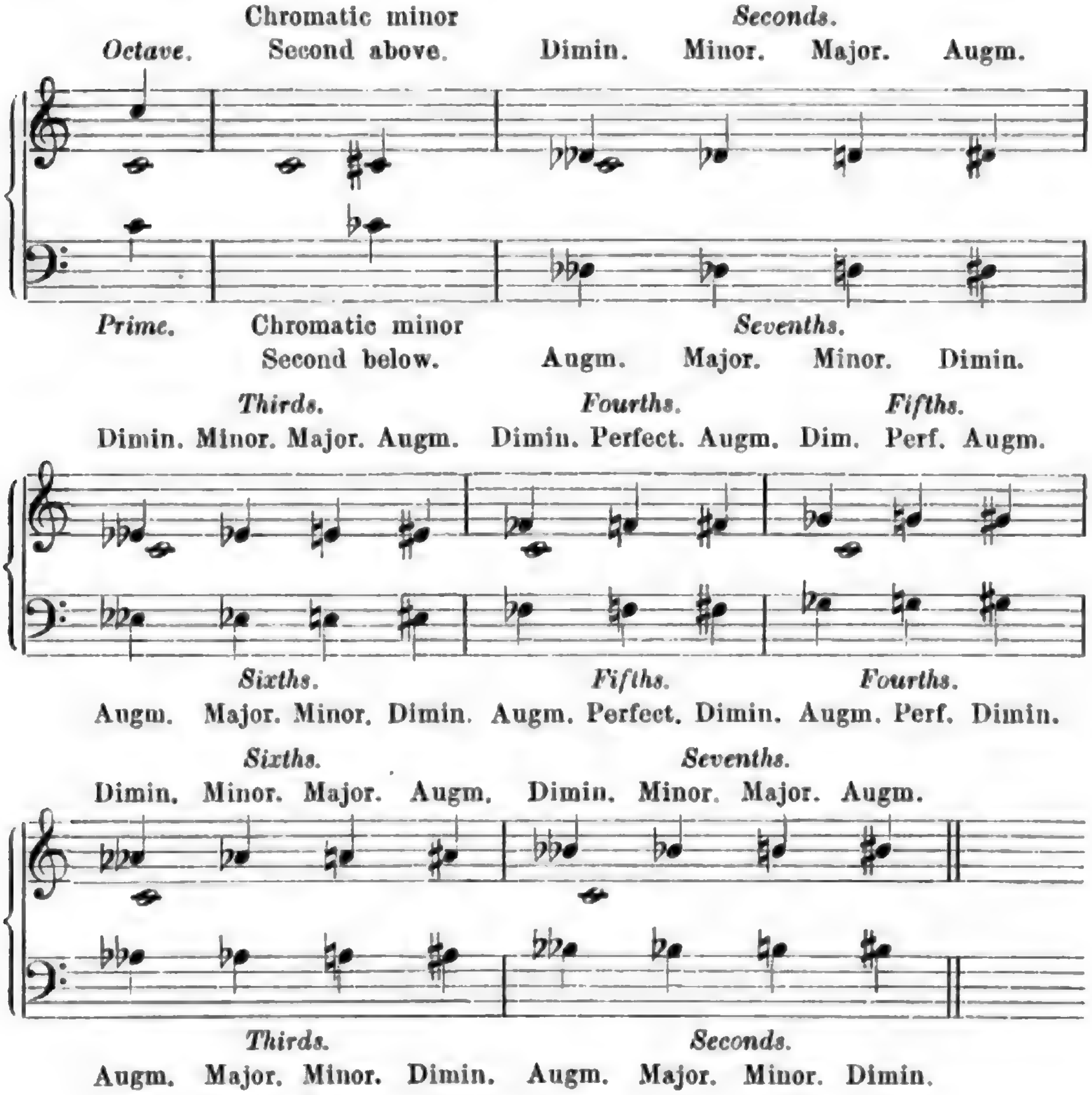


The impossibility of such motion lies in the organic interconnection of the chords; thus the cardinal precept in the manuals of harmony, to "avoid parallel octaves and fifths", is made to appear superfluous. An unpracticed eye will nevertheless be often deceived when parallel fifths and octaves occur between the middle parts, or between one middle and one outer part.

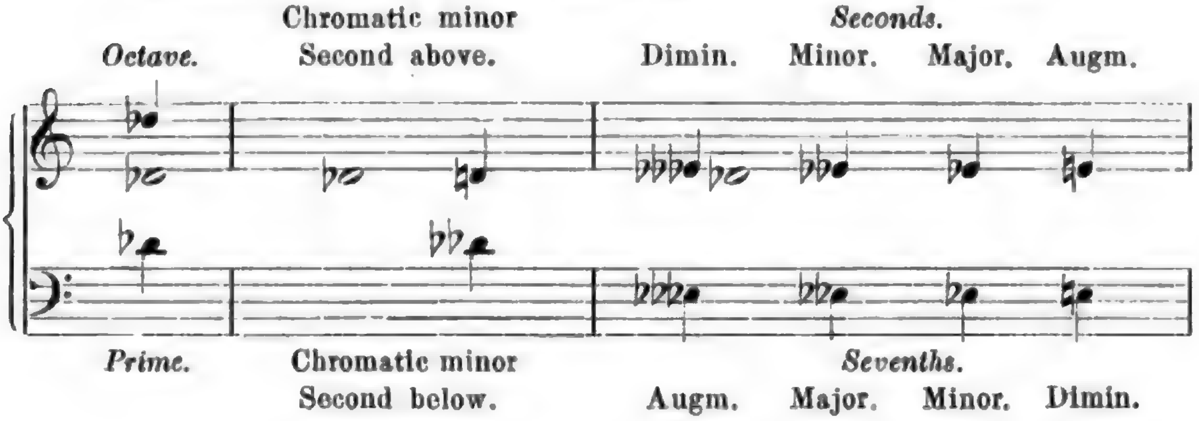
The pupil should now review the following principal points:

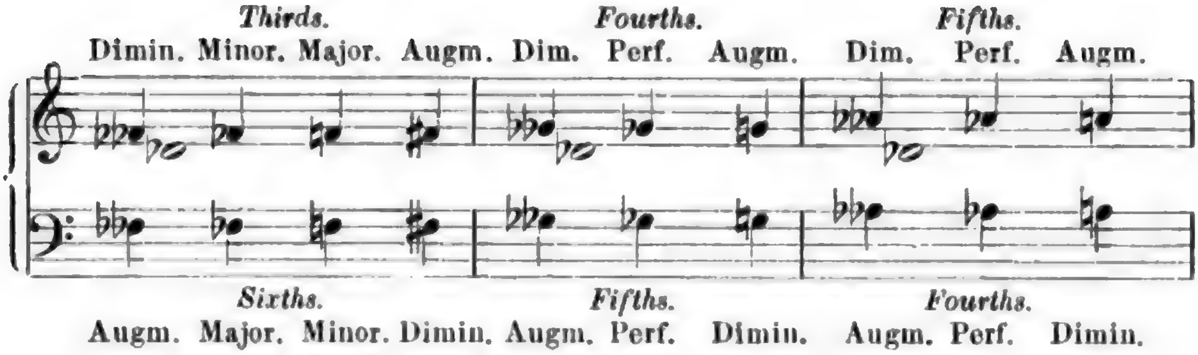
1. Write all the intervals, taking any given note as basis, and making sure that the explanations on the subject of consonances and dissonances have been thoroughly understood. *E. g.*:

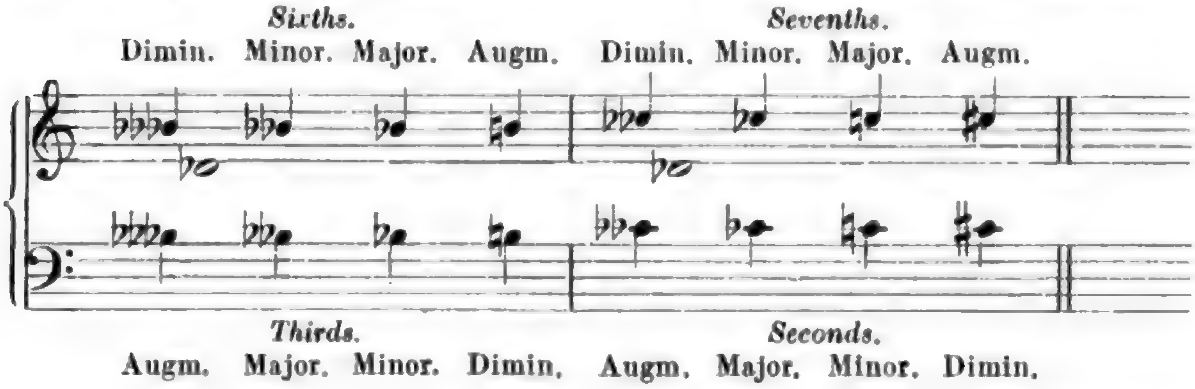
a) INTERVALS FORMED WHEN *C* IS TAKEN AS BASIS.



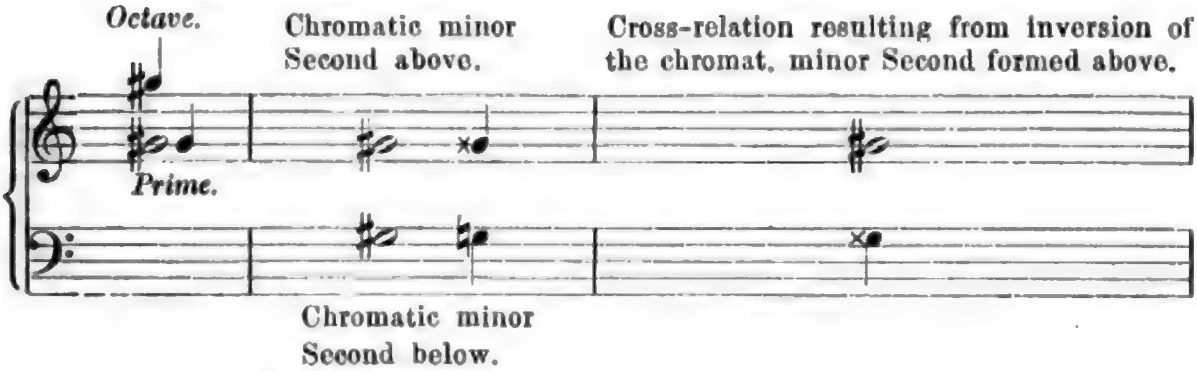
b) INTERVALS FORMED WHEN *D♭* IS TAKEN AS BASIS.

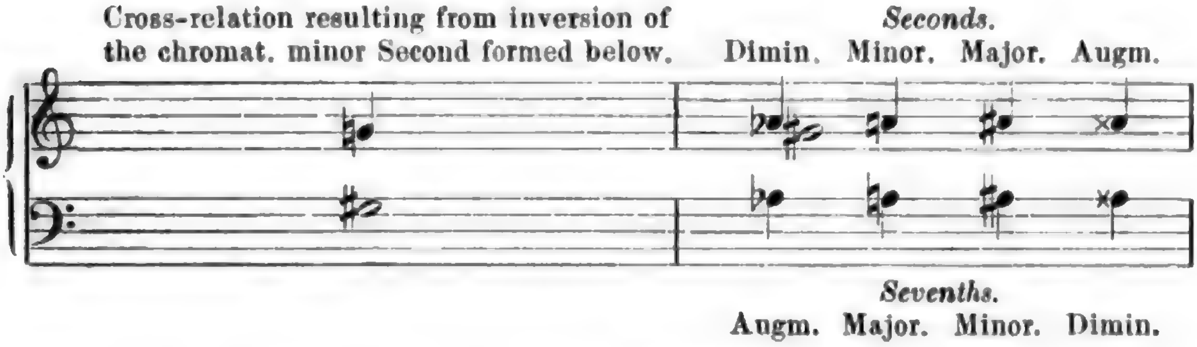


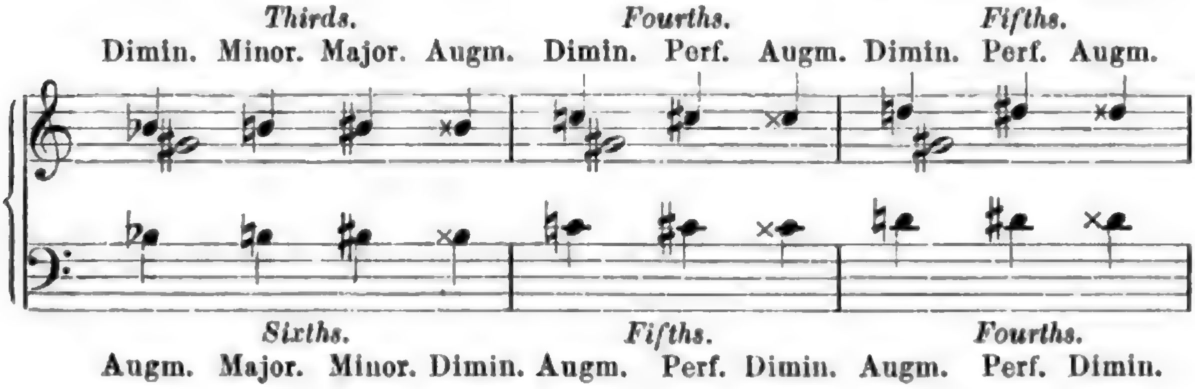


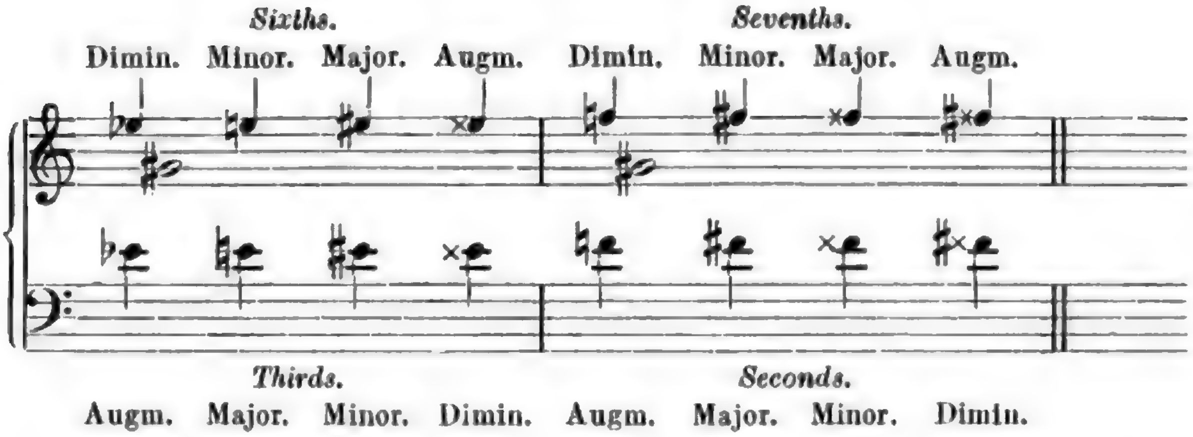

Sixths.
Dimin. Minor. Major. Augm.
Sevenths.
Dimin. Minor. Major. Augm.
Thirds.
Augm. Major. Minor. Dimin.
Seconds.
Augm. Major. Minor. Dimin.

c) INTERVALS FORMED WHEN G♯ IS TAKEN AS BASIS.


Octave.
Prime.
Chromatic minor Second above.
Cross-relation resulting from inversion of the chromat. minor Second formed above.
Chromatic minor Second below.


Cross-relation resulting from inversion of the chromat. minor Second formed below.
Seconds.
Dimin. Minor. Major. Augm.
Sevenths.
Augm. Major. Minor. Dimin.





These examples will suffice to render the principles involved in the formation of intervals intelligible.

2. Write out the fundamental triads, in all the keys, in letters and notes, beginning with *C*-major.

3. Point out the subordinate triads on the third and sixth degrees, and explain the formation of the compound triads on the second and seventh degrees of the scale.

4. Show in what manner the triads are connected with each other (through the third, through the fifth, or indirectly).

5. Write out all major keys in letters and notes.

6. Analyse the triads on the second and seventh degrees, and make sure that the distinction between the fundamental and subordinate triads, as regards their major or minor character, is thoroughly comprehended.

7. Explain the influence of the interconnection of the chords on the progressions.

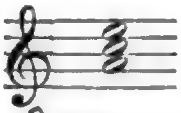
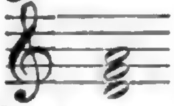
8. Review exercises on progressions between chords connected through the third and fifth, and indirectly connected chords. These exercises afford, at the same time, practice in forming closes (cadences), in avoiding parallel fifths and octaves, and the unmelodic progression of the tritone, and in treating the sequences.

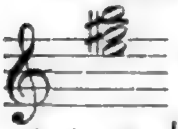
CHAPTER IV.

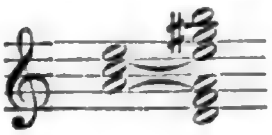
The Minor Mode and its Triads.

The minor character is the opposite or contradiction of the major. The derivation of the minor mode from the major is a matter of purely historical interest. These two modes are now perfected, and completely different one from the other; the minor mode is therefore to be regarded as a separate and independent system or formula, and not as derived from the major. For this reason Hauptmann's alphabetical formula is to be preferred, in musical practice, to that of Helmholz [see Note XI]. This latter is more convenient for acoustic experiments in connection with the major scale, the tonic triad in minor having precisely the same acoustic relations as the triad on the sixth degree in major. Among the minor keys *A*-minor is that most nearly related to *C*-major. In this latter the triad on the sixth degree appears, in letters, as *a-C-e*; whereas the tonic triad of *A*-minor would be written, in order to exhibit the minor mode as an independent system, *A-c-E*. Here the fundamental and the fifth are designated by capitals, as in major, and the third by a small letter. Thus the tonic triad in minor appears at once to the beginner as a fundamental chord; the triad on the sixth degree, on the contrary, appears as a subordinate chord. The entire alphabetical formula for this minor key would be the following:

B/D-f-A-c-E-g♯-B/D

We find, on analysing this formula, the triad *A-c-E*  as tonic triad with minor character, and that the triad *D-f-A*  the sub-dominant triad, also has a minor character. It should be remembered, that the difference in the form of major and minor triads lies in the difference of the relative positions of the major and minor thirds constituting these triads; in the former the major third is below the minor third, and in the latter the minor third is below the major.

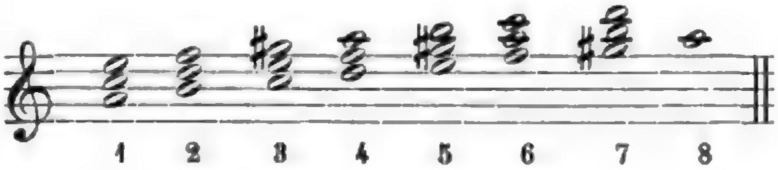
The minor character of the minor mode is sustained by the tonic and sub-dominant triads alone, the remaining triads being of a different nature. Of these the third fundamental triad, the dominant triad *E-g♯-B*,  first claims our attention. Its form shows it to be a major triad, *E-g♯* being a major third, and *g♯-B* a minor third. Through this difference in the character of the three fundamental triads, two being minor and the third major, an element of disunion or contradiction makes itself felt in the minor mode. This peculiarity of the fundamental triads is also the cause that the minor mode is the exact opposite of the major. It is probably a generally received opinion among musicians that the character of the major mode is active, the minor mode being of a more passive nature. The major keys are therefore adapted to the expression of joyful feelings; the minor, on the other hand, for expressing suffering or sorrow. On placing the three fundamental triads in minor together

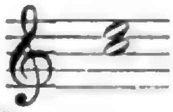
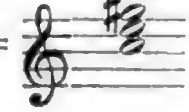
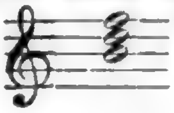
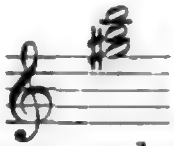
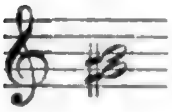
D-f-A-c-E-g♯-B 

we recognize the mutual relations of the several tones, and can see whether they are, as regards their positions in the triads, fundamentals, thirds or fifths. If we finish or close the formula in the same manner as given above for the major mode, by setting *B*, the fifth of the dominant triad, next to *D*, and *D*, the fundamental of the sub-dominant triad, next to *B*, we obtain a complete formula for the minor mode and all triads belonging to the same:

B/D-f-A-c-E-g♯-B/D.

The fundamental triads are easily to be distinguished from the subordinate by the aid of the curves, and we perceive at a glance that the three triads already mentioned, *i. e.* the tonic triad *A-c-E*, the sub-dominant triad *D-f-A*, which are both minor, together with the dominant triad *E-g♯-B*, which is major, are the fundamental triads, the others being subordinate. Of these subordinate triads those connected through the third first claim our attention. On the sub-dominant side of the formula we find *f-A-c* connected through the third with *A-c-E* and *D-f-A*, having the tones *A* and *c* in common with *A-c-E*, and the tones *f* and *A* in common with *D-f-A*. The chord *f-A-c* is a major triad; whereas the chord occupying the same position in the major formula, *i. e.* the triad which is connected through the third with both the tonic and sub-dominant triads, has been shown to be a minor triad. In order to exhibit the chords of the minor mode in regular succession, it may be observed here that in the scale

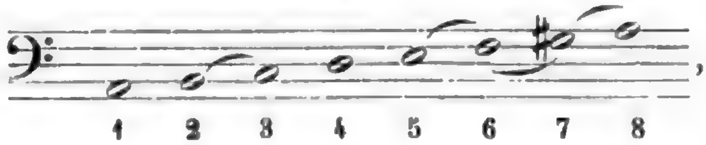


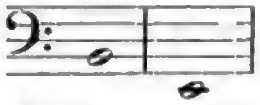
the tonic triad lies on the first degree, the sub-dominant triad on the fourth degree, and the dominant triad on the fifth degree; *f-A-c*, which is connected through the third with the tonic and sub-dominant triads, lying on the sixth degree. The triad *c-E-g♯*, on the dominant side of the formula, which is connected through the third with the tonic triad *A-c-E* and the dominant triad *E-g♯-B*, differs wholly in structure from all the others. It is called the "augmented triad", being formed by the conjunction of two major thirds, which together constitute an augmented fifth. In this case the augmented fifth is *c-g♯* = , the two major thirds being *c-E* =  and *E-g♯* = . The entire chord *c-E-g♯* =  lies on the third degree of the harmonic minor scale. All augmented intervals being dissonant, this triad, with its augmented fifth, is named a dissonant triad. As explained above, indirectly connected triads are such as have, although belonging to the same key, no note in common; their connection being effected by means of other triads connected with them through the fifth and third. The two diminished triads in the key of *A*-minor, which are found at either extremity of the alphabetical formula, are *B*/*D*-*f* =  on the second degree, and *g*♯-*B*/*D* =  on the seventh degree; they have the tones *B* and *D* in common, and are consequently connected with each other through the third. *B*/*D*-*f* is connected with the sub-dominant triad *D*-*f*-*A* through the third, and a like relation subsists between *g*♯-*B*/*D* and the dominant triad *E*-*g*♯-*B*; *B*/*D*-*f* and *g*♯-*B*/*D* are connected through the fifth with *f*-*A*-*c* and *c*-*E*-*g*♯ respectively; whereas neither of the diminished triads has a tone in common with the chord of the tonic *A*-*c*-*E*, with which latter they are therefore only indirectly connected. *B*/*D*-*f* and *g*♯-*B*/*D* are named diminished triads, because they are both formed by the conjunction of two minor thirds, which constitutes a diminished fifth. *B*/*D*-*f* is formed by uniting the two minor thirds *B*/*D* = and *D*-*f* = ; *g*♯-*B*/*D* from *g*♯-*B* =  and *B*/*D* = .

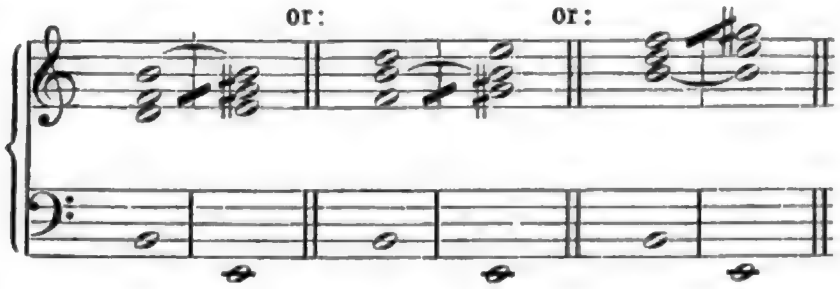
CHAPTER V.

Progressions between the Triads in Minor.

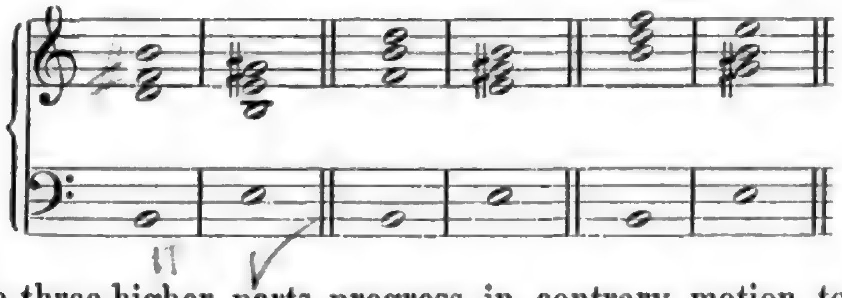
The progressions between the triads of the minor keys are effected in the same manner as in the major keys; but in view of the fact that augmented intervals should not be employed in melodic progressions in the strict style, particular attention must be paid to the augmented second occurring, in minor, between the sixth and seventh degrees of the scale. In the harmonic *A*-minor scale



which is superseded, in simple melody, by the melodic minor scale (see Chap. I), the semitones, marked here by curves above the notes, are found between the second and third, fifth and sixth, and seventh and eighth degrees; the augmented second, marked by a curve below, lies between the sixth and seventh degrees. This second cannot be used, in harmonic progressions from one triad to another, as a melodic step, *i. e.* from *f* to *g*♯, or *vice versa*. This chromatic interval has, to be sure, been latterly employed with good effect in polyphonic compositions without instrumental accompaniment. It is nevertheless necessary for the student to confine himself, at first, strictly to diatonic composition, and thus to obtain a thorough acquaintance with the strict style of writing. Only a master of harmony and form may be allowed to combine chromatic harmony and melody with diatonic, in order to give expression to the beautiful conceptions of his imagination. In order to avoid employing this chromatic interval melodically in progressions between the triads of the minor scale, it should be borne in mind, that whenever the second and fifth degrees follow each other in the bass the progression between the triads should be effected as if they were indirectly connected; because an observance of the principle of connection through the fifth would in this case make the use of the chromatic augmented second as a melodic step unavoidable. If, for instance, in *A*-minor the bass  be given, and the progression be executed by the help of the connection through the fifth, the result would necessarily be:

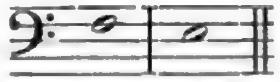
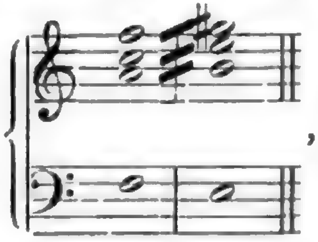
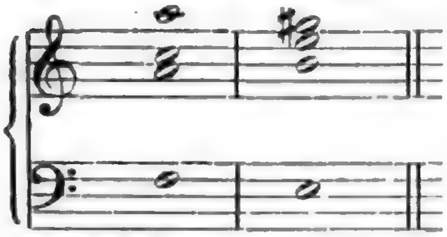
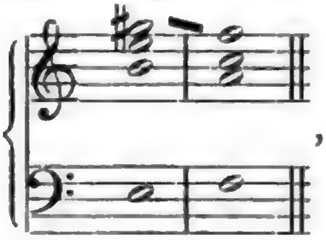
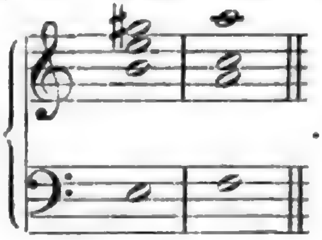


In all such cases the step *f*-*g*♯ does not give us the desired diatonic connection. The progression would therefore be better thus:

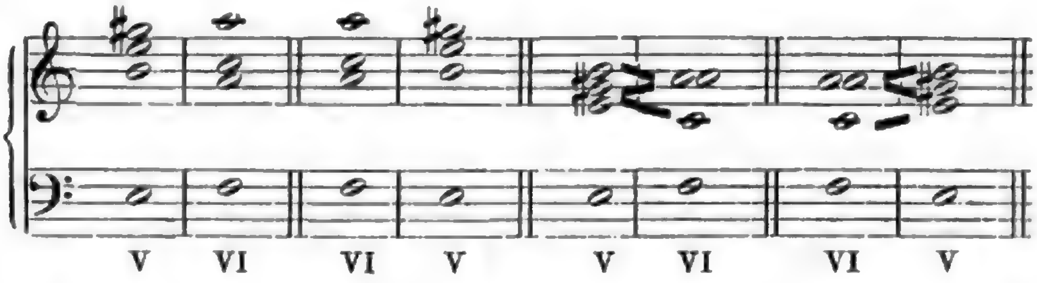


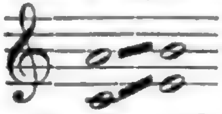
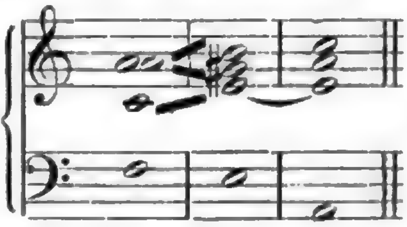
Here the three higher parts progress in contrary motion to the bass, as has been shown in the case of indirectly connected chords.

The augmented second also requires special notice in progressions between the two triads based on the sixth and fifth degrees of the scale.

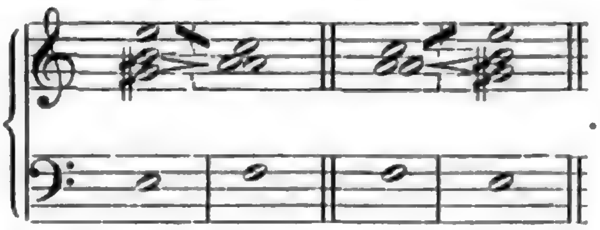
When the bass in *A*-minor moves from *f* to *E* , and the triads based upon these tones are to be correctly connected in harmonic progression, the progression , would *not* be diatonic, the step from *f* to *g*♯, the augmented second, being chromatic. Instead of the fundamental *f*, in the soprano, the third *A* should be written , by which means the diatonic progression *A*–*g*♯ results. Conversely, the same rule holds good where the sixth degree follows the fifth, in which case we should not write: , but rather .

According to the position of the triads the progressions would be accomplished as given below:



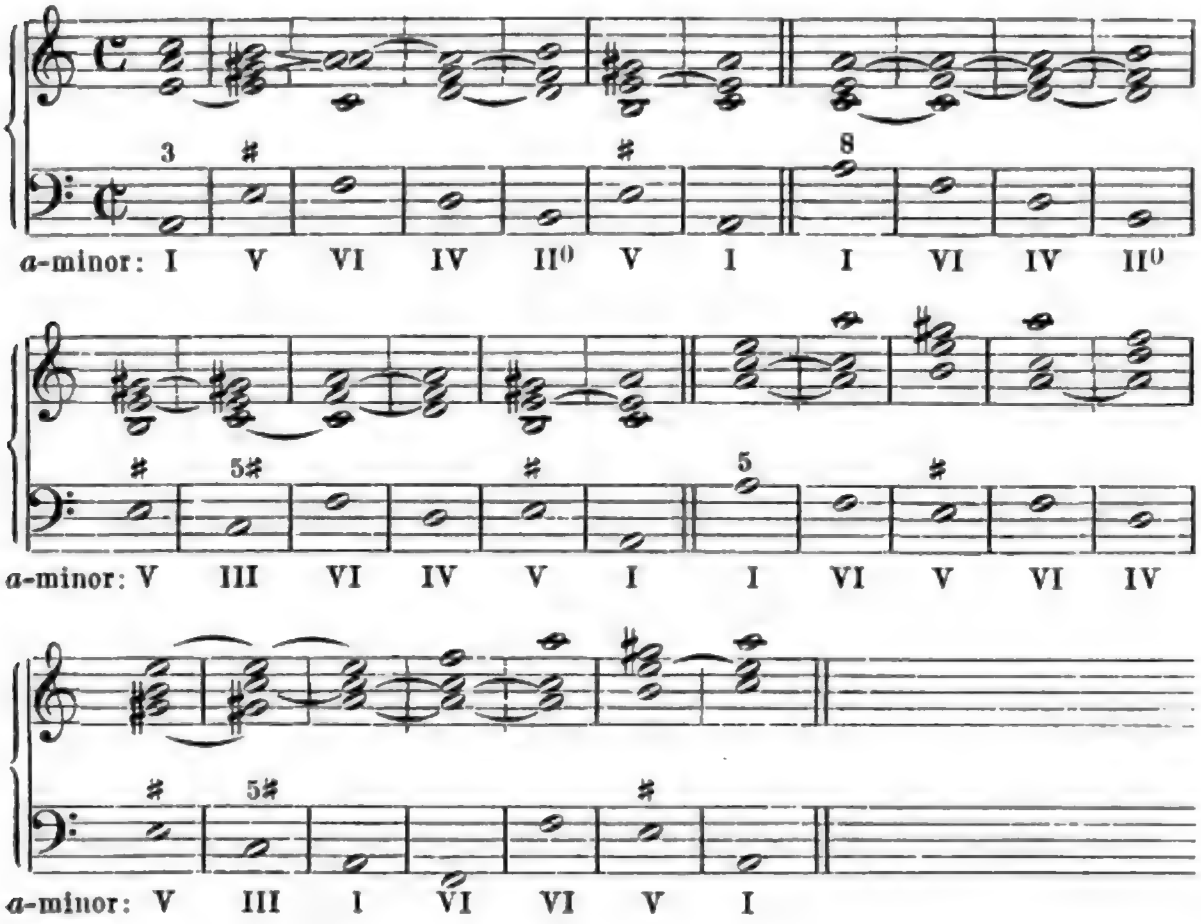
Many theorists might take exception to this last progression from VI to V on account of the so-called *covered fifths*: ; as, however, no argument based upon harmonic laws can be brought forward for excluding such a progression as , its employment in strict harmony must be regarded as justifiable.

Finally, in case the triad on the fifth degree should occur with the octave in the soprano, the progression would be

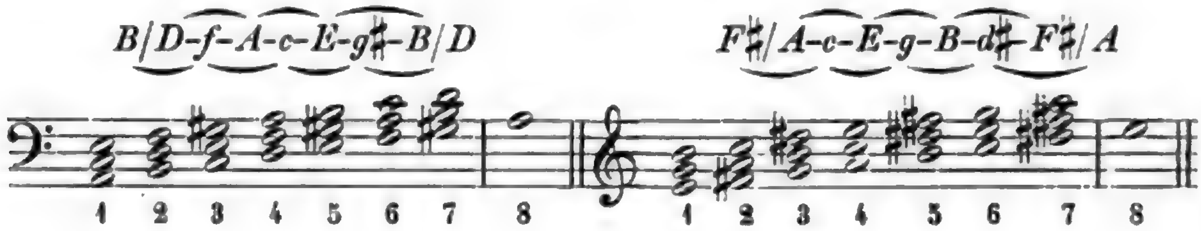


It may, in short, be given as a general rule, that when the triads of the fifth and sixth degrees follow each other, whichever may come first, the third of the triad on the sixth degree should be doubled.

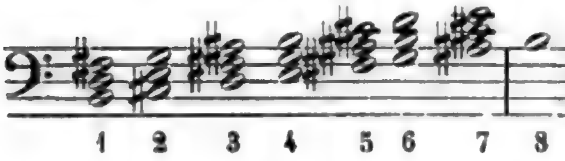
After these explanations, progressions between the triads of the minor keys will offer no difficulties, provided that the laws of the affinity of the chords be properly observed. The examples here given may serve as an introduction to the exercises, for which the bass only is written.


3 # # 8
a-minor: I V VI IV II⁰ V I I VI IV II⁰
# 5# # 5 #
a-minor: V III VI IV V I I VI V VI IV
# 5# #
a-minor: V III I VI VI V I

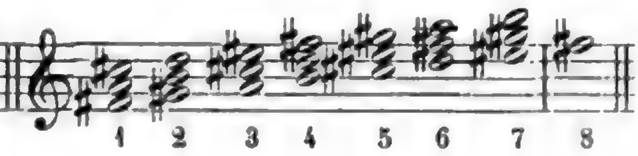
The pupil should now write out the various minor keys in full, both in notes and letters.



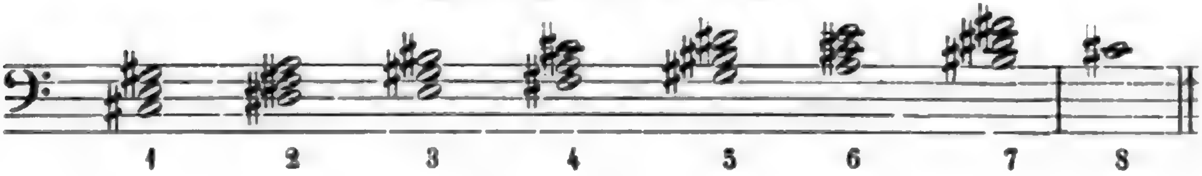
C♯/E-g-B-d-F♯-a♯-C♯/E



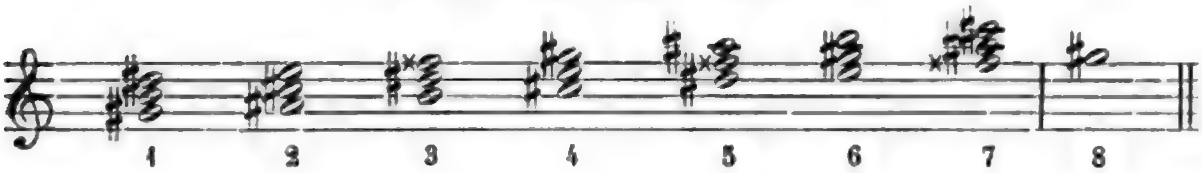
G♯/B-d-F♯-a-C♯-e♯-G♯/B



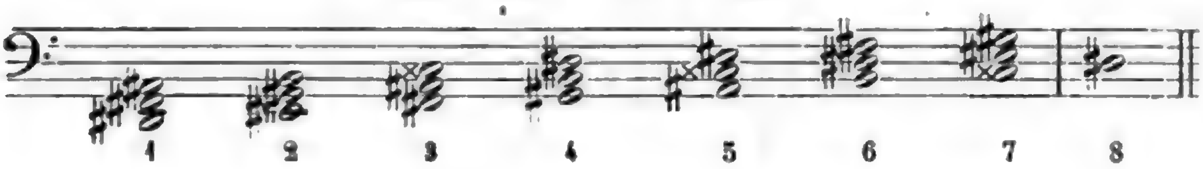
D♯/F♯-a-C♯-e-G♯-b♯-D♯/F♯



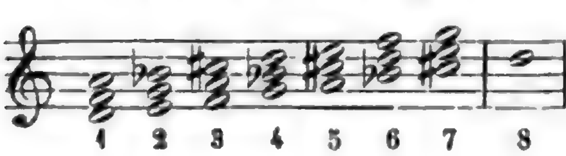
A♯/C♯-e-G♯-b-D♯-f×-A♯/C♯



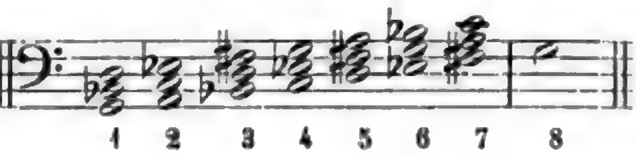
E♯/G♯-b-D♯-f♯-A♯-c×-E♯/G♯



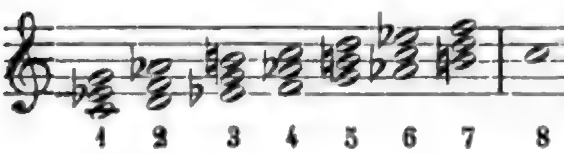
E/G-b♭-D-f-A-c♯-E/G



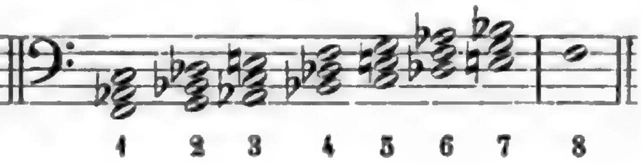
A/C-e♭-G-b♭-D-f♯-A/C



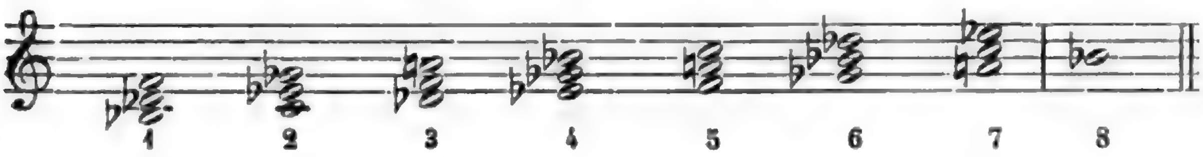
D/F-a♭-C-e♭-G-b-D/F



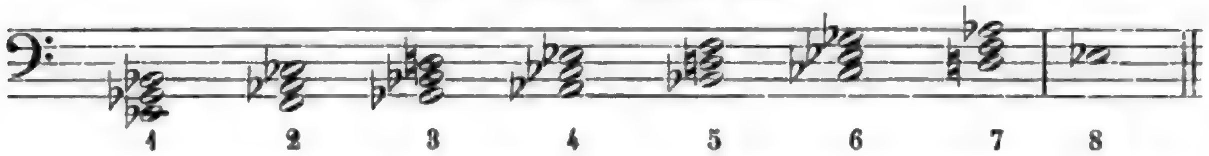
G/B♭-d♭-F-a♭-C-e-G/B♭



C/E♭-g♭-B♭-d♭-F-a-C/E♭

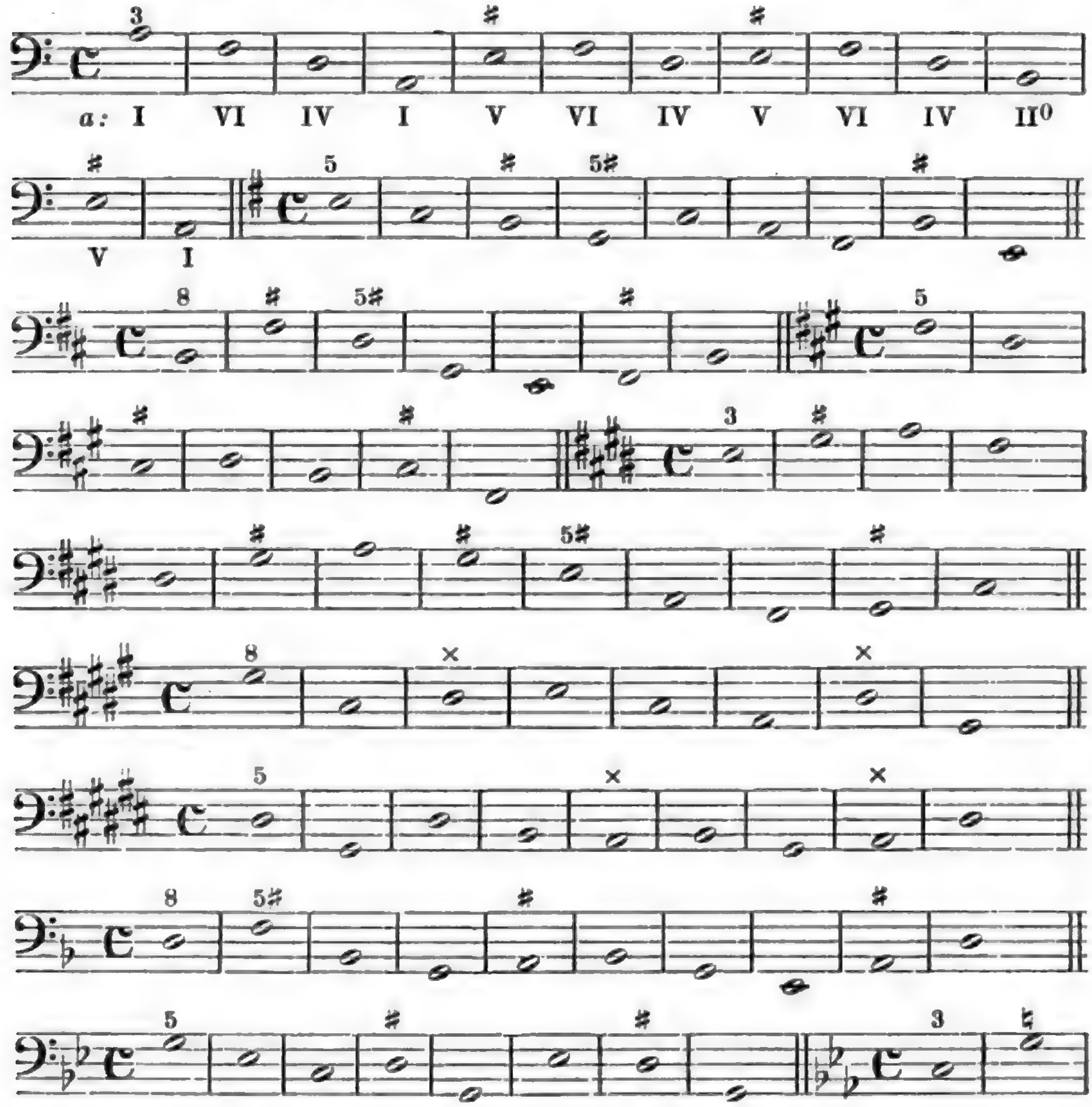


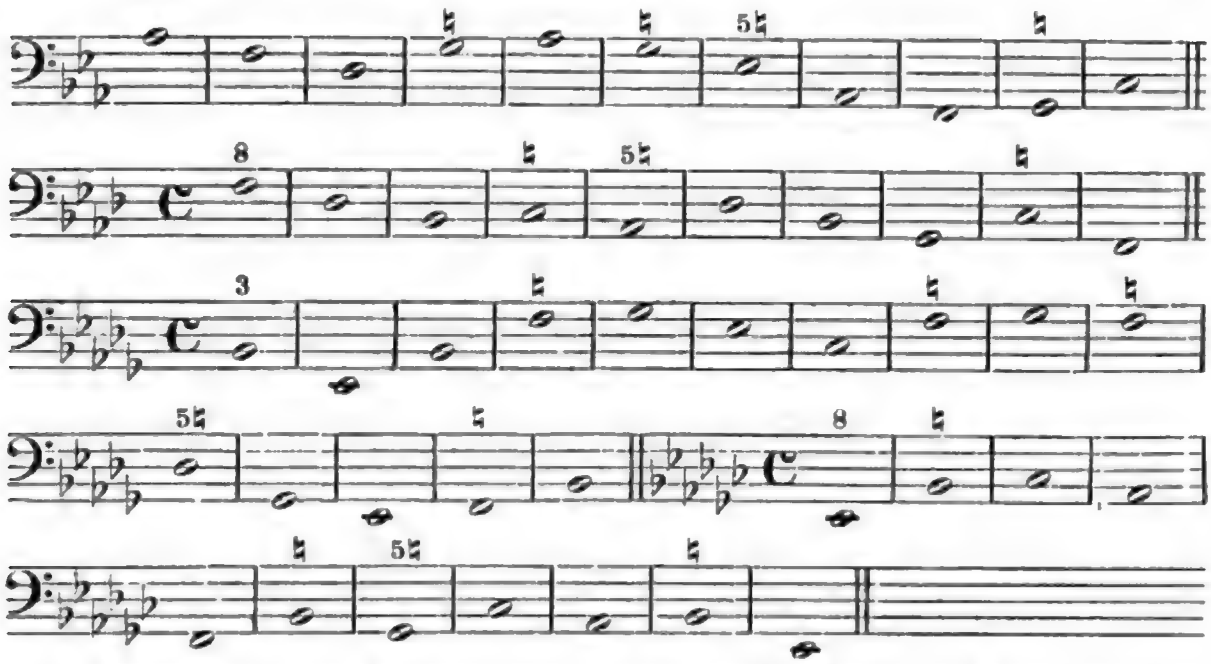
F/A♭-c♭-E♭-g♭-B♭-d-F/A♭



After learning these thoroughly the exercises in progressions between the triads may be worked out. To the basses given below the three higher parts are to be set.

NB. When a sharp (♯) stands alone over a bass-note it signifies that the third of that triad, of which the bass-note is the fundamental, is to be raised a semitone; *i. e.* that the major third is required. In some keys the double sharp (×) is needed to raise the natural tone to the desired major third; in the flat keys the natural (♮) is employed for the same purpose. [Where a sharp occurs in conjunction with a figure (5♯) the tone indicated by the figure is to be raised a chromatic semitone.]





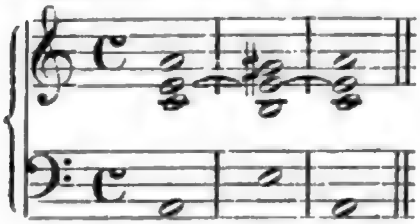
After working out the exercises, which will afford sufficient practice if the learner be possessed of any talent, the following principal points should be carefully reviewed:

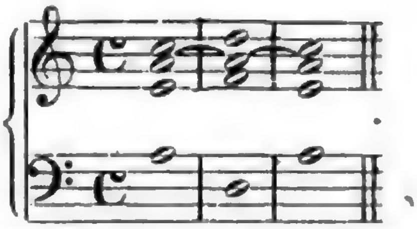
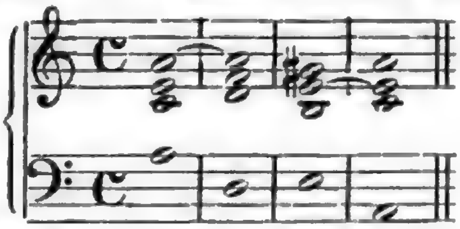
1. The character of the three fundamental triads in minor, particularly the formation of the dominant triad in contradistinction to those of the tonic and sub-dominant.
2. The formation of those triads which are connected through the third with the tonic triad, and their relation to the two other fundamental triads; taking special notice of the tones and intervals which the various triads have in common, and of the structure of the augmented triad on the third degree, and of the triad on the sixth degree.
3. The structure of the harmonic minor scale, and of the augmented second between the sixth and seventh degrees of the same.
4. The progressions between the triads on the second and fifth degrees, and also between those on the fifth and sixth degrees.

CHAPTER VI.

On the Formation of Closes with the Triads of the Minor Mode.

The various forms of the close have, as in major, different names. The "authentic close" is formed between the dominant and tonic triads, thus:

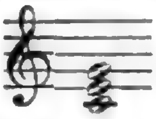


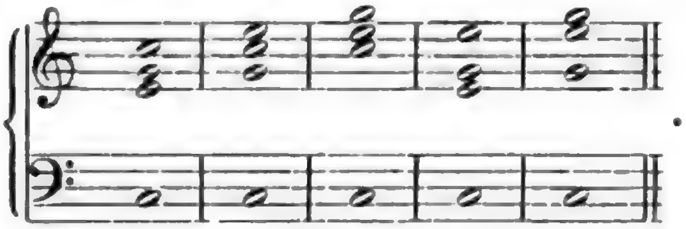
. The so-called "plagal close" is formed between sub-dominant and tonic, as here given: . [See Note VII.] Finally, the "perfect close" is formed by combining the three fundamental triads, *i. e.* those of the sub-dominant, dominant and tonic, and may be written thus: . The progression from the fifth to the sixth degree, the composition closing further on with the chord of the tonic, is often called a "deceptive" cadence or close. It does not, however, appear necessary to name a simple progression of chords which may be followed by various others a "deceptive" close, *i. e.* a close which deceives the ear. The ear, when correctly apprehending this progression, is not deceived; such a progression is not felt in the least to be a close, this feeling being awakened only by a return to the chord of the tonic, in which the key has its beginning and its end. The "deceptive" closes of the manuals of harmony are, it is true, generally given in combination with the chord of the seventh, to be explained further on, and not as progressions between simple triads. All closes being however founded, in principle, upon progressions between these triads, the "deceptive" close had to be noticed here.

A succession of the triads of the second, fifth and first degrees can be employed as close in minor as well as in major.

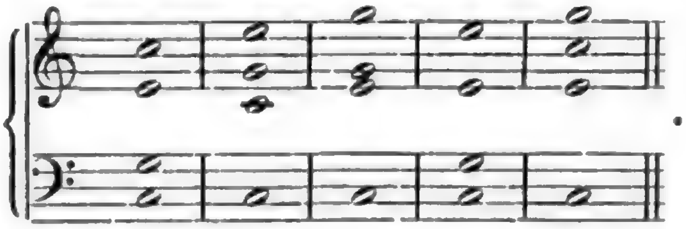
CHAPTER VII.

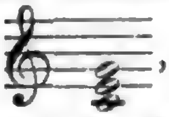
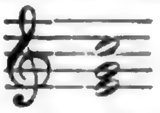
The Inversions of the Triads.

Every triad is composed of fundamental, third and fifth. If the fundamental tone of the triad be given in the bass, the other parts being arranged at pleasure, the triad is said to be in the fundamental position. If for instance the triad *C-e-G*  be written in four-part harmony, with the fundamental *C* in the bass, the three upper parts may be arranged at will without inverting or changing the fundamental harmony. The following examples show the fundamental harmony of a triad, the fundamental tone in each case lying in the bass, and the other parts being arranged in various positions:



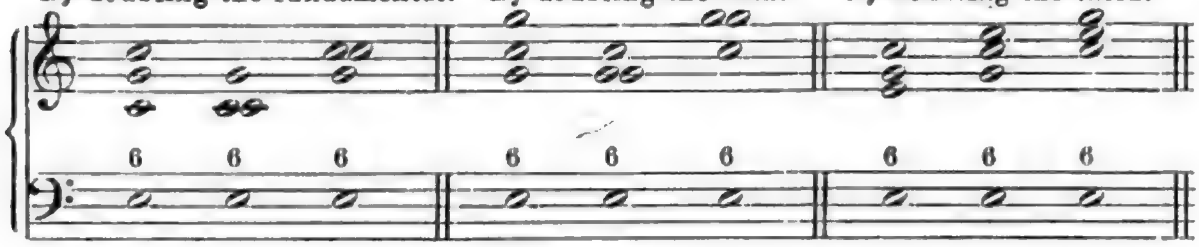
This arrangement of the three upper parts within the compass of an octave is called "close position". In "open position" on the other hand the three upper parts are spread out beyond the compass of an octave:



The third, as well as the fundamental, can be used as bass, *i. e.* as the harmonic basis, or foundation of the chord. In this case the chord is called a "chord of the third and sixth," or, as abbreviated, "chord of the sixth." In a chord of the sixth either the fundamental, the fifth or the third may be doubled in four-part harmony. The chord of the sixth derived from the triad *C-e-G* = , which is *e-G-C* = , is indicated by the figuring $^6_3$, or 6 for short, *G* being the third and *C* the sixth of *e*, and may be written in the following positions:

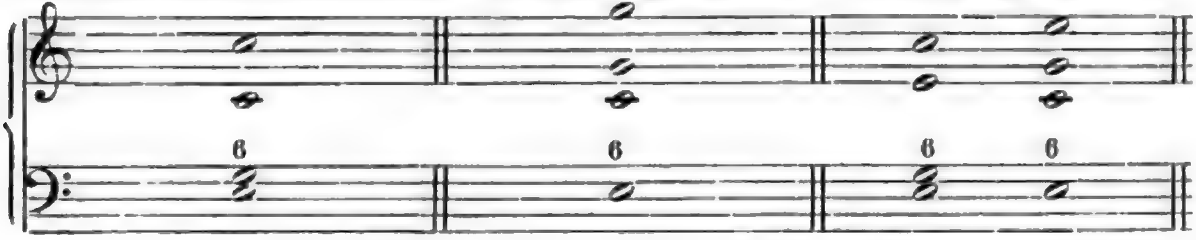
1. *Close Position.*

By doubling the fundamental. By doubling the fifth. By doubling the third.

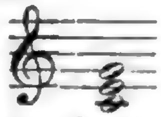
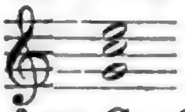


2. *Open Position.*

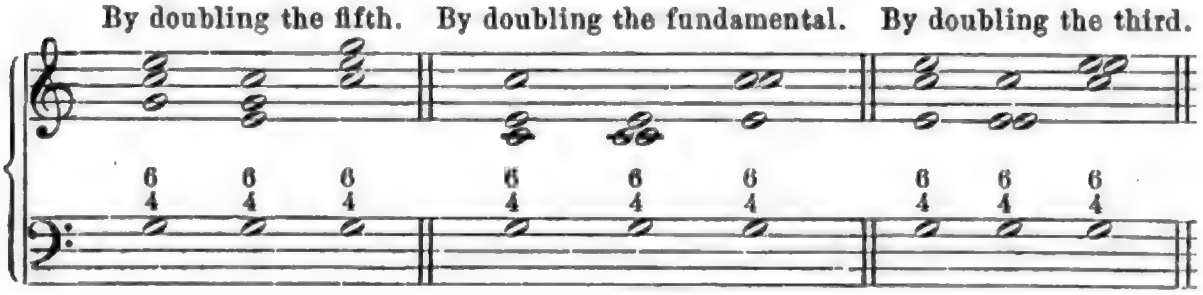
By doubling the fundamental. By doubling the fifth. By doubling the third.



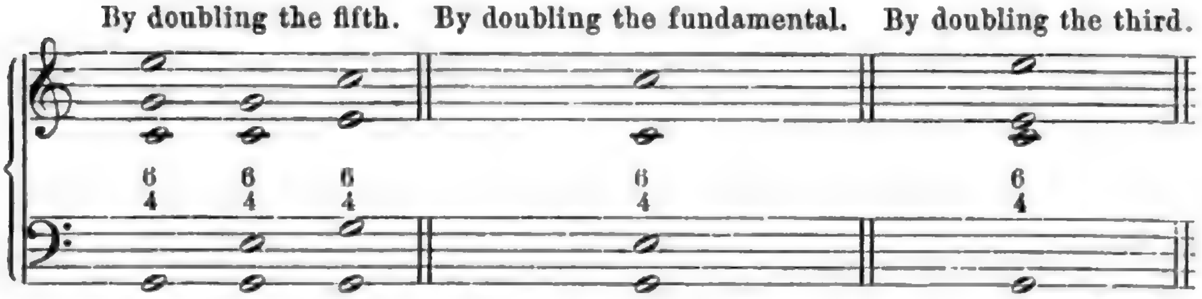
It should be remarked here that the figuring always indicates the intervals as reckoned from the bass-note.

If the fifth of the triad *C-e-G* =  be taken as bass, or basis of the harmony, the chord of the fourth and sixth, expressed by the figures $^{6}_{4}$, is formed. *G-C-e* =  would therefore be the chord of the fourth and sixth derived from *C-e-G*. In four-part harmony the chord of the fourth and sixth may be written in various positions, in which either the fifth, fundamental or third may be doubled, as the following examples show:

1. *Close Position.*



2. *Open Position.*

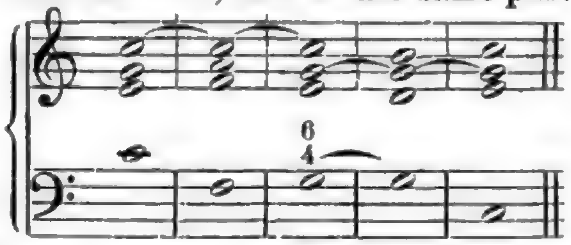


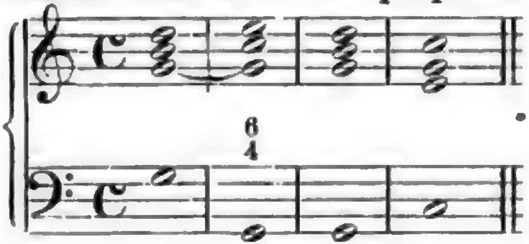
Of these forms that in which the fifth is doubled is oftenest employed, the distinctive character of the chord being brought out most strongly in this position.

When the chord of the fourth and sixth is used in the formation of a close, it is necessary that it should be *prepared* in the chord preceding.

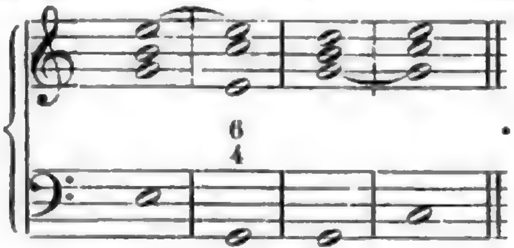
A distinction is made between harmonic and melodic preparation.

The chord of the fourth and sixth is prepared harmonically when it has a tone in common with, and in the same part as, the preceding chord.

The example  presents the chord of the fourth and sixth as prepared by the fundamental tone of the fundamental chord, which is *C*, and which appears in the same part (soprano) of the chord *F-a-C*, immediately before the entrance of the chord of the fourth and sixth. The next example exhibits this chord as prepared by the fifth of the fundamental chord:

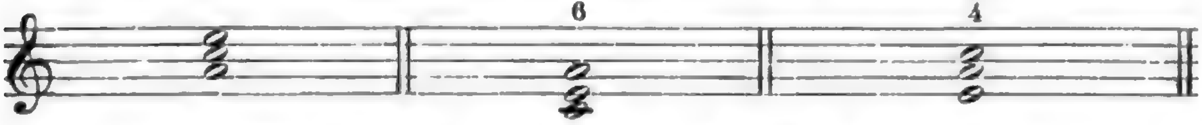


Finally, the third may also be employed for the same end:

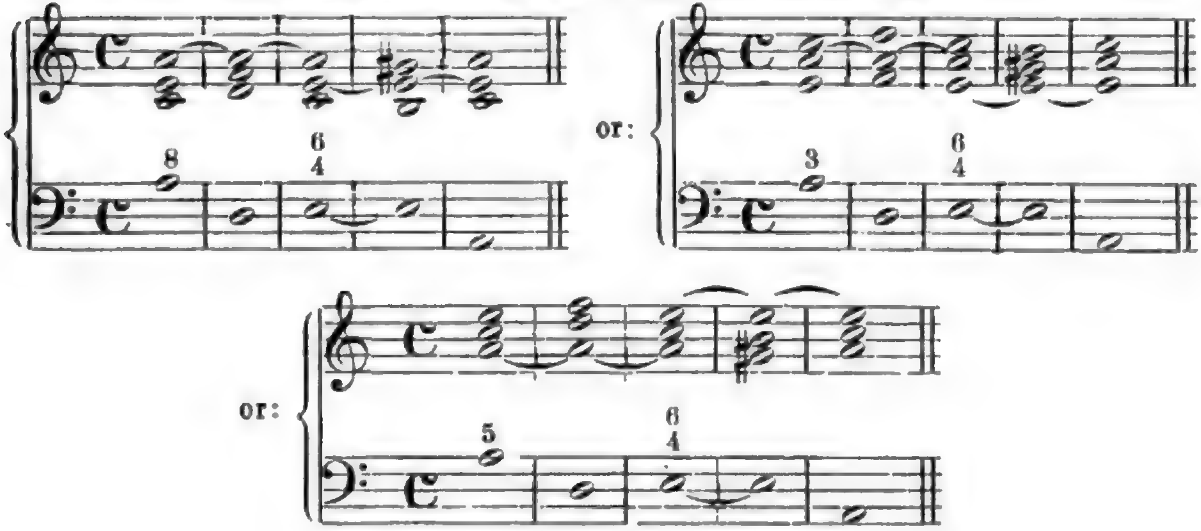


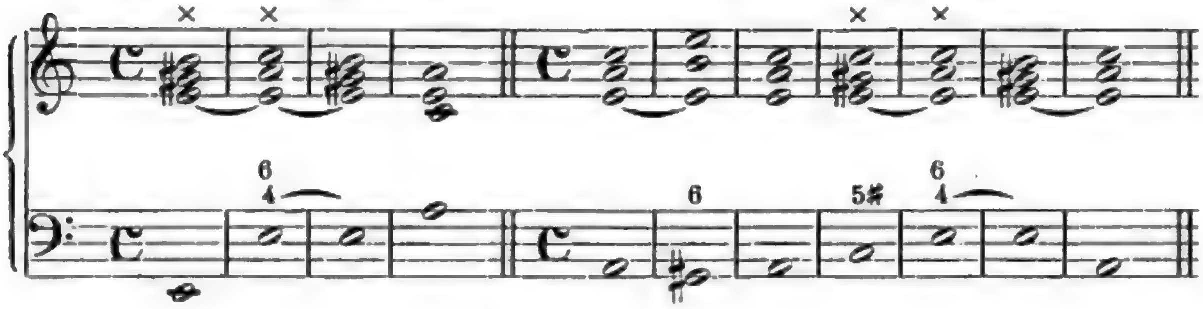
The first of these examples, in which the fundamental tone (the fourth in the chord of the fourth and sixth) is used in preparation, is the best for the formation of the close; the basis of the entire key [*i. e.* the *tonic*] being thus prepared, and the principal constituent elements of the key being brought into play through the combination of the sub-dominant triad, the chord of the fourth and sixth, the dominant triad, and the tonic triad [see Note X]. This is the most complete closing cadence that can be formed by the use of triads in combination with the chord of the fourth and sixth derived from the tonic triad. The same forms of the close can also be employed in minor, as shown in following examples:

Fundamental chord. Chord of the sixth. Chord of the fourth and sixth.

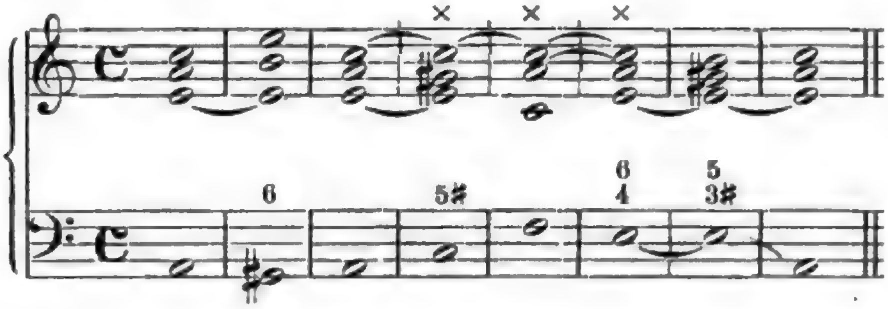


Closing cadences in which the chord of the fourth and sixth is prepared by the fundamental note of the fundamental chord

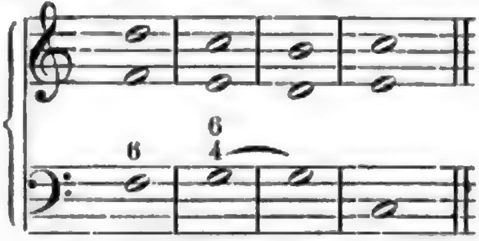
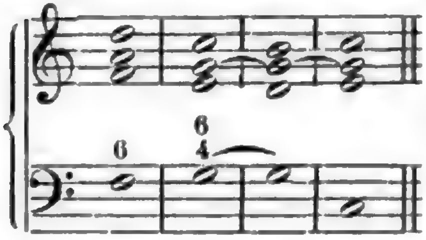


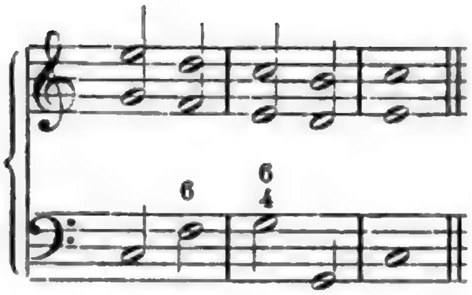

Preparation of the fifth of the fundamental chord.
Preparation of the third of the fundamental chord.

In the last example the resolution of the augmented triad through the chord of the fourth and sixth is unsatisfactory, which necessitates the interpolation of the triad on the sixth degree, whereby the chord of the fourth and sixth appears as prepared through the fundamental tone of the fundamental chord, *e. g.* :



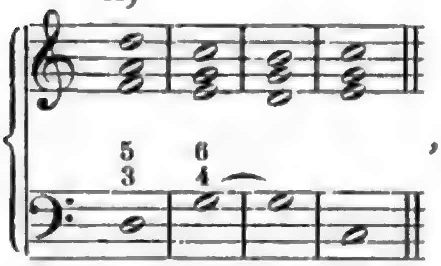
From these examples the harmonic preparation by means of the fundamental tone of the fundamental chord appears to be decidedly the best in forming closes; the other preparations leave an impression of incompleteness which is dissipated only when they are followed by the perfect close with prepared tonic.

Besides the harmonic preparation we have the melodic preparation, which Moritz Hauptmann rightly derives from the Italian opera-music of the seventeenth and eighteenth centuries. We have merely to add a fourth part to the three-part close:  in general use at that period to obtain the now customary close: , which made its appearance at the beginning of the last century. In the following closing cadence:

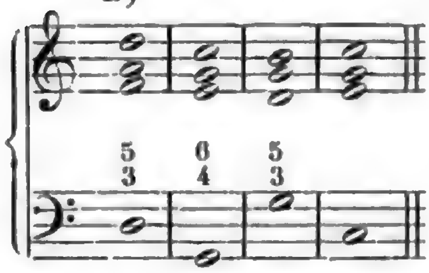


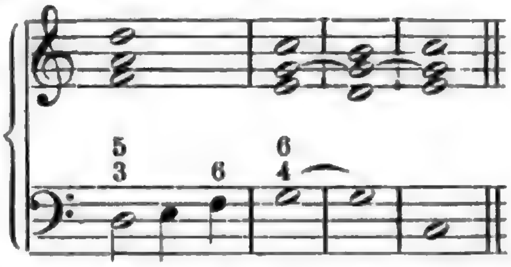
found in the dramatic music of the seventeenth and eighteenth centuries the middle part is, it is true, usually left out on account of the parallel fourths and, by doubling (*e. g.* in orchestral composition) parallel fifths which this fourth part would form with the soprano. The practice of writing parallel fourths accompanied by the third, which latter mitigates the harshness of such progressions, has, however, been long since sanctioned by usage, and the four-part phrase given above is now recognized as correct by all musicians. The chord of the fourth and sixth derived from the tonic triad may also be preceded by the triad of the second degree in its fundamental position. The last example shows the progression from the chord of the sixth derived from the triad of the second degree. This phrase likewise came into vogue in the following form, with the chord in its fundamental position: , the bass progressing by a skip to the chord of the fourth and sixth. No objection can be raised to this form of the phrase, although the following progression of the bass is inadmissible . In the progression unter *A*) the notes *e* and *F* may be interpolated, to complete the melodic upward progression of the bass, in this manner . On the other hand, by filling out the downward progression of the bass-skip *D-G* in

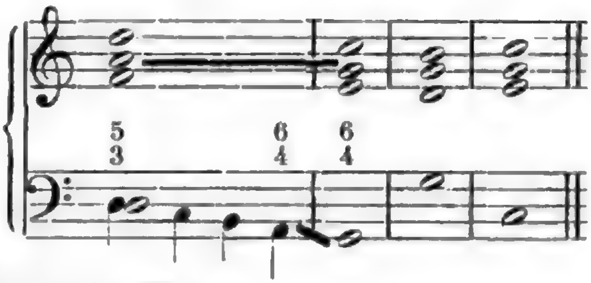
A)

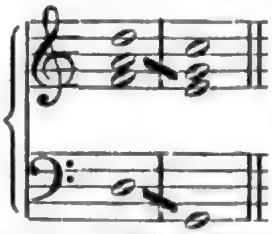


B)



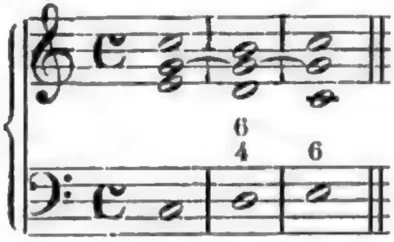


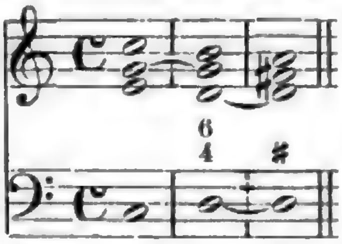




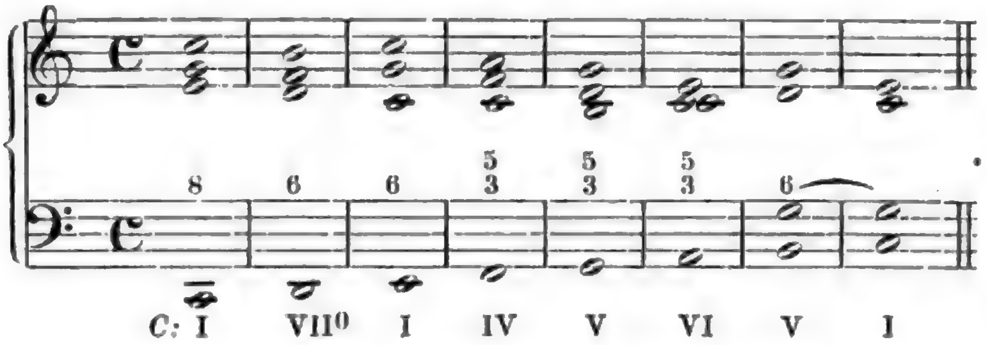
the same way, with the notes *C b a* the so-called "covered octaves" are brought to light, as shown by the dashes from *A* to *G*. This progression is however less faulty than the entirely incorrect succession of two chords of the fourth and sixth. In progressions between triads the chord of the fourth and sixth is always to be regarded as the connecting link between two other triads, each of which must appear either in the fundamental position or as a chord of the sixth. "In opposition to this rule," says Moritz Hauptmann, "that a succession of chords of the fourth and sixth is not allowable, isolated exceptions may be found, not in the exercises of learners, but in masterpieces of composition, contrary to rule and yet harmonious; nevertheless, these do not overthrow the law upon which the rule is founded".

The proper employment of the chord of the fourth and sixth often gives the beginner much trouble; he sometimes writes it where it is quite uncalled for, and omits it where necessary. Talented pupils, guided by correct musical feeling, make fewer mistakes in free composition. Uncertainty as to the correctness or incorrectness of progressions often arises, however, when the harmony is to be written to a given melody or part, a so-called "*cantus firmus*". In such exercises the mistake is frequently met with, that the chord of the fourth and sixth is written on the second degree of the scale, as the following example with a "given" bass shows:



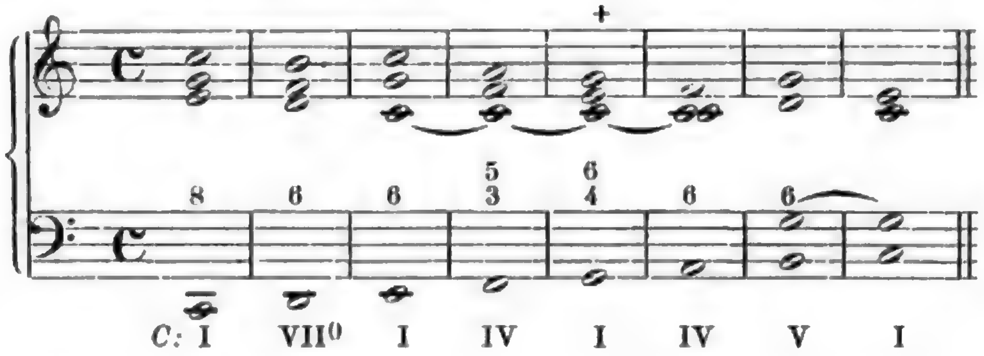
. The chord of the fourth and sixth is quite inadmissible in such a progression. The fact that the fourth is prepared and that the parts progress according to rule, does not in this case prove the progression to be correct. The reason for the admissibility of the chord of the fourth and sixth is founded not alone upon the preparation and accurate movement of the parts, but lies in the nature of the key itself, which requires that this chord should appear in a decided and unequivocal character. In the above progression, in which the chord of the fourth and sixth derived from the dominant triad occurs on the second degree, this chord suggests the progression 

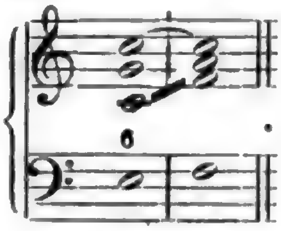
; through the employment of the chord of the fourth and sixth derived from the dominant triad our apprehension of, or mental hold upon, the original key is consequently weakened. Instead of this chord of the fourth and sixth the triad on the second degree, or the chord of the sixth on the seventh degree, should be employed. After these explanations it will be easy to write the three upper parts to the major scale as bass, using only fundamental triads or chords of the sixth; *e. g.*:

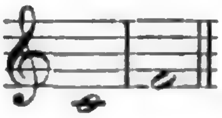
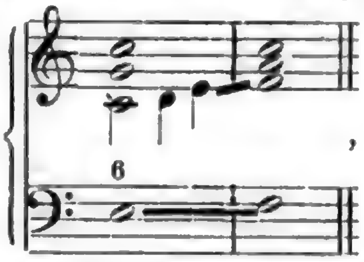


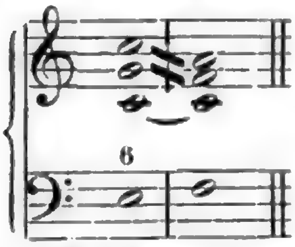
In this example the key of *C*-major is indicated by *C*, the degrees of the scale by Roman numerals, and the position of the triads by Arabic numerals.

The chord of the fourth and sixth derived from the tonic triad might be employed with good effect on the fifth degree of the scale:



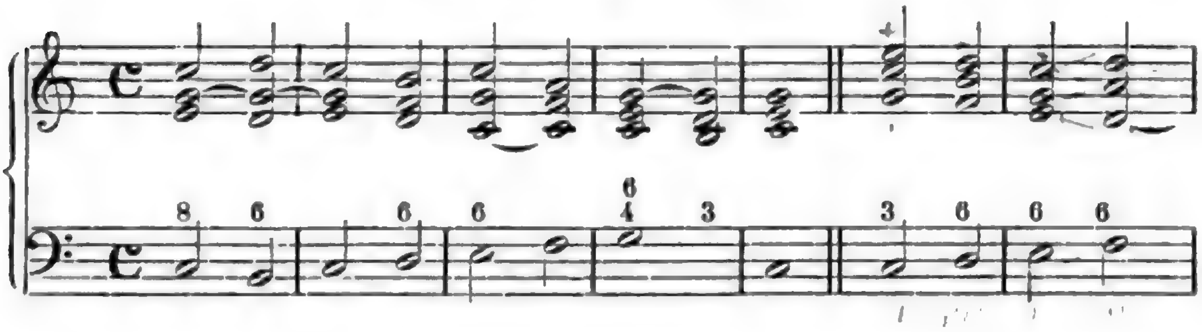
It will now suffice to direct the pupil's attention to the fact, that in progressions between directly connected triads the tones common to both form the connecting link in passing from one to the other, as shown in the last example in four consecutive measures, from the third to the sixth inclusive, which are connected through *C*, and between the seventh and eighth measures, connected through *G*. As the progression between the third and fourth measures shows, it is preferable, if the harmony admit of two different progressions, to lead the upper parts in contrary motion to the bass, this giving them a certain degree of independence. The third and fourth measures might also be written thus: 

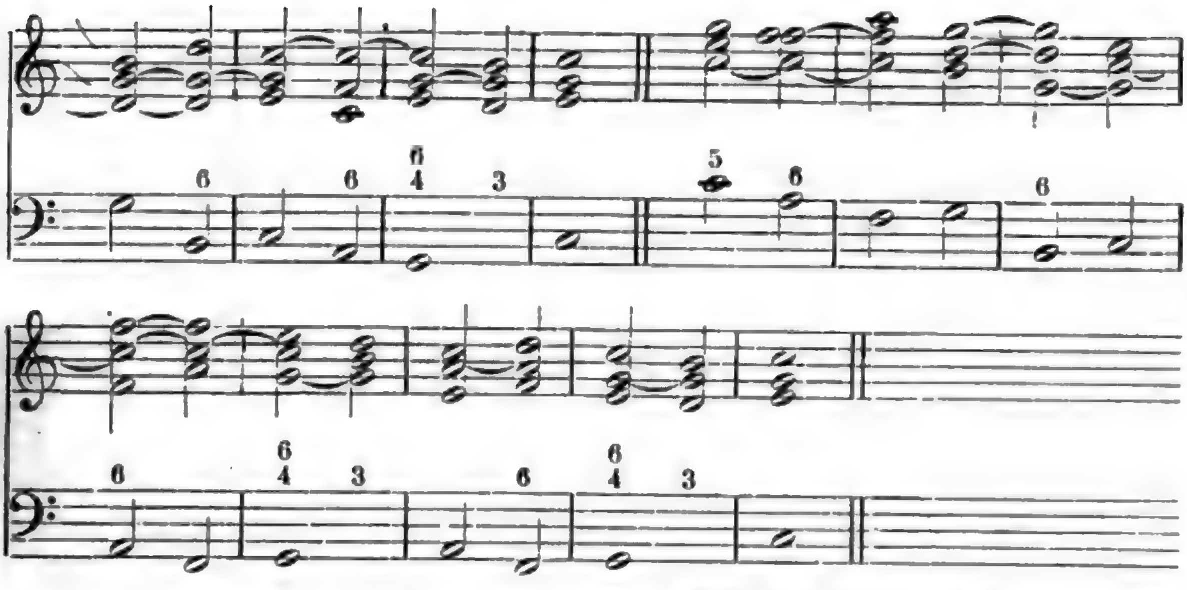
Apart from the occurrence of covered octaves, which are clearly shown by filling out the skip  in the tenor thus: ,

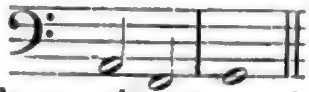
the independence of the several parts is much better preserved by a "contrapuntal" leading (note against note = point against point = *nota contra notam* = *punctus contra punctum*), and forms in the example  the decisive consideration.

The three examples worked out below will suffice to show the learner in what manner the exercises, for which the bass alone is given, are to be written. It should be observed that the figure 3, when it occurs after the chord of the fourth and sixth $^6_4$, is used as an abbreviation of $^5_3$, and consequently indicates the fundamental position of the triad. The figure 3 has therefore a wider signification here than in the first measure of an exercise, where it indicates that the third of the tonic triad is to be taken by the soprano; just as the figures 8 and 5 signify that the soprano is to take the fundamental or fifth respectively. These various cases are illustrated by the following examples:

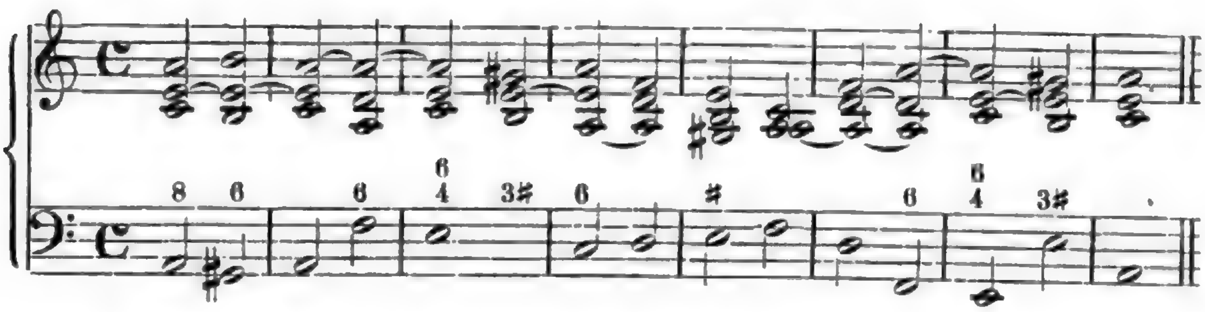
Examples in Major.

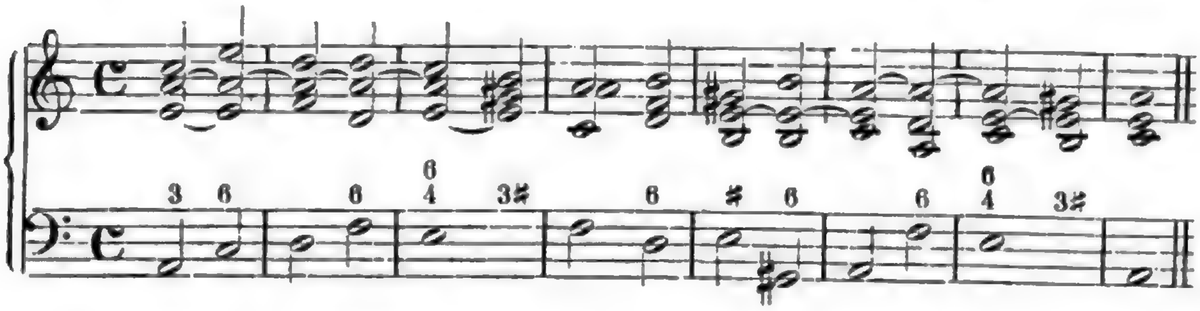


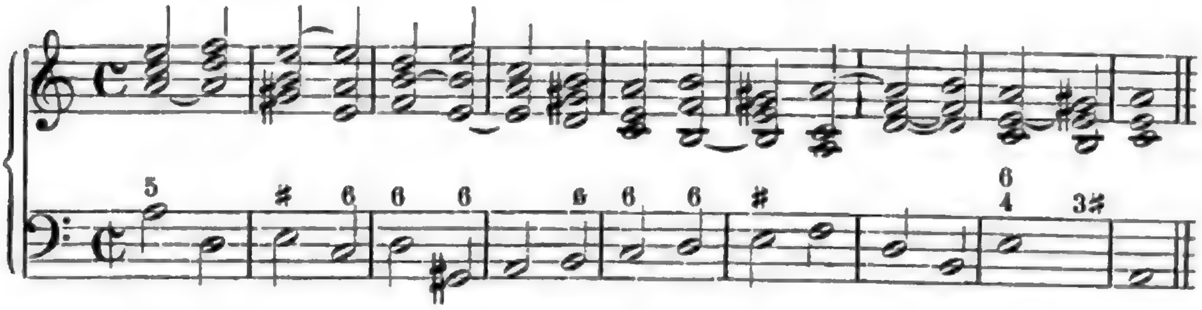


In the last exercise the bass-progression  is repeated intentionally, in order to exhibit different harmonic progressions in combination with the chord of the fourth and sixth.

Examples in Minor.

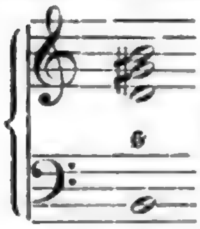




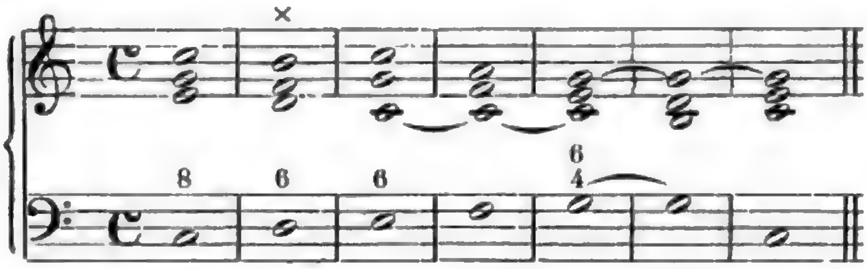


In these examples in minor the manner in which progressions between the second and fifth, and fifth and sixth degrees should be effected must again be called to mind. In regard to the thorough-bass notation, *i. e.*

the figured basses, which are the foundation of the harmony, or the exponents of the chords, it should be remembered, that a sharp beside or a line drawn through a figure signifies that the tone is to be raised a chromatic semitone. In *A*-minor for instance the chord of the sixth derived from the triad of the seventh degree is indicated by a 6̸:



, the sixth of *B* being *g*♯, and not *g*. In this particular chord it is best to double the third *B* in four-part harmony, this tone forming, as third to *D* and *g*♯, a consonance with these latter, and being consequently unhindered in its progression, the dissonant tones *D* and *g*♯ appearing, on the contrary, dependent upon each other. In consonances the separate tones appear as independent; in dissonances as dependent one upon the other. For this reason, when the triad on the seventh degree in major is employed, the third is generally doubled, as, for instance, in the triad *b-D/F* on the seventh degree in *C*-major, the third *D*, as shown below:



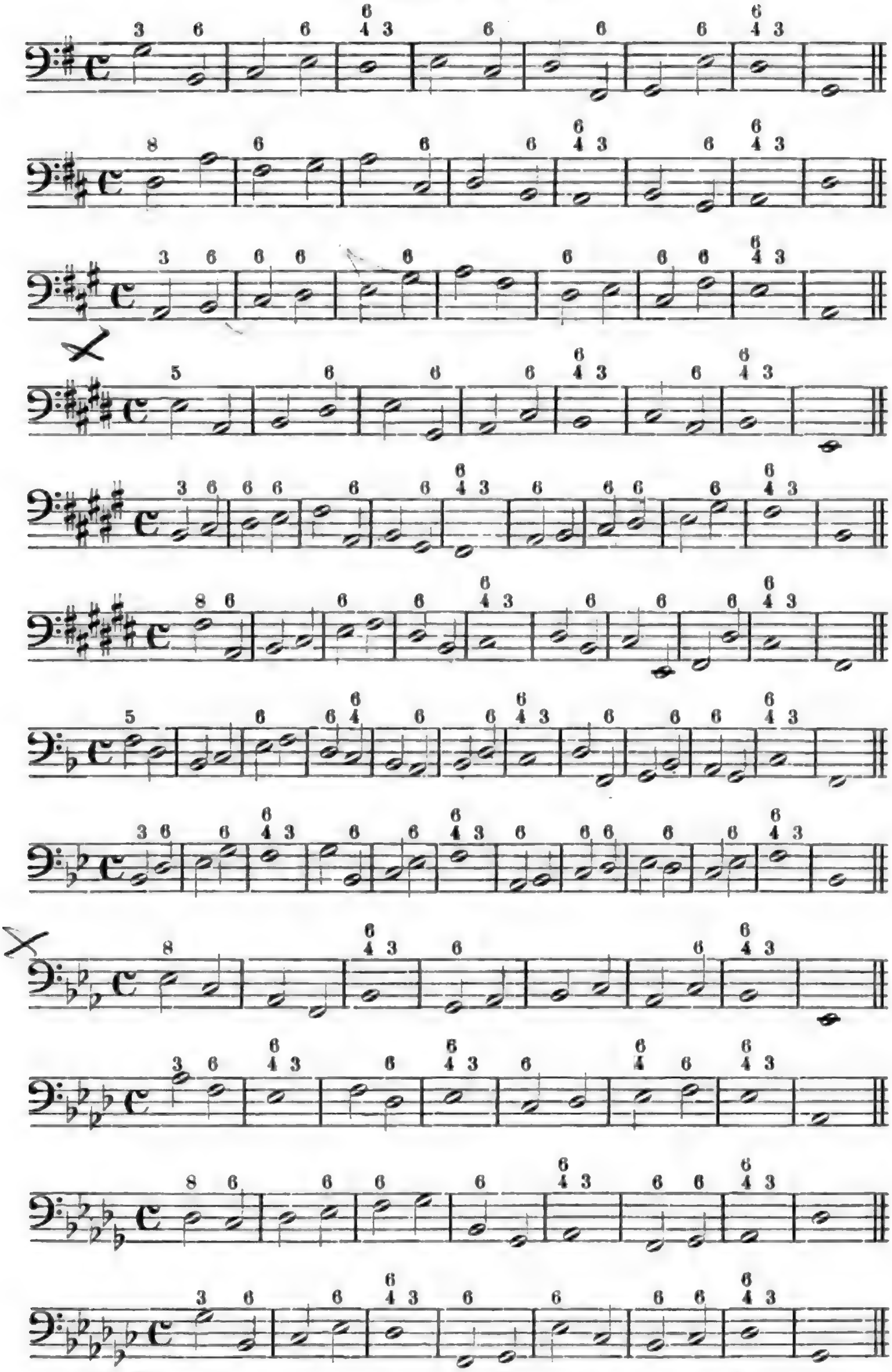
It may also be observed that the seventh degree of the scale is sometimes called the *leading-note* for the eighth.

The pupil should now write out the chords of the sixth, and of the fourth and sixth, *i. e.* the inversions of all the triads, in every key, bearing hereby in mind:

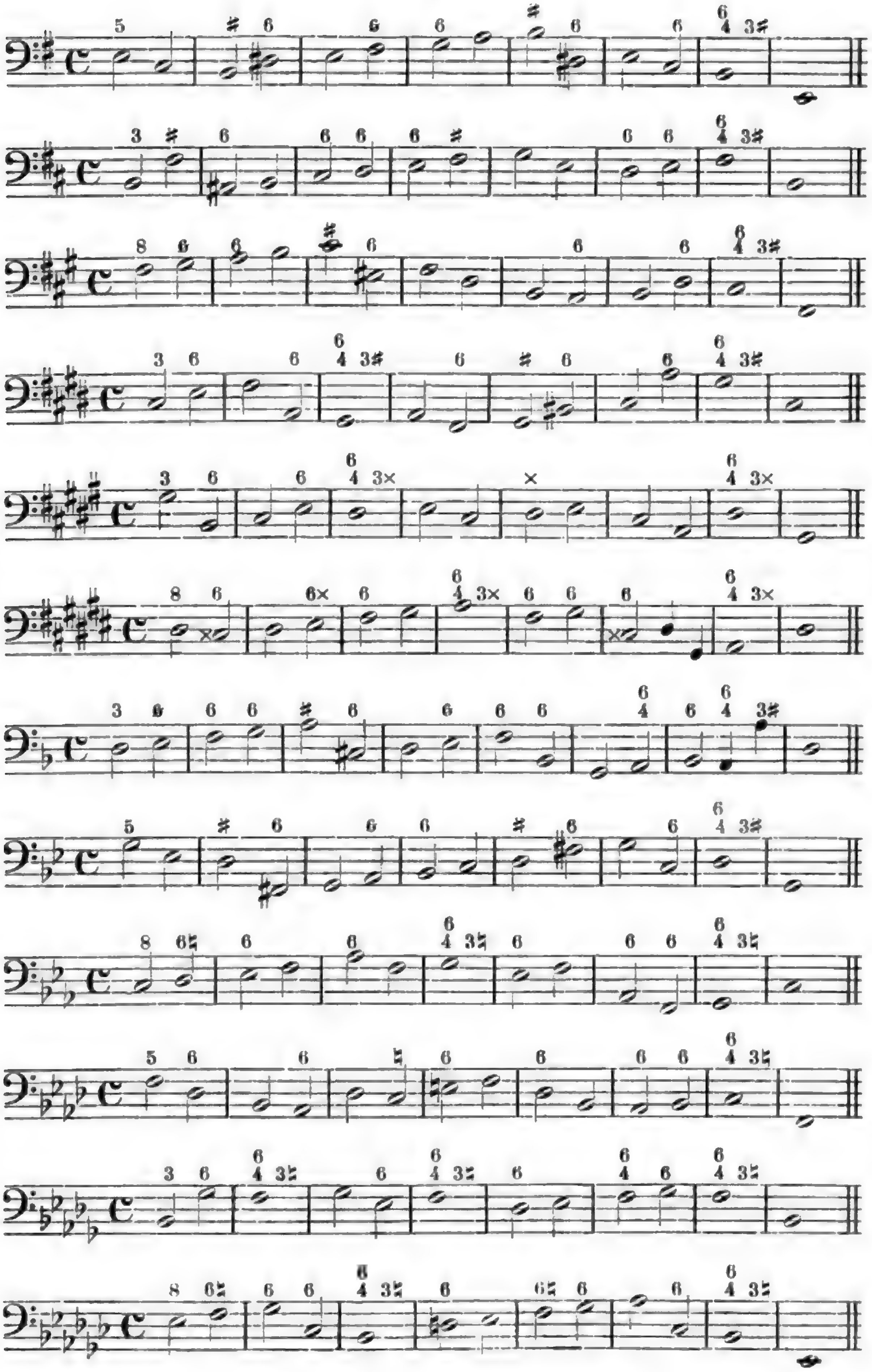
1. The dependence of the other parts on the bass, and the precise designation of the chords by means of the figures 6 and $^{6}_{4}$.
2. The harmonic and melodic preparation of the chord of the fourth and sixth.
3. The position of the chord of the fourth and sixth in the harmonic examples, and the formation of closes in combination with this chord.
4. The character of the diminished triad on the seventh degree, of which the third is most frequently set in the bass, because it forms a consonance with the two remaining tones. This chord therefore generally appears as a chord of the sixth.

After this review the pupil should write the three upper parts to the following basses.

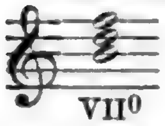
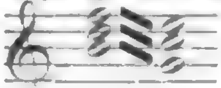
Exercises in Major.



Exercises in Minor.



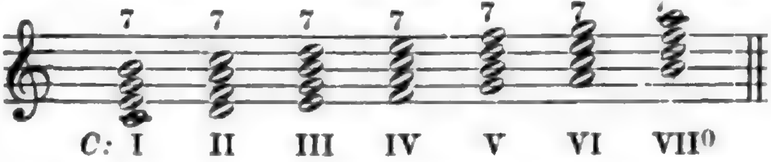
After finishing these exercises the pupil should again examine the structure of the diminished triad.

The diminished triad on the seventh degree of the major scale, , which is indicated, with reference to its position in the scale, by VII⁰, the ⁰ signifying that the fifth is diminished, progresses to the tonic triad thus: . Upon this three-part resolution is based a progression of chords, which must be noticed in treating of the dominant chord of the seventh.

CHAPTER VIII.

Chords of the Seventh.

An examination of the alphabetical formula for the major key *D/F-a-C-e-G-b-D/F* shows us, that any two triads connected through the third form, when combined, a group of four tones*). Such a group is called a "chord of the seventh". The two triads *C-e-G* and *e-G-b* are connected through the third, and constitute, when united, the chord of the seventh *C-e-G-b* on the first degree of the *C*-major scale. In like manner may be formed: — on the second degree the chord of the seventh *D/F-a-C*, by uniting *D/F-a* and *F-a-C*; on the third degree the chord of the seventh *e-G-b-D* from *e-G-b* and *G-b-D*; on the fourth degree the chord of the seventh *F-a-C-e*, from *F-a-C* and *a-C-e*; on the fifth degree the chord of the seventh *G-b-D/F*, from *G-b-D* and *b-D/F*; on the sixth degree the chord of the seventh *a-C-e-G*, from *a-C-e* and *C-e-G*; and on the seventh degree the chord of the seventh *b-D/F-a*, from *b-D/F* and *D/F-a*. As given in notes, in regular order, the chords of the seventh in the key of *C*-major appear as follows:



The chord of the seventh based upon the fifth degree is called the dominant or principal chord of the seventh. The name "dominant" chord of the seventh is derived from its position and structure, it being formed

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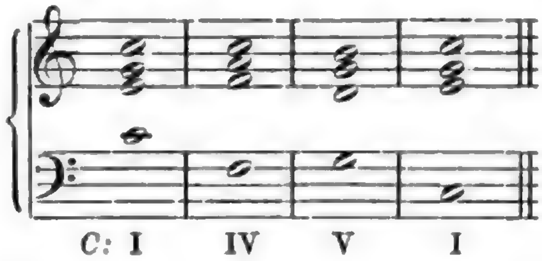
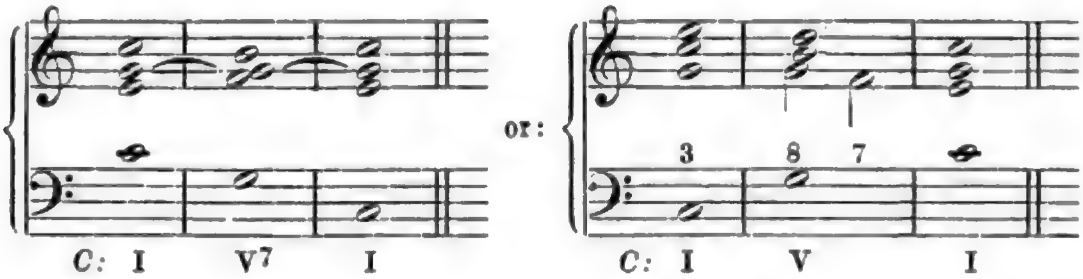
*) The German "Vierklang" might well be rendered by *tetrad*, as "Dreiklang" is translated by the analogous word *triad*. Tr.

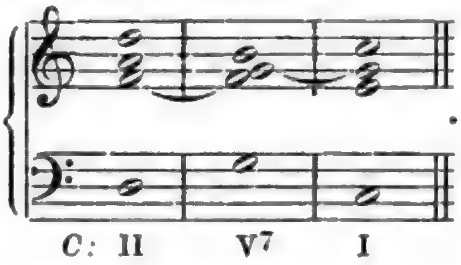
by uniting two triads connected through the third, *G-b-D* and *b-D/F*; *G-b-D* is the dominant triad, and *b-D/F*, besides containing the third (*b*) and the fifth (*D*) of the dominant triad, has also *F*, the fundamental of the sub-dominant triad. This chord is therefore rightly named "dominant" chord of the seventh. It is called "principal" chord of the seventh on account of its character, which adapts it for use in the formation of perfect closing cadences. If we abridge the perfect close:



but still desire to employ the principal constituent elements of the dominant and sub-dominant triads, the close may be formed by the aid of the dominant chord of the seventh, thus:



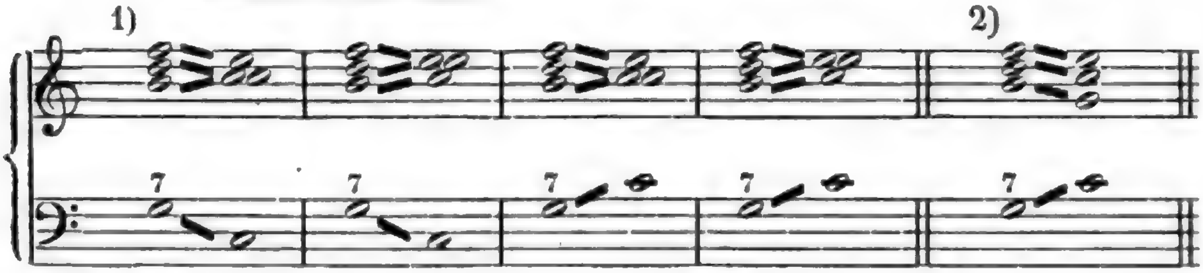
In thorough-bass the figuring 8 7 indicates that the octave is to be followed by the seventh. The triad on the second degree, containing tones in common with the sub-dominant and dominant triads (fundamental and third of the former, and fifth of the latter), may be used advantageously in forming closes, as shown in the following example:



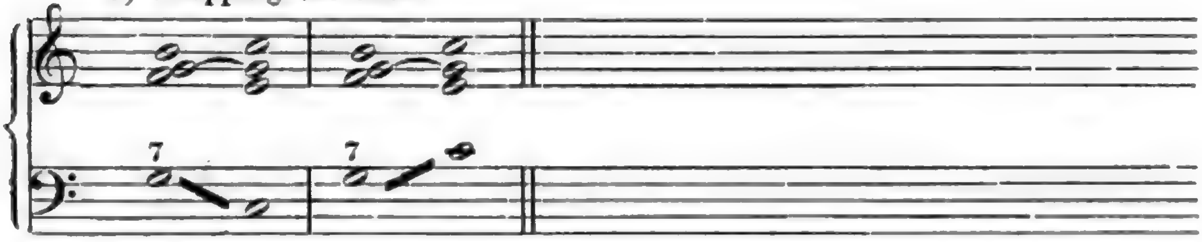
In the above examples the dominant chord of the seventh is prepared, either the fundamental or seventh being present in the chord immediately preceding. In the strict style such progressions are best effected through the above preparations; in free composition this chord is often prepared through the fifth, or may even enter without preparation. Such an employment of the chord would, however, be out of place in harmonic exercises in which the principle of the interconnection of the chords is to be rigidly observed. In free composition, where the composer frequently wishes to take his hearers by surprise, his inventive faculty may be allowed full scope. If he knows and respects the laws of harmony, the liberties he takes can be shown to be based upon those laws. Only such a derivation from fundamental principles, which presupposes a thorough acquaintance with the same, can justify the liberties to be found in the works of the great masters, which do not annul the laws, but rather serve to confirm them.

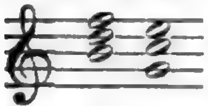
After learning the preparation of the chord of the seventh, its resolution may be taken up. Every dissonance seeks to resolve itself into, or progress to, a consonance. The relations of the separate tones to each other must here be carefully considered, the mutual dependence of the tones forming a dissonance being the cause of their effort to gain that independence which is characteristic of the consonance. For this reason it is also rightly maintained, that the nature of the dissonance is unrestful, while that of the consonance is restful. The resolution of the dominant chord of the seventh may be accomplished in either of the following ways:

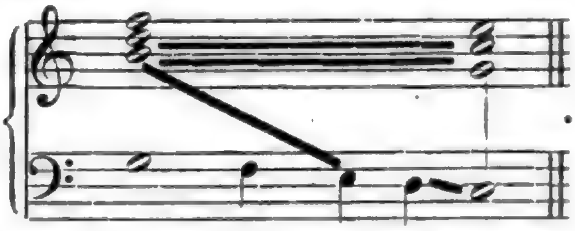
Resolution to the tonic:



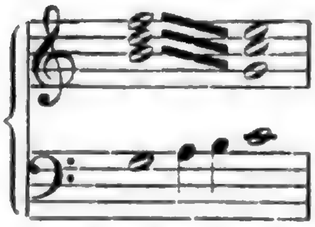
3) **Dropping the fifth:**



In this resolution towards the tonic in major we should observe, that the fundamental tone of the chord of the seventh progresses either a perfect fifth downwards or a perfect fourth upwards; also that the seventh progresses a semitone downwards, the third a semitone upwards, and that the fifth, forming as it does a consonance with all the other tones of the chord, is at liberty to move in either direction, a whole tone upwards or a whole tone downwards. In ex. 2 the third progresses a third downwards, which movement is explained by the progression of triads  given before, the bass to which is led in contrary motion. It would here not be consistent with our ideas of pure harmonic progression to lead the bass a perfect fifth downwards; because, even leaving the covered fifths and octaves, marked by dashes, out of consideration, the resolution of the seventh *F* to *e* is to a certain extent anticipated in the progression of the bass, as shown by the interpolated notes:



The resolution resulting from leading the bass in contrary motion

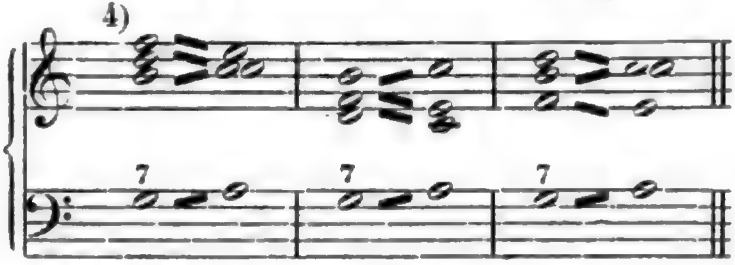


is open to none of these objections.

Finally it may be remarked that the fifth is often dropped from the dominant chord of the seventh in forming a close, it being necessarily followed by a full triad when employed in this form.

Besides these resolutions to the tonic the resolution to the sixth degree of the scale claims our notice.

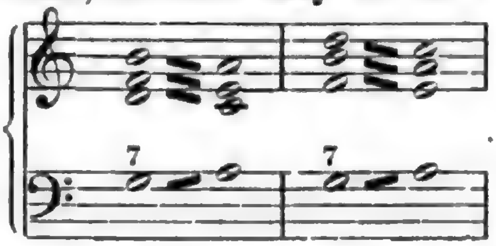
Resolution to the triad on the sixth degree.



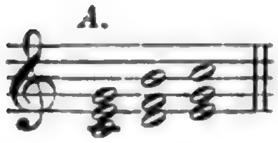
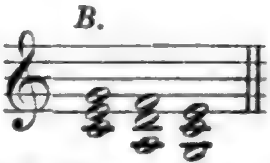
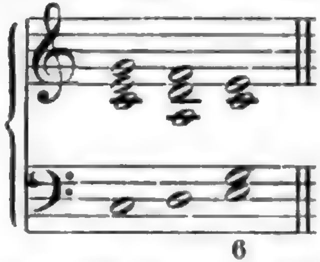
Here the bass progresses a whole tone upwards, the seventh a semitone downwards, the third a semitone upwards, and the fifth a whole tone downwards.

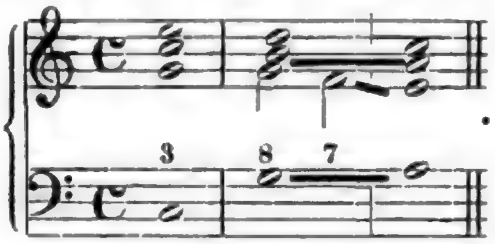
If the parts be so arranged that the third and seventh form the interval of the augmented fourth, the third may be led a whole tone downwards

in the resolution:

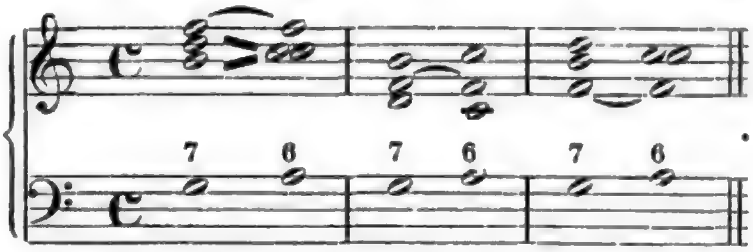


This resolution is based upon the usage of employing successions of fourths accompanied by the third in strict harmony, a custom which may be regarded as justifiable in three-part harmony even where the chord of the seventh is not employed, *e. g.* in the following progression:

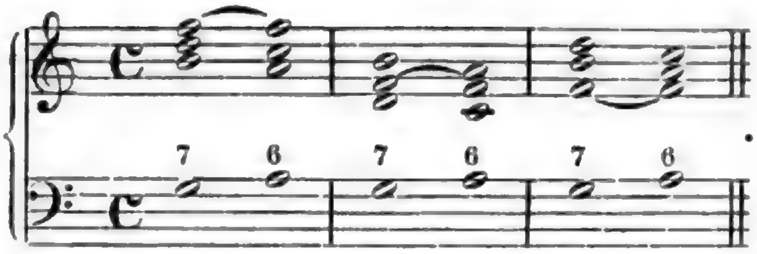
A. , whereas this progression: B. , on account of the succession of chords of the fourth and sixth, requires a bass. Successions of chords of the sixth occur in the earliest attempts at harmonic composition; they are established in practice as an 'agreeable harmonic progression, which can, however, as all the parts move in the same direction and consequently without the variety obtained through their opposition, be but sparingly used in polyphonic composition. Two chords of the fourth and sixth cannot, on the other hand, be used successively without a bass, by reason of the modulatory power peculiar to this chord; by adding a bass the progression marked *B* may be given thus: . A basis always being present when the chord of the seventh is employed, it is easier in such cases to introduce successions of fourths accompanied by the third, as for instance in this progression:



The resolution of the dominant chord of the seventh to the triad of the fourth degree has also an agreeable effect, and may be exhibited in the following forms:

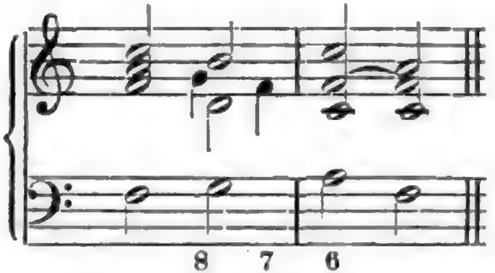


In this case also the resolution in which the third moves downwards is correct:

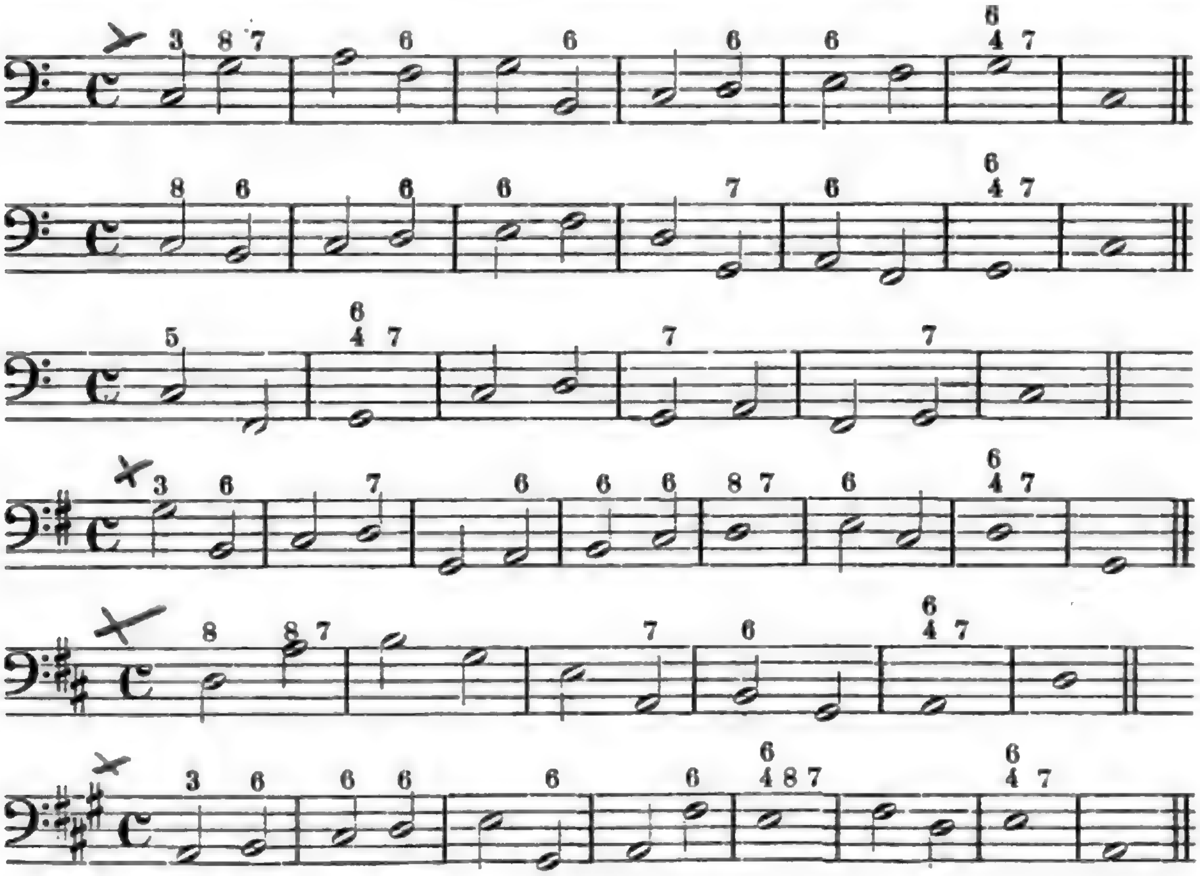


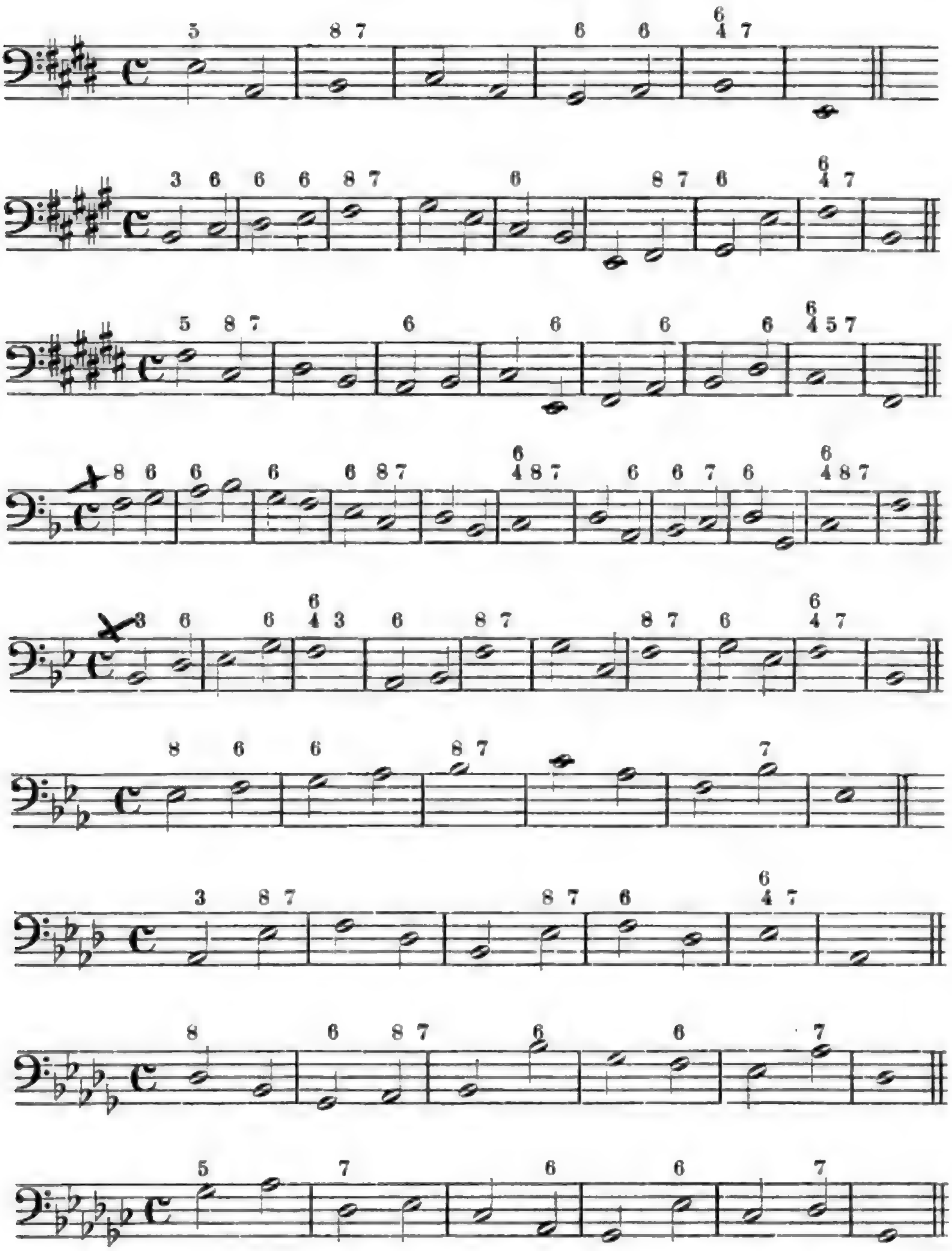
Respecting the resolution to the triad of the fourth degree we should therefore observe, that the bass moves downwards a whole tone, the seventh remains stationary, the fifth moves a whole tone upwards, and that the third may progress either a semitone upwards or a whole tone downwards. After sufficient practice in the various resolutions of the dominant chord of the seventh, in all major keys, the pupil should work out the exercises given below. It may be remarked here that the resolutions of the dominant chord of the seventh to the fourth and sixth degrees were formerly regarded as deceptive closes.

In working out the following exercises it must be noticed particularly, with reference to their *rhythmical* form, that the use of the ligature is only allowable: (1) between two notes of like value, as 𝅝 | 𝅝, or 𝅗𝅥 | 𝅗𝅥, or ♩ | ♩; (2) in case the second note is of shorter duration than the first, *e. g.* : Here it would be incorrect to tie the quarter-note *f′* in the first bar to the half-note *f′* immediately following.



Exercises in the Resolutions of the Dominant Chord of the Seventh in Major.



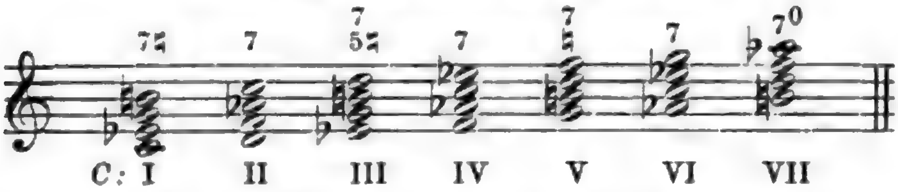


In minor keys the dominant chords of the seventh are the same as in the major keys having the same tonic, and the progressions of this chord are alike in major and minor. Special attention must, however, be paid to the third, which, in resolutions to the triads on the fourth and sixth degrees, can never progress downward; the step of the augmented second resulting from such a progression not being allowable in strict harmony. The following examples confirm the above statements.

Key of *C*-minor.

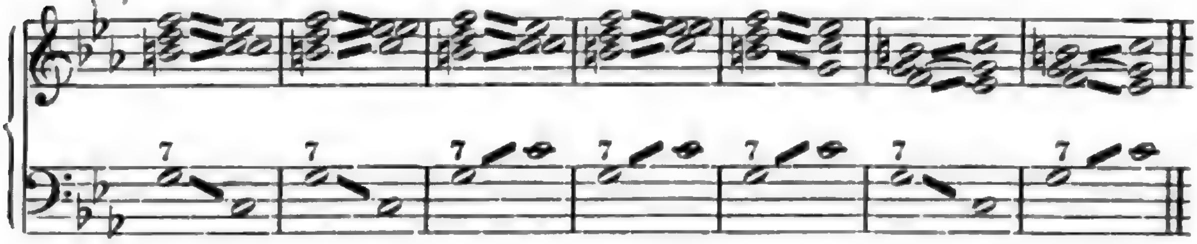
D/*F*-*a*♭-*C*-*e*♭-*G*-*b*-*D*/*F*

Chords of the seventh in *C*-minor.

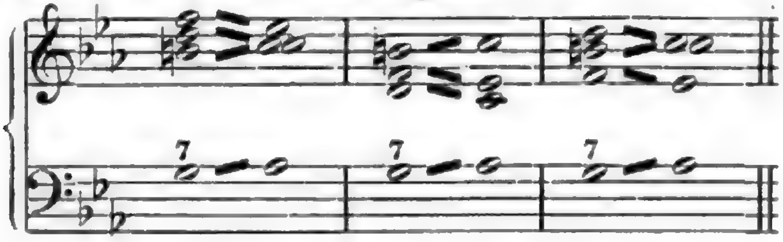


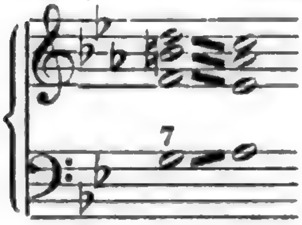
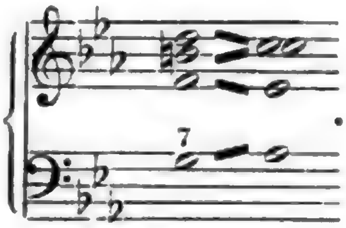
Resolutions of the dominant chord of the seventh in *C*-minor.

(1) To the tonic.

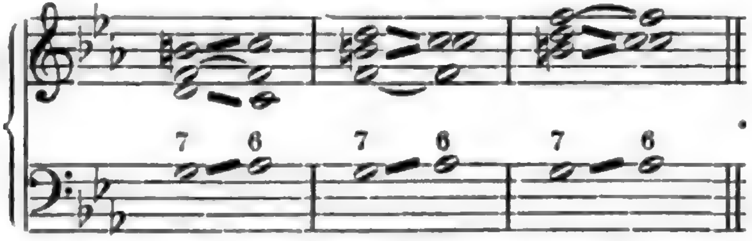


(2) To the triad on the sixth degree.



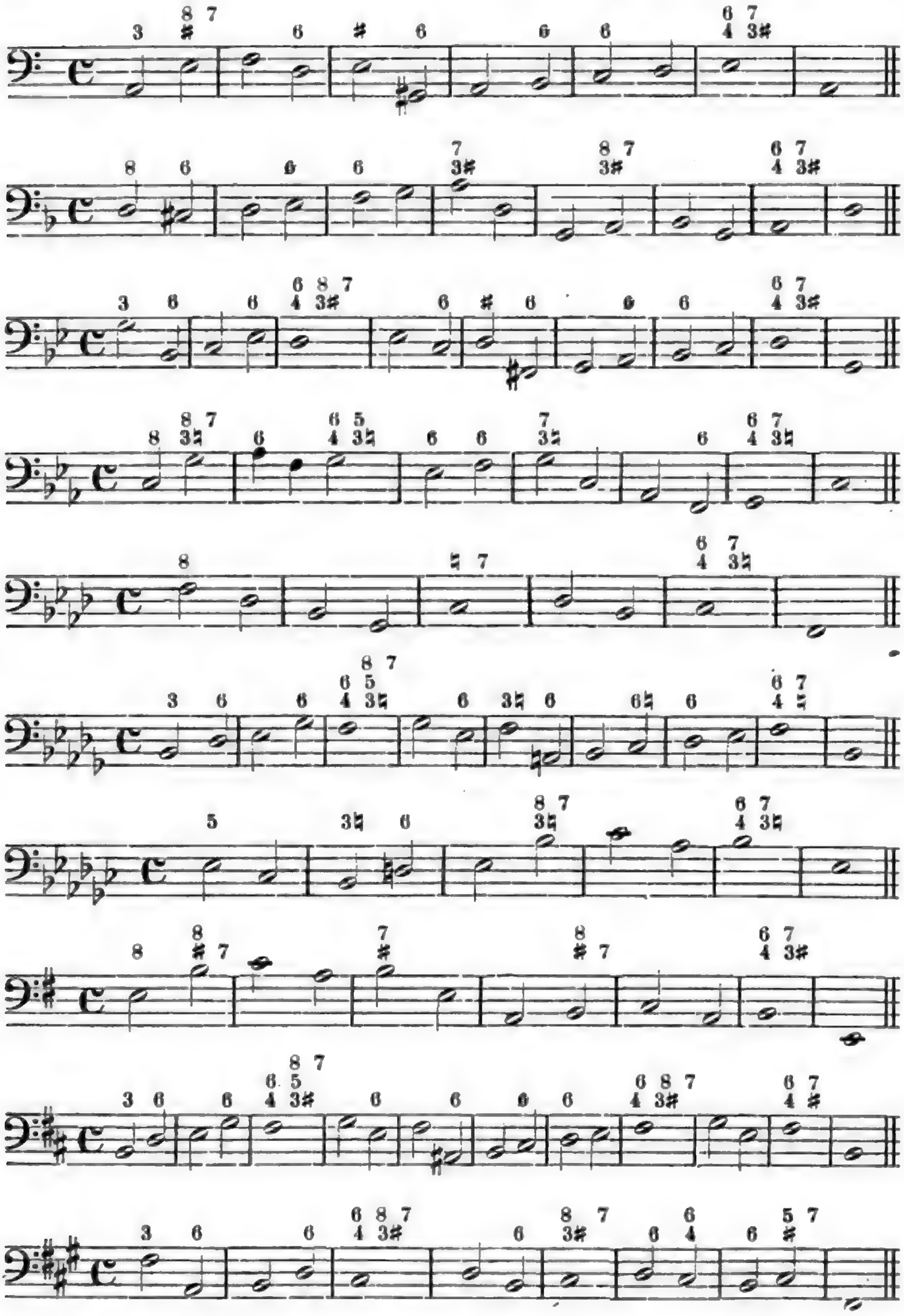
It should be carefully noticed, that the third *b* can never progress to *a*♭, *b* - *a*♭ forming an augmented second; such a progression as  , is therefore entirely incorrect, although often written by learners instead of the correct progression  .

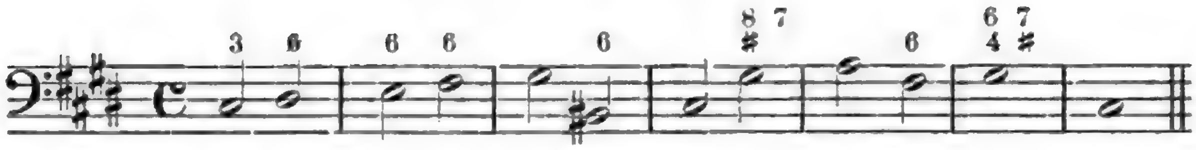
(3) To the triad on the fourth degree.

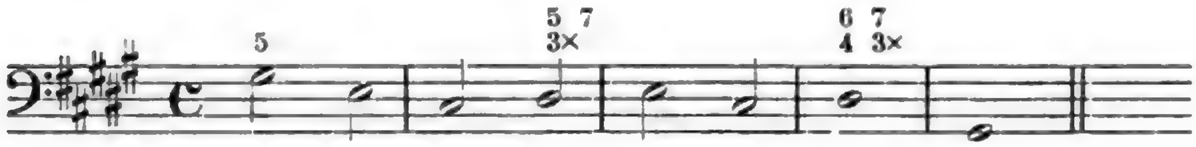


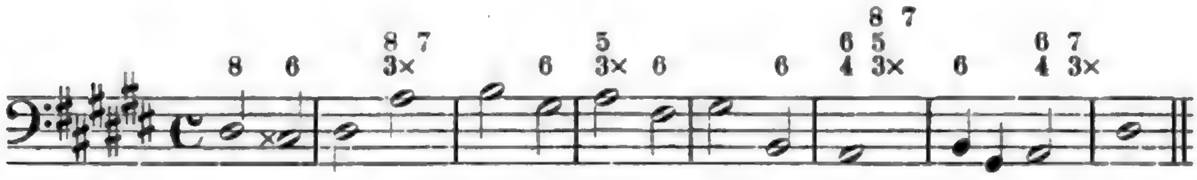
After making himself thoroughly acquainted with the chords of the seventh in all minor keys, by writing them out, the pupil should practice the resolution of the dominant chord of the seventh given above. He will then be able to work out the exercises without difficulty.

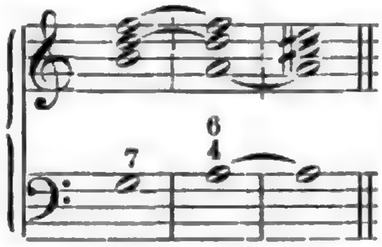
Exercises in the Resolutions of the Dominant Chord of the Seventh in Minor.



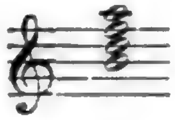


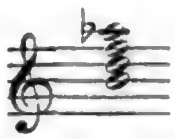




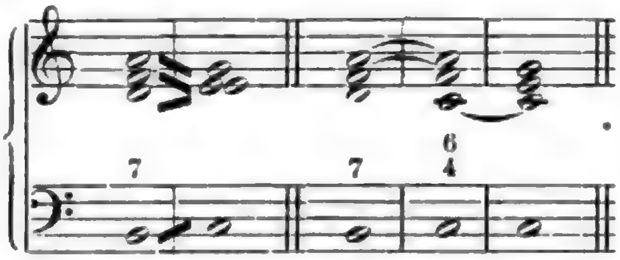
It remains to be remarked, that the dominant chord of the seventh may also progress to the chord of the fourth and sixth derived from the triad on the second degree. Instead of this second degree the fourth degree of the relative minor key [*i. e.* that minor key, the tonic triad of which is connected through the third with the tonic triad of the original major key on the sub-dominant side] is frequently written for the purpose of modulating, *e. g.*:  . As the subject of modulation will be treated at length further on, the progressions of the other chords of the seventh are now given without modulation.

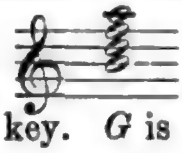
Of these chords, sometimes called "subordinate chords of the seventh", that based upon the seventh degree is to be first examined. It is, both in major and minor, a diminished chord of the seventh.

In the key of *C*-major *b-D/F-a*:  .

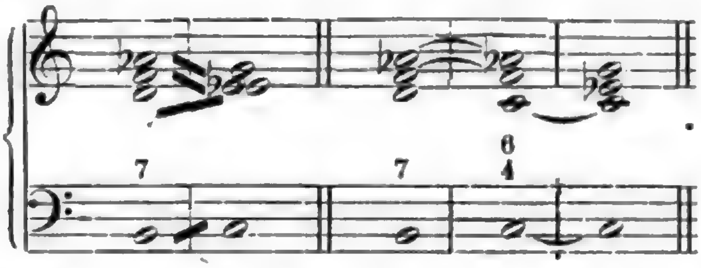
In the key of *C*-minor *b-D/F-a♭*:  .

The chord of the seventh on the seventh degree in *C*-major must be carefully distinguished from the chord of the seventh on the second degree of the relative minor key (*A*-minor), *B/D-f-A*. The alphabetical formula for the key of *A*-minor, *B/D-f-A-c-E-g♯-B/D*, shows the difference immediately, which cannot be done by means of notes. In practice the difference is dependent upon the modulation. The chord of the seventh on the seventh degree needs no preparation, and finds its resolution in the tonic triad. This resolution may be retarded by the chord of the fourth and sixth derived from the sub-dominant triad. These resolutions are shown in the following example:



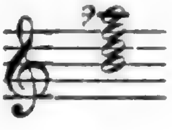
It was formerly a generally received opinion, that the seventh should be employed only in the highest part; but this practice was not supported by any harmonic law. The prevailing uncertainty as to the nature of this chord probably gave rise to the doubts regarding its correct employment. As already shown by the alphabetical formula, it is a decided mistake to consider the chord of the seventh on the seventh degree in any major key as the same chord as the chord of the seventh on the second degree of the relative minor key. In *C*-major *b-D/F* and *D/F-a* are diminished triads; in *A*-minor, on the contrary, only *B/D-f* is a diminished triad, *D-f-A* being a minor triad, and, as based on the subdominant, one of the three fundamental triads of the key. In the key of *A*-minor the chord of the seventh  cannot progress to *C-e-G*, this triad not being found in that key. *G* is excluded in harmonic combination by *g*♯, the third in the dominant triad *E-g*♯*-B*. A confounding of these two chords is therefore a blunder, which is unfortunately often made by celebrated teachers of harmony.

In minor the chord of the seventh on the seventh degree is formed from two diminished triads, *e. g.* in *C*-minor (= *D/F-a*♭*-C-e*♭*-G-b-D/F*) from *b-D/F* and *D/F-a*♭, the latter being more decidedly diminished than *D/F-a* in *C*-major. Its resolution is governed by the same laws:

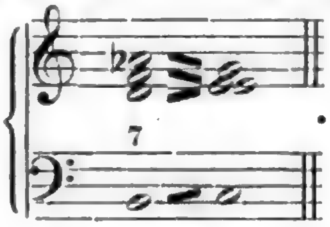


This diminished chord of the seventh is also found in the *combined* major and minor mode [see p. 71] for which reason its resolution to the *major* tonic triad *C-e-G* is allowable.

Combined key of *C*: *D/F-a*♭*-C-e-G-b-D/F*.

Dimin. chord of the seventh on the seventh step: .

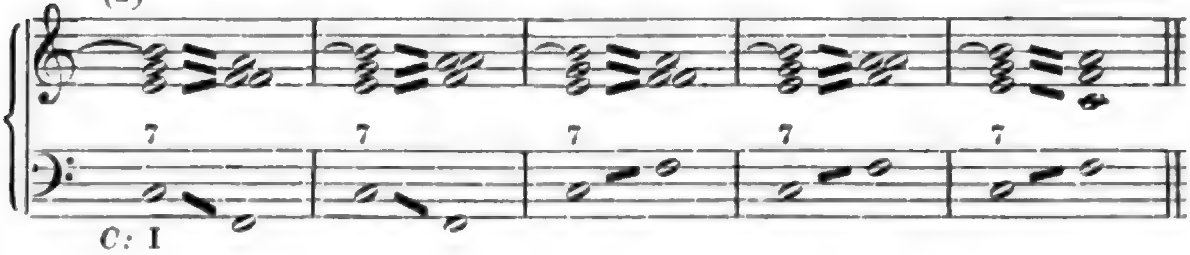
Resolution of this chord in the combined major and minor key of *C*:

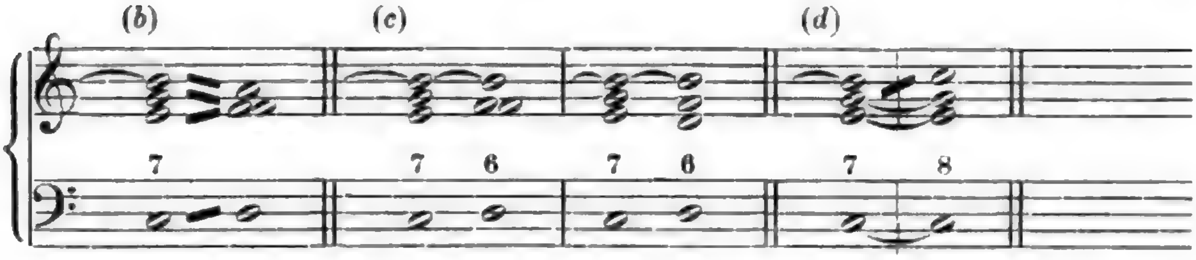


The remaining chords of the seventh are distinguishable one from the other chiefly through difference in compass. In the major mode subordinate chords of the seventh with major seventh are found on the first and fourth degrees; with minor seventh on the second, third and sixth degrees. We have already noticed that the dominant or principal chord of the seventh has a minor seventh. This chord contains a diminished triad, whereas the chords of the seventh on the first, third, fourth and sixth degrees are formed by uniting a major with a minor triad. The compass of any chord of the seventh is dependent upon the relative positions of these triads; if the major triad be below and the minor triad above, the chord has a major seventh; if the minor triad be below the major, the chord has a minor seventh. The chord of the seventh on the second degree, the compass of which is a minor seventh, contains the slightly diminished triad *D/F-a*, above which lies the major triad *F-a-C*. The dissonant element being stronger in the subordinate chords of the seventh than in the dominant chord of the seventh, the former must be more carefully prepared. The diminished chords of the seventh, which can enter without preparation, form an exception to this rule; the dominant chord of the seventh needs only to be prepared through the fundamental, the presence of which in the same part of the preceding chord permits the free entrance of the seventh. The subordinate chords of the seventh on the first, second, third, fourth and sixth degrees must, on the contrary, be prepared through the seventh, the entrance of the dissonances otherwise appearing too sudden. The resolution takes place in accordance with the rules for the resolution of the dominant chord of the seventh. The seventh can be led upwards only in chords having a major seventh, the fundamental in this case remaining stationary and fixing, as it were, the character of the key as harmonic basis of the chord.

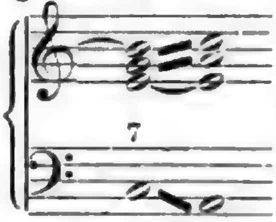
The progressions of the subordinate chords of the seventh to the triads in major are given below, the notes prepared being marked by ligatures (‿).

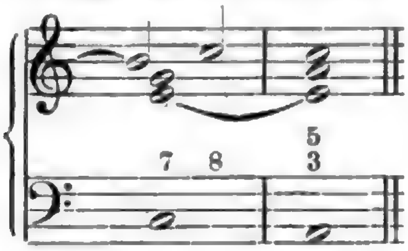
1. Chord of the seventh on the first degree.



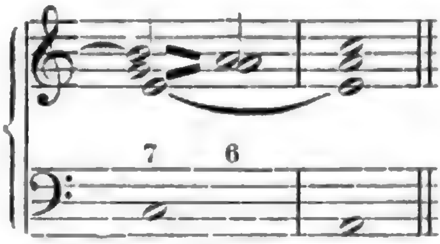


The succession of chords under *c* cannot be regarded as a resolution, but rather as a simple progression to a dissonant triad.

Various progressions are often given, in manuals of harmony, under the head of "exceptions". On closer inspection these prove to be, not exceptions, but contracted progressions based upon the rules of strict composition. The progression  for instance is frequently designated as permissible, although an "exception". Even a scientific theorist would not consider this progression as incorrect in free composition, being able to explain it thus:



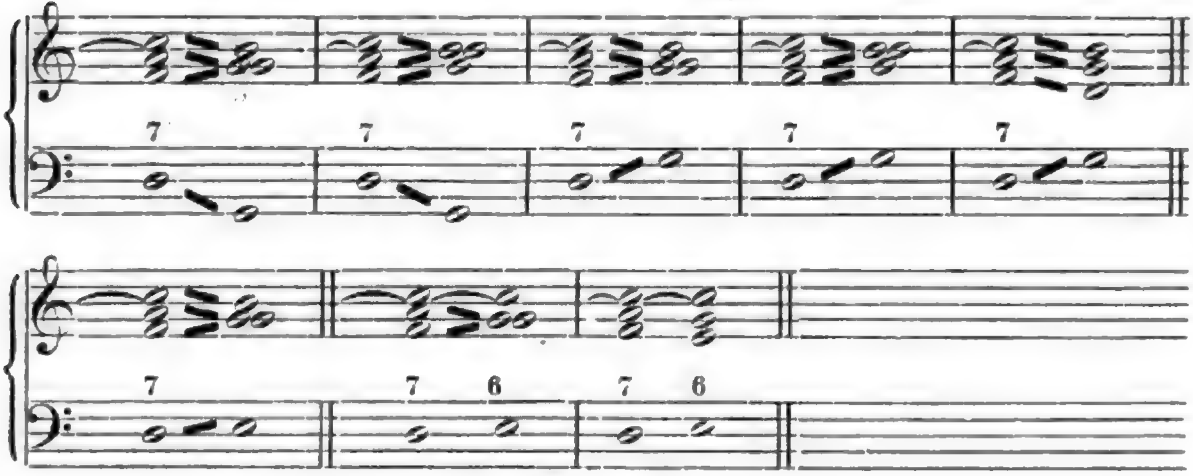
, or thus:



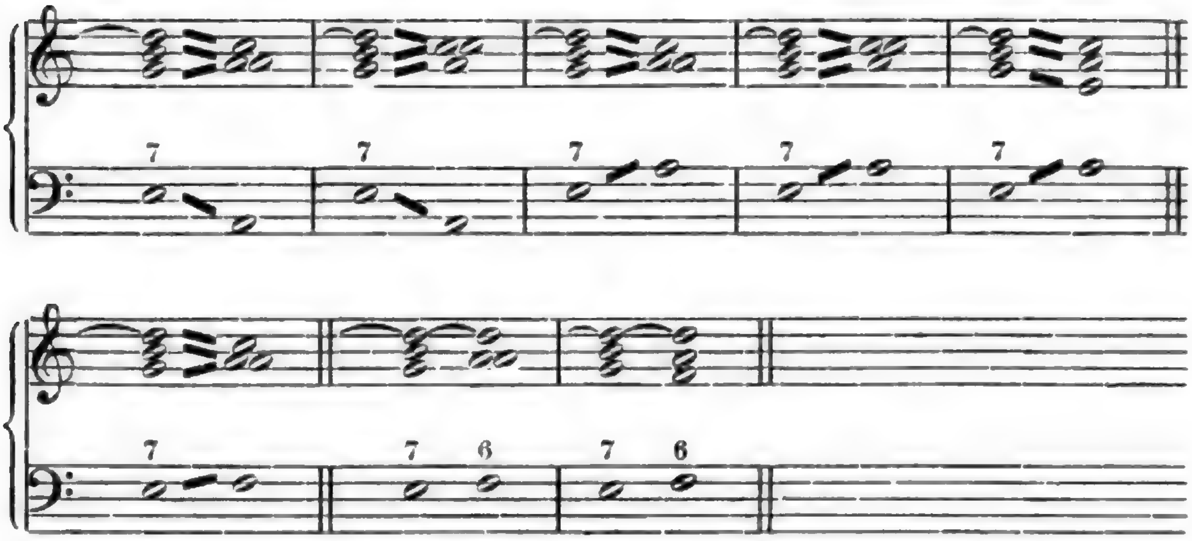
.

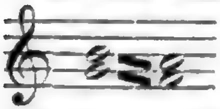
In the first progression a connecting link, which would appear necessary in strict composition, is merely left out; this progression may therefore be said to be based upon, or derived from, a strict harmonic progression. Only such liberties as can be proved scientifically to be based upon the laws of strict composition are allowable in free composition. Arbitrary and unlawful digressions can no more be suffered in the realm of harmony than in the body politic.

2. Chord of the seventh on the second degree.

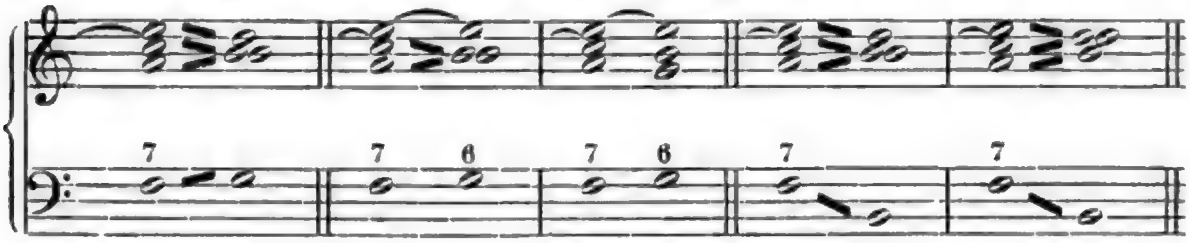


3. Chord of the seventh on the third degree.

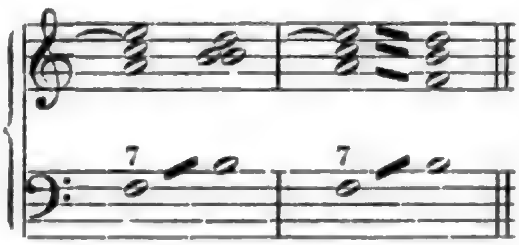


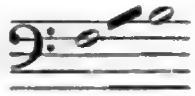
In the last resolution we find a succession of two major thirds: , the correctness of which was denied by old-fashioned theorists. This view was probably a tradition handed down from the period in which the antique musical system prevailed, and is based on no scientific principle. At that time the major third was considered a melodic, but not a consonant, interval, the Pythagorean proportion of 64 : 81 being then accepted as correct, whereas in modern music the proportion of 4 : 5 is established.

4. Chord of the seventh on the fourth degree.

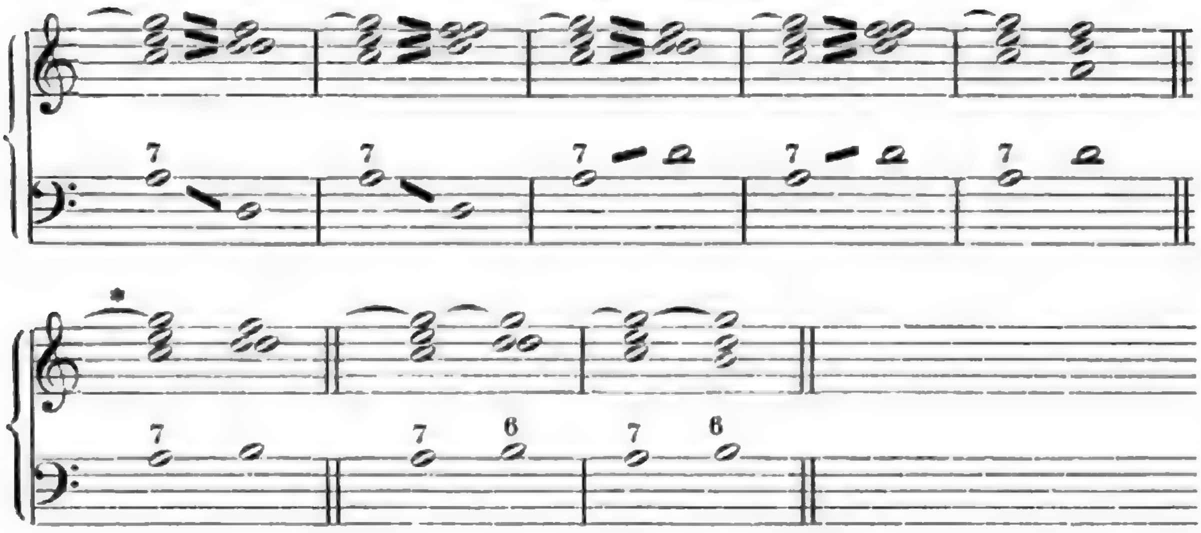


The last progression is undoubtedly that most seldom employed, although entirely correct. It would, however, be very faulty to lead the bass upwards in the following manner:



the tritone thus formed in the melodic progression of the bass  being decidedly incorrect.

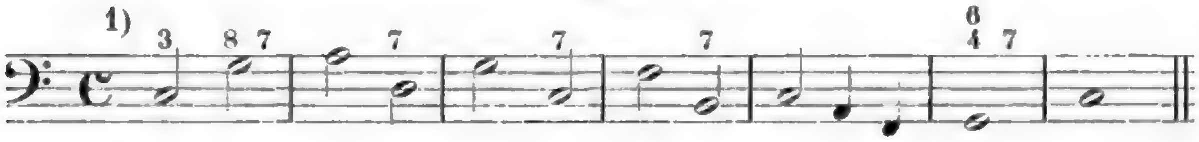
5. Chord of the seventh on the sixth degree.



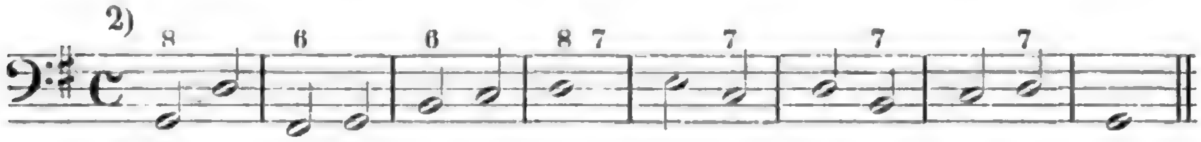
The progression marked * cannot be regarded as a resolution; for *b-D/F* is a dissonant triad, and it is a general rule that a progression to a dissonance is not a resolution.

Exercises in progressions between chords of the seventh and the triads in major.

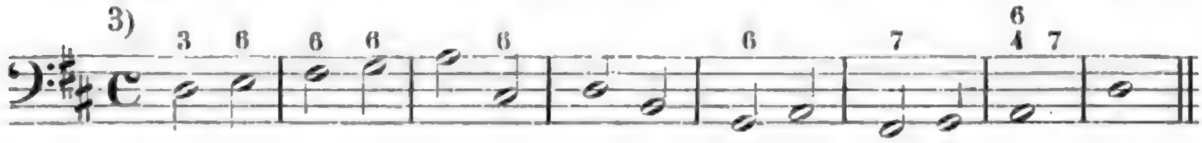
1)



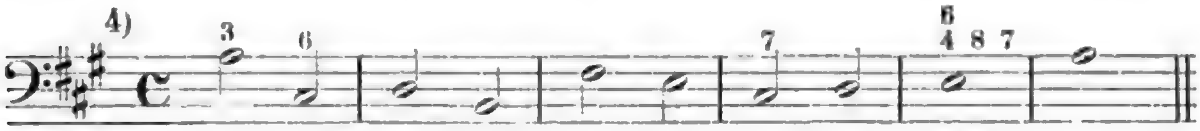
2)



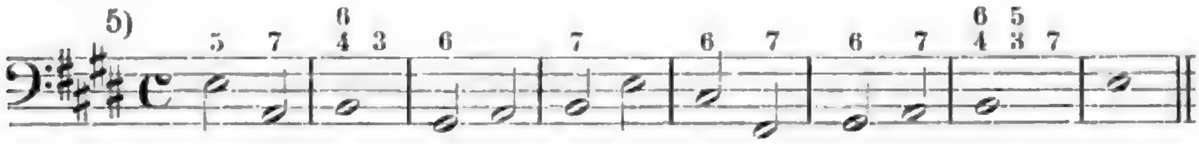
3)



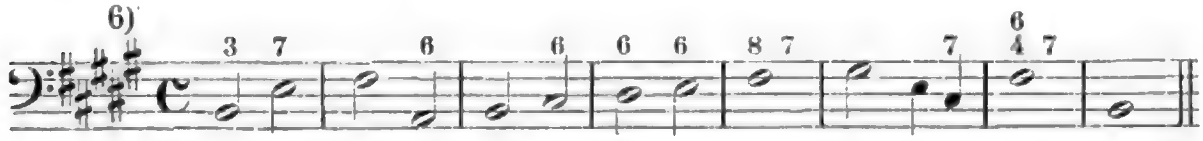
4)



5)



6)



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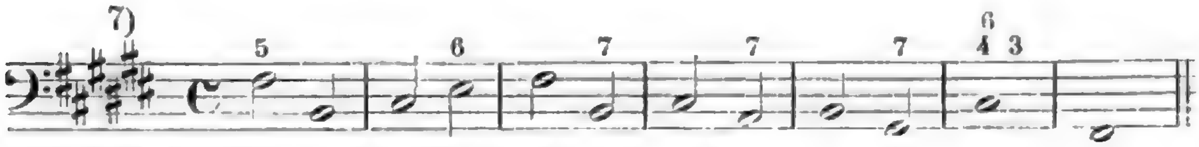
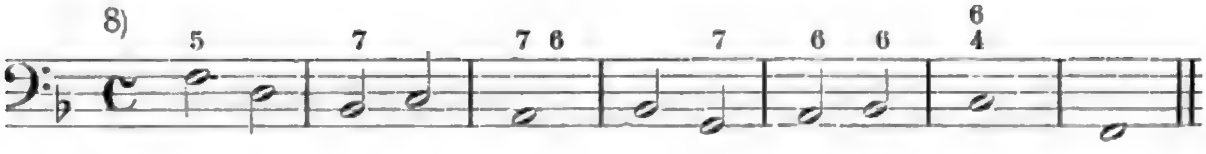


8)



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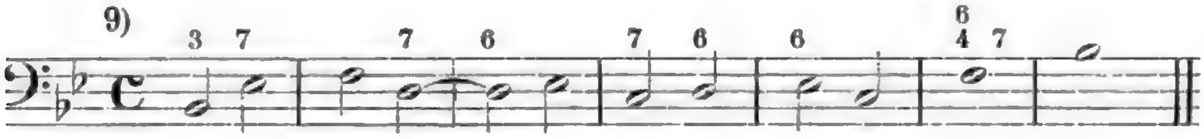
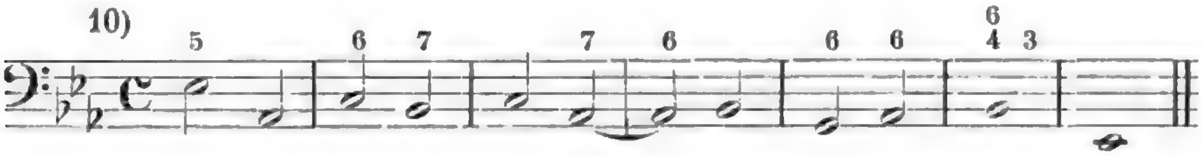
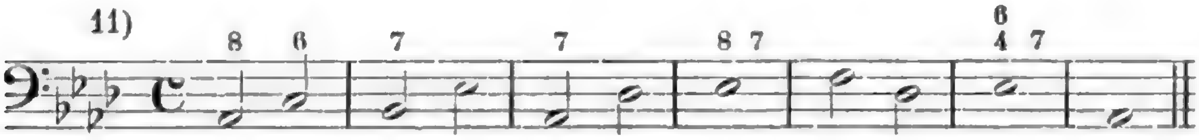


10)



11)



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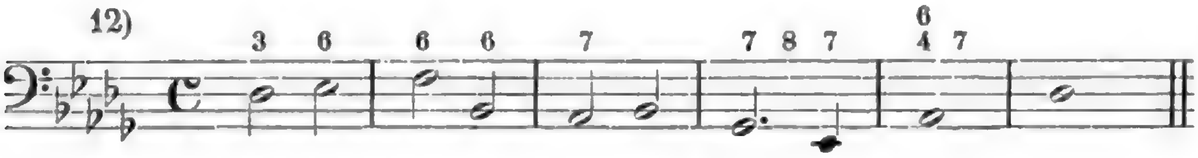
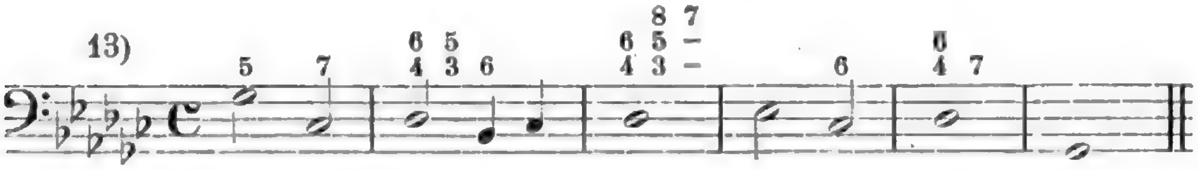


13)

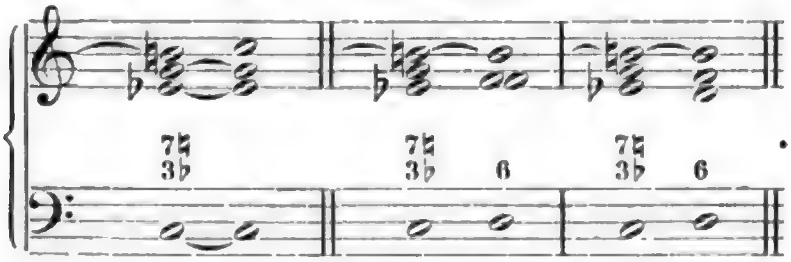


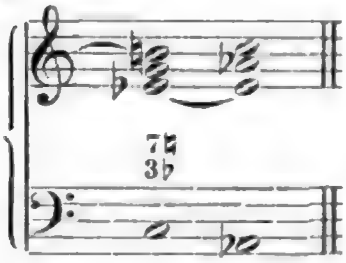
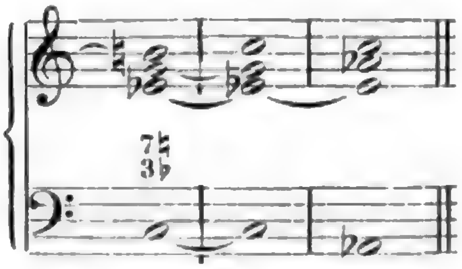
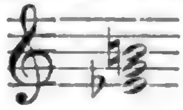
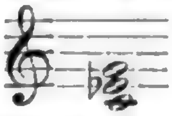
CHAPTER IX.

Chords of the Seventh in Minor.

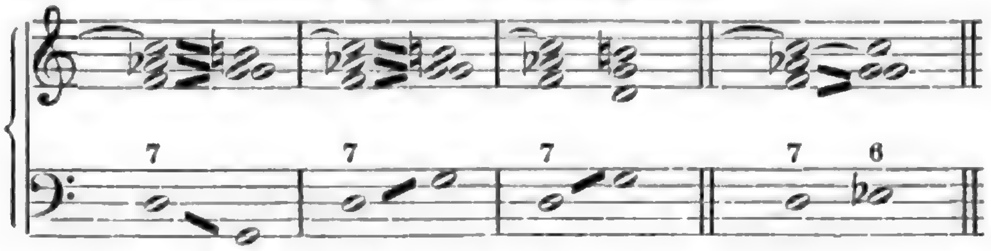
The structure of the chords of the seventh in the minor mode is, of course, also dependent upon the form of the triads, every chord of the seventh being composed of two triads connected through the third. The chord of the seventh is, to quote an expression of HAUPTMANN'S, a coalescence, so to speak, of two triads connected through the third. The dominant chord of the seventh, in major and minor keys beginning on like tones, is the only chord of the seventh which these keys have in common, as was observed above; the remaining chords of the seventh differing from each other, because composed of triads of different structure. In minor the progressions of the subordinate chords of the seventh to triads are as follows:

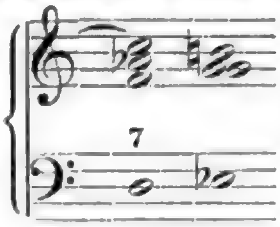
1. Chord of the seventh on the first degree.

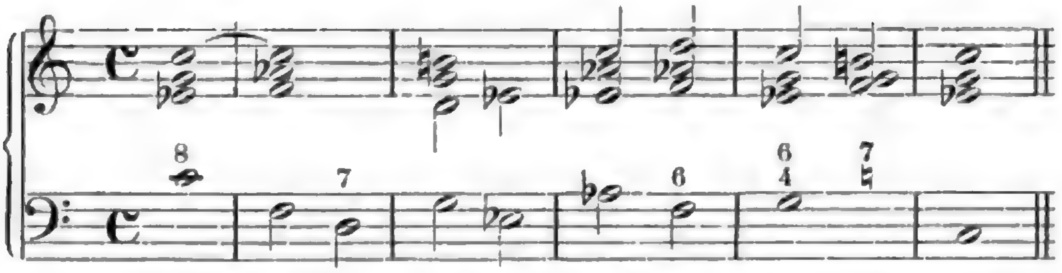


In the harmonic minor mode no further progressions to triads are possible in strict composition, as the seventh cannot be led downwards on account of the augmented second *b-a*♭. In free composition the progression  which is based upon  is frequently written; it is all the more allowable because the augmented triad , which, together with the triad  connected with it through the third, forms this chord of the seventh, may progress quite correctly to the triad connected with it through the fifth, *i. e.* *a*♭-*C*-*e*♭ on the sixth step.

2. Chord of the seventh on the second degree.

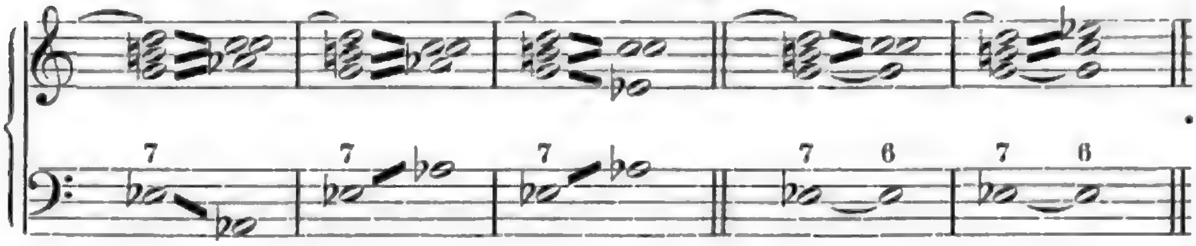


The progression to the triad of the third degree, which at once strikes our musical perception as insufficiently prepared, should be avoided as incorrect in the strict style, on account of the free entrance of the augmented triad. The example  shows the free entrance of this triad, which requires preparation. A succession of chords like the following, the augmented triad entering after preparation, must be admitted to be lawful, even if the sequences in the bass may not suit the individual taste:

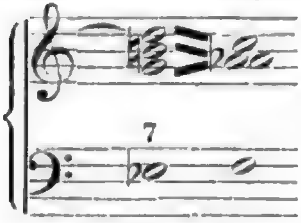


The progression of the chord of the seventh on the second degree to the augmented triad is here effected through the mediation of the dominant triad *G-b-D*. This chord of the seventh, formed by uniting the triads *D/F-a♭* and *F-a♭-C*, is of so mild a character, that it is sometimes employed without preparation, by eminent composers, in free or even in *a cappella* composition.

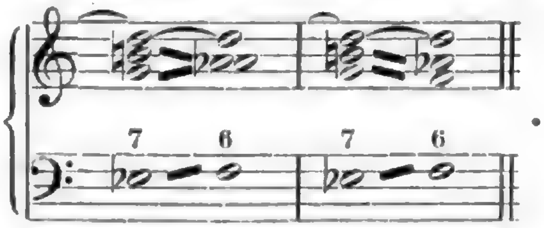
3. Chord of the seventh on the third degree.



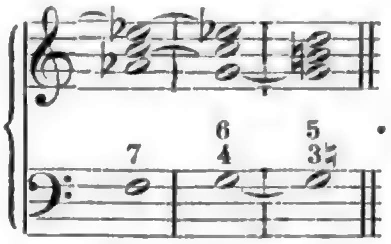
A progression to the fourth degree of the scale is here impossible, because of the augmented second, as the following faulty example shows:



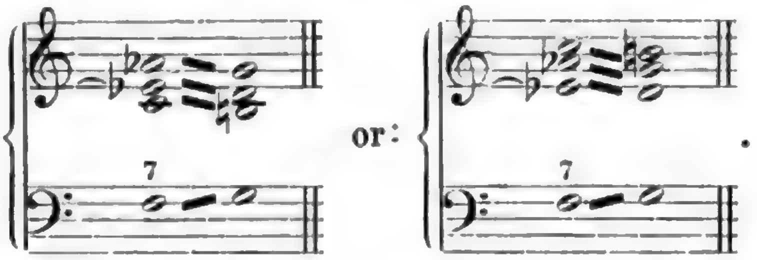
, *b-a♭* being, as augmented second, an unmelodic step; nor would a progression to the chord of the sixth derived from the triad on the second degree be allowable, for the same reason, as the following faulty examples show:



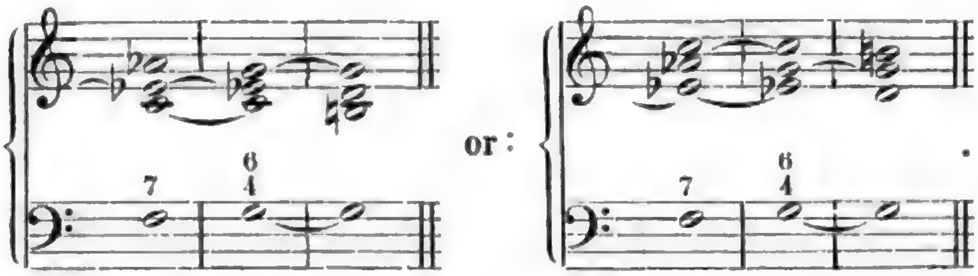
4. Chord of the seventh on the fourth degree.



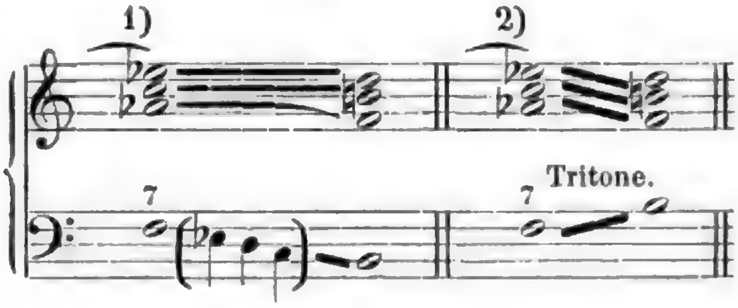
This chord of the seventh, which should not be confounded with that on the second degree in the relative major key (*E♭*-major), has no resolution besides the one given above. In order to reach the dominant triad the chord of the fourth and sixth derived from the tonic triad must be interpolated. If the seventh lie in one of the middle parts a direct progression to the dominant triad is possible:



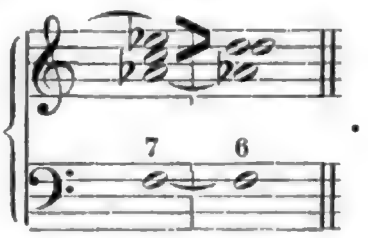
The parallel fourths *e♭-a♭* and *D-G* are accompanied by the thirds of *a♭* and *G*, *i. e. C* and *b*. This progression is an abbreviation of the one in which the chord of the fourth and sixth is interpolated. With the chord of the fourth and sixth derived from the tonic triad the examples would appear thus:



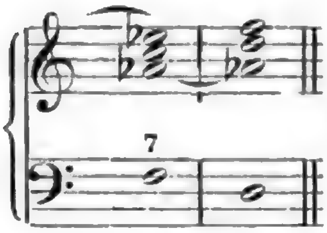
Progressions like the following:

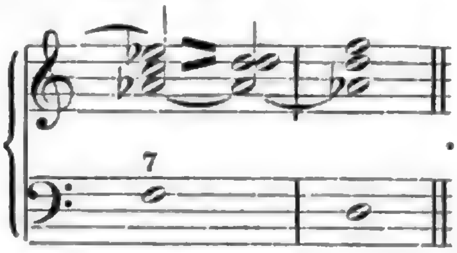


are incorrect; under 1) the interpolated notes show the progression to be unmusical; under 2) the tritone appears as an unmelodious interval. On the other hand, the following progression is quite correct:



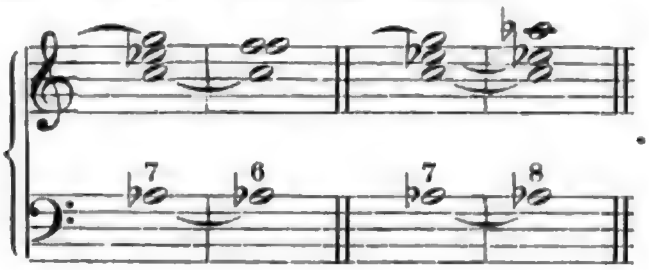
This progression is no resolution, the triad *D/F-a♭* being dissonant.

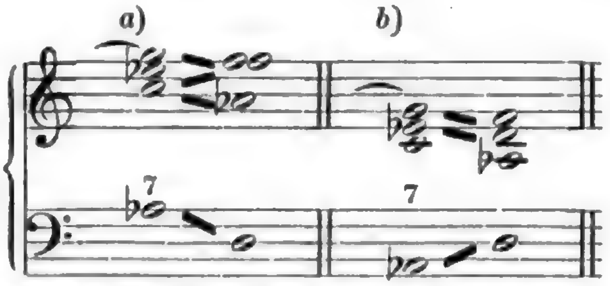
That mode of progression employed in free composition, in which the seventh moves upwards 

, is based upon the following progression in strict style: 

.

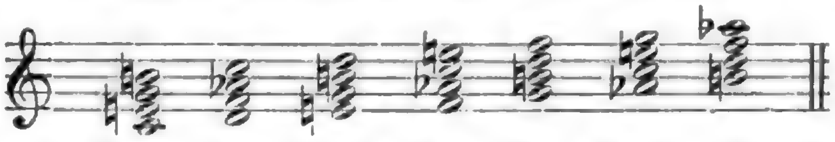
5. Chord of the seventh on the sixth degree.



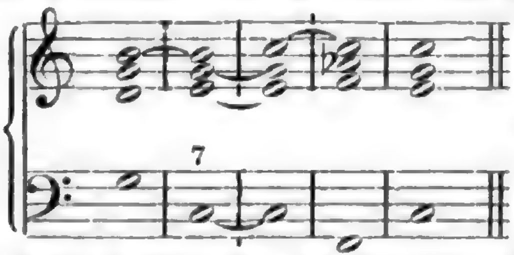
Of the following progressions: 

that under *a*) is allowable; the second, marked *b*, is faulty, because of the tritone *a♭-D*.

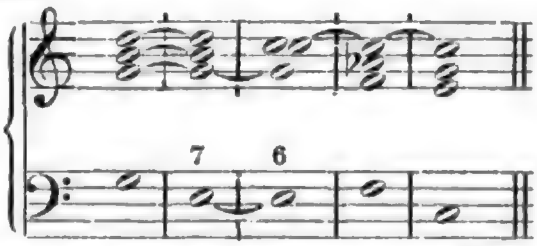
The *combined* major and minor key *F-a♭-C-e-G-b-D*:



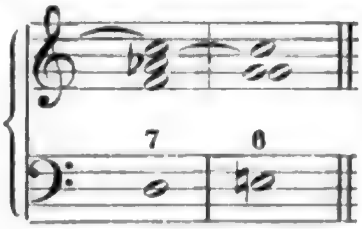
has the same dominant chord of the seventh as the major and minor keys beginning on the same tone; the chords of the seventh on the first and third degrees are identical with those in major, whence may be drawn an explanation of the fact, that after striking these chords the minor sub-dominant triad is sometimes sounded:



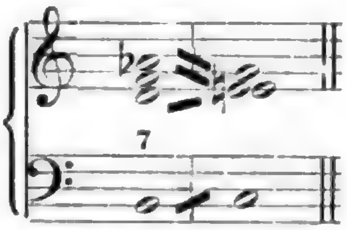
or:



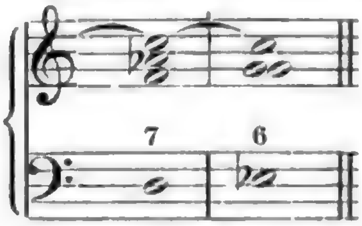
Besides this, the combined mode has the chords of the seventh on the second and seventh degrees in common with the minor mode; they can therefore progress to either the major or minor tonic triad. Such progressions as:



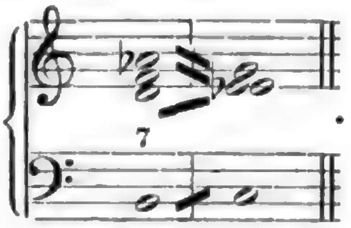
and



are consequently quite as justifiable as the following:



and



Through this presentation of the chords with the aid of the combined mode an easy explanation is found for the oft-recurring major closes of minor fugues; otherwise a theoretical and scientific exposition of this phenomenon could scarcely be given.

The chords of the seventh on the fourth and sixth degrees of the combined mode are identical with those on the first and third degrees in minor, the latter lying respectively a fourth above and a fifth below the former. In the combined key of *C* the chords of the seventh on the fourth and sixth degrees are the same as those on the first and third degrees of *F*-minor. The resolutions of the chords of the seventh on the first and third degrees in minor have already been shown in *C*-minor; the transposition to *F*-minor is purely mechanical, and easily performed.

Progressions between chords of the seventh in their fundamental position may be accomplished if they are connected through the fifth. The major mode is peculiarly adapted to such progressions, as it does not contain the inconvenient step of the augmented second. The following sequences in major:

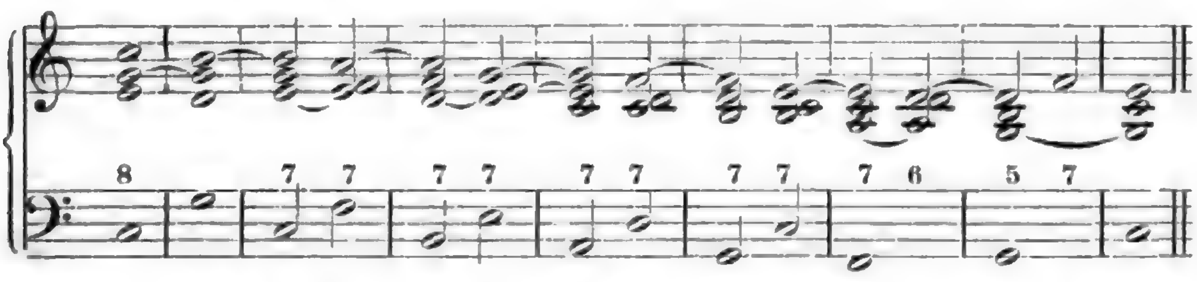
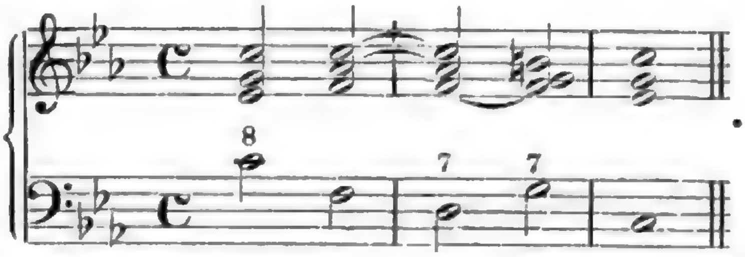
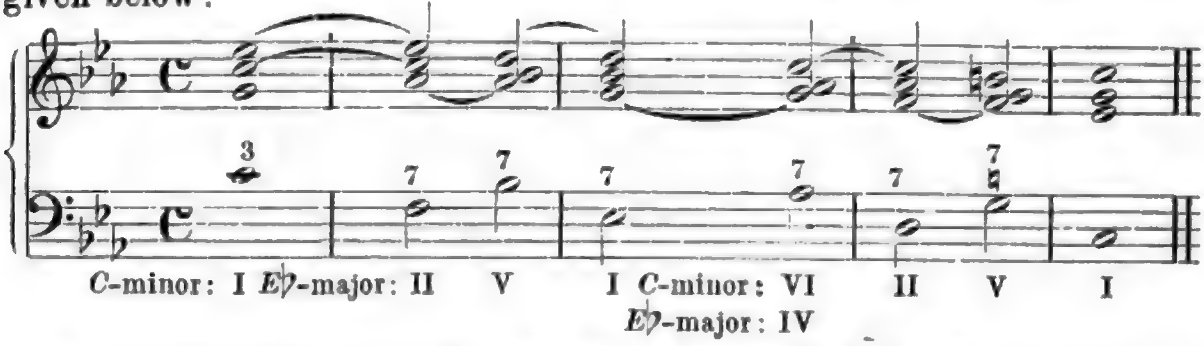


exhibit such a succession of chords; it should be noticed, that the third of one chord of the seventh is employed to prepare the seventh of the following chord, and that a full chord of the seventh is in every case succeeded by one in which the fifth is lacking.

A like succession of chords is impossible in minor if the key be strictly adhered to, because the augmented second between the sixth and seventh degrees cannot be used in melodic progression; a progression between chords of the seventh can nevertheless be brought about, as shown in the example:



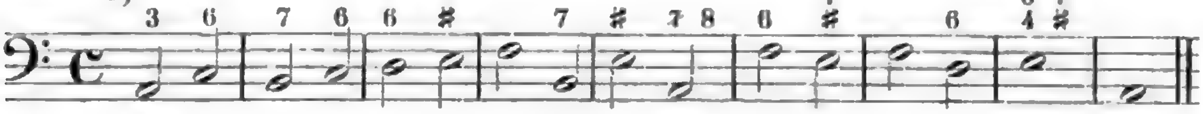
By modulating to the relative major key, *e. g.* from *C*-minor to *E♭*-major, a full sequence of chords of the seventh can be written, as given below:



The interconnection of the chords of the seventh is still closer when their inversions are employed.

Exercises in progressions between chords of the seventh in the fundamental position, and the triads, in minor.

1)



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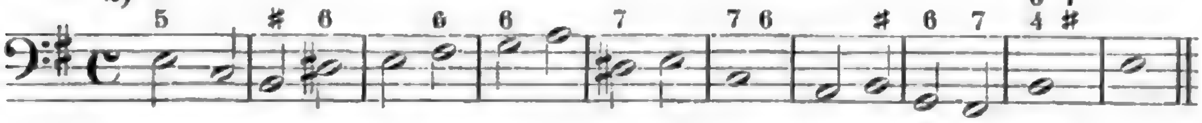

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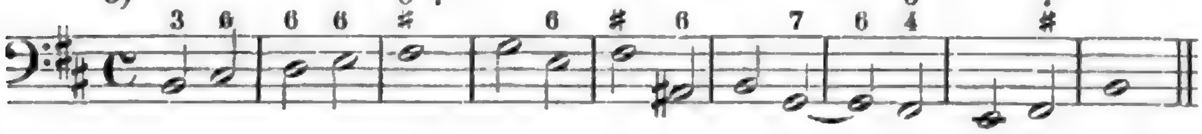

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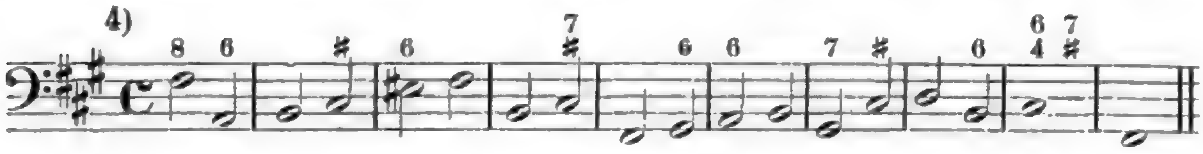

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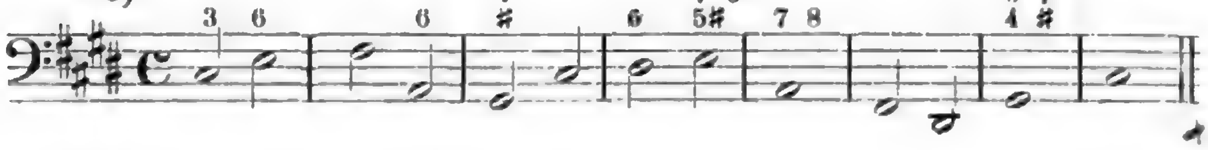

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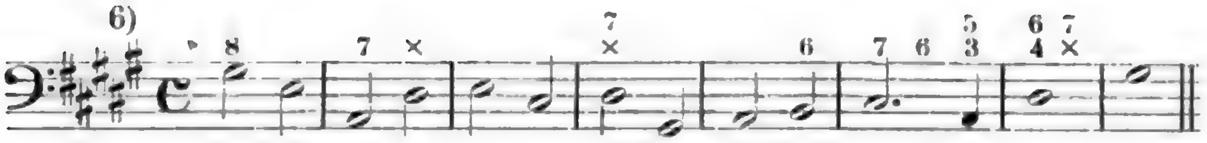

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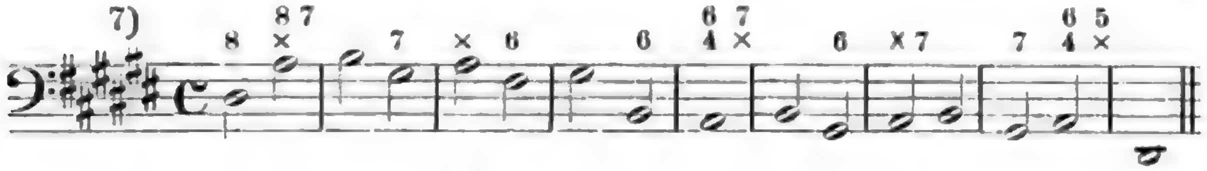

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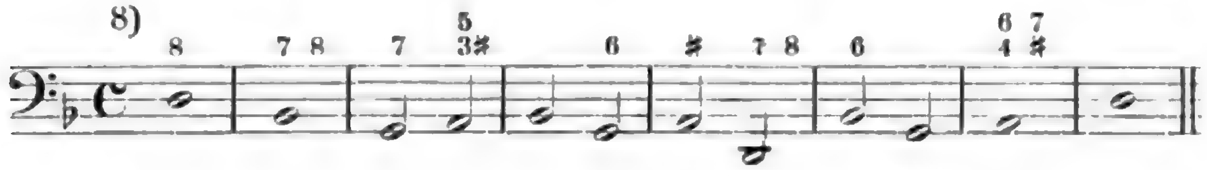

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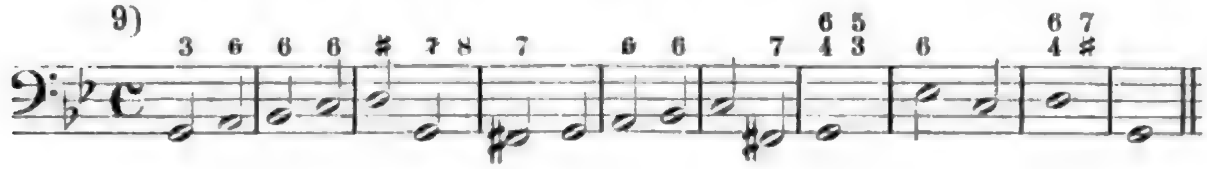

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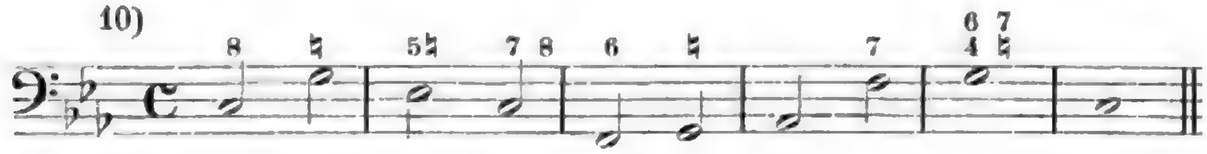

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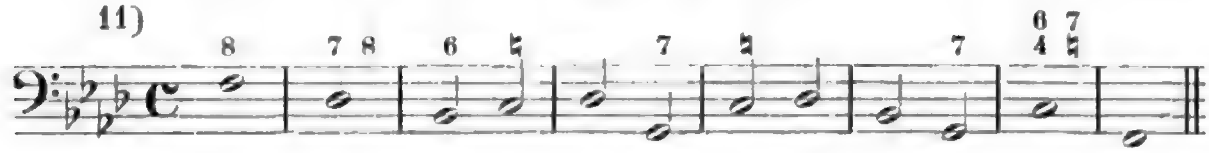

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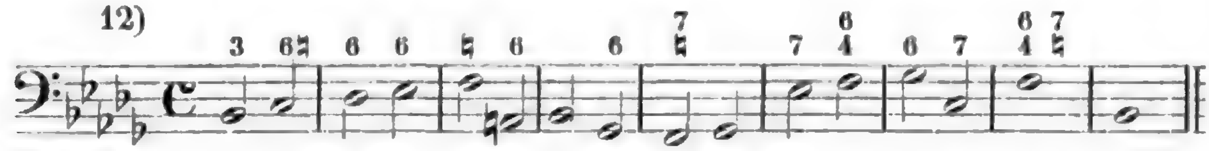

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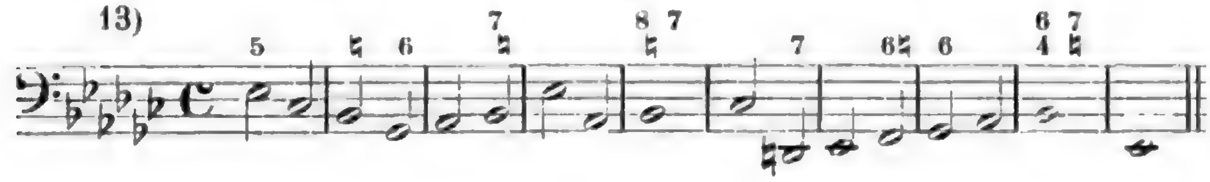


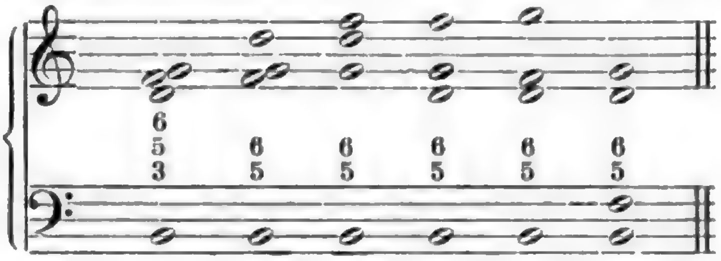
CHAPTER X.

Inversions of the Chords of the Seventh.

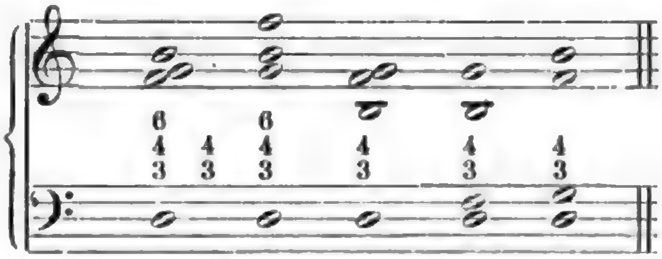
Like the triads, any chord of the seventh may be inverted, *i. e.* the third, fifth or seventh may be taken as the harmonic basis, or bass, instead of the fundamental tone.

The inversions of the chord of the seventh are: the chord of the third, fifth and sixth, or, as abbreviated, chord of the fifth and sixth; the chord of the third, fourth and sixth, abbreviated to chord of the third and fourth; and the chord of the second, fourth and sixth, abbreviated to chord of the second. The names of these chords are derived from the intervals which their higher tones form with the bass, the latter, as in the case of the triads, determining the name of the chord, no matter how the other parts are arranged. The inversions of the dominant chord of the seventh in *C*-major are here given.

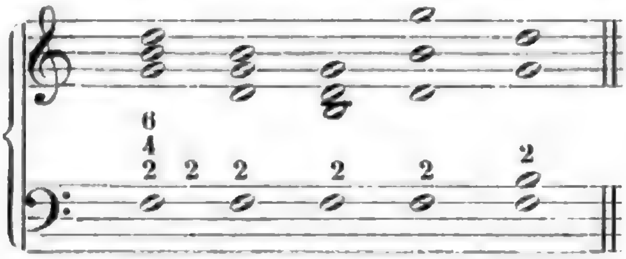
A. Chord of the third, fifth and sixth (= chord of the fifth and sixth).



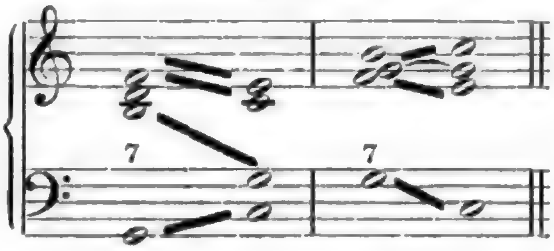
B. Chord of the third, fourth and sixth (= chord of the third and fourth).



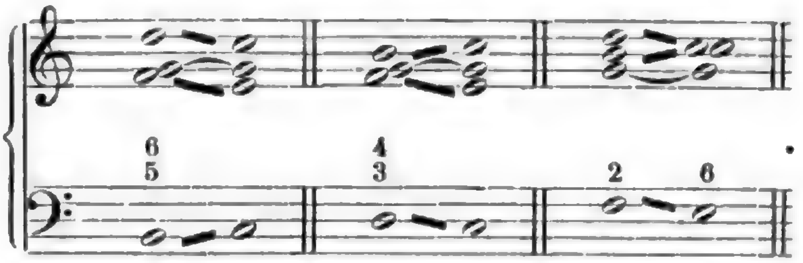
C. Chord of the second, fourth and sixth (= chord of the second).



The progressions from inverted chords of the seventh to triads are the same as those from the fundamental position; only that the fundamental remains stationary in such cases as, were the chord in its fundamental position, it would have moved a fourth upwards or a fifth downwards. The dominant chord of the seventh may be taken as an illustration. In the progression of this chord from its fundamental position to the tonic triad, the bass moves a fourth upwards or a fifth downwards:

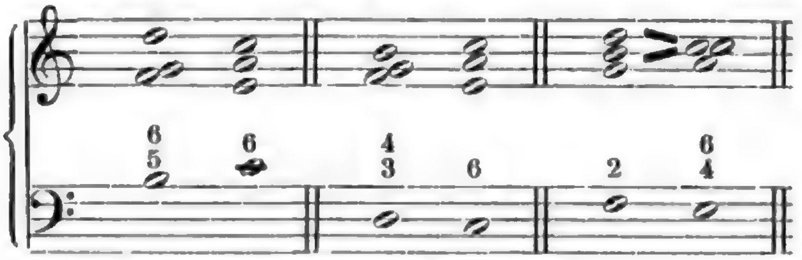


in the progression from an *inversion* to the tonic triad the fundamental remains stationary:

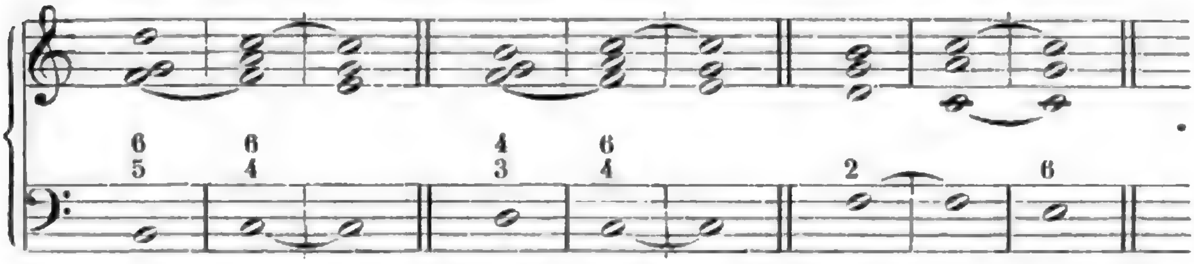


In all other progressions from inverted chords of the seventh to triads the leading of the fundamental tone is the same as if the chord of the seventh were in its fundamental position; *e. g.*:

A. Progressions to the sixth degree.

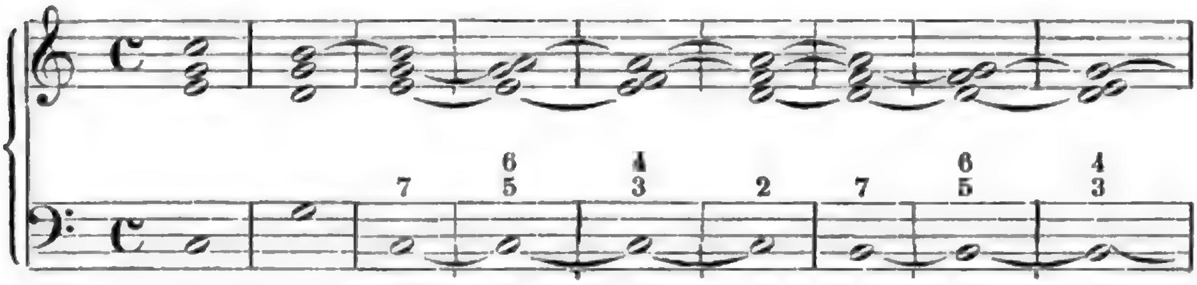


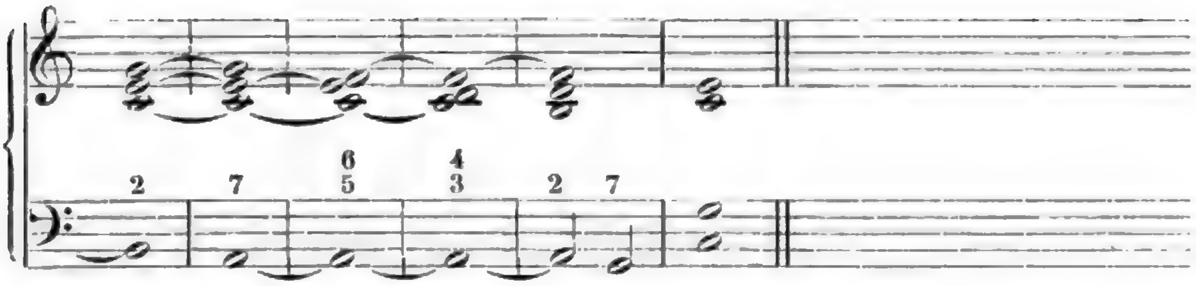
B. Progressions to the fourth degree.



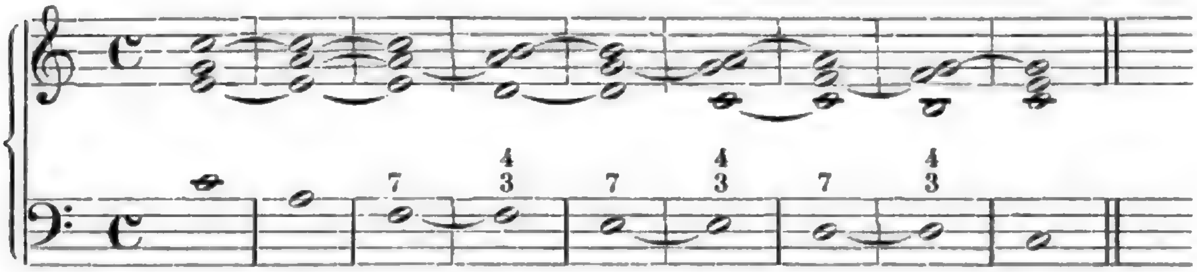
The progressions of the remaining chords of the seventh, when inverted, are subject to the same laws, and to the rules given above for the fundamental position. The augmented second in minor must always receive special attention.

Finally, the progressions between chords of the seventh connected through the third or fifth are to be noticed. Here a wide field is opened for the employment of this chord. The examples given below sufficiently illustrate such progressions between chords of the seventh connected through the third in major.

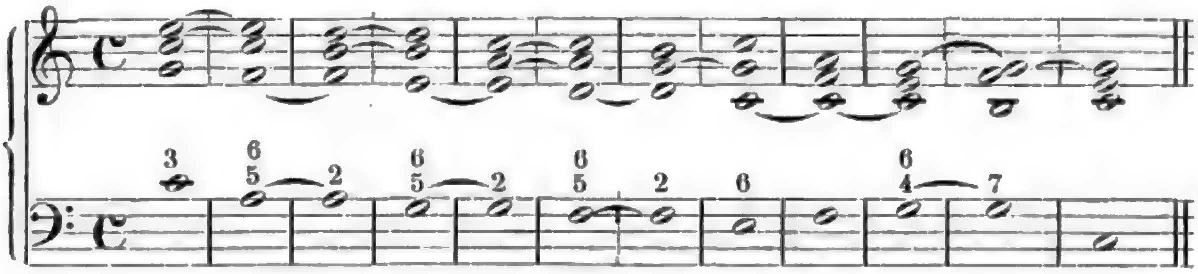




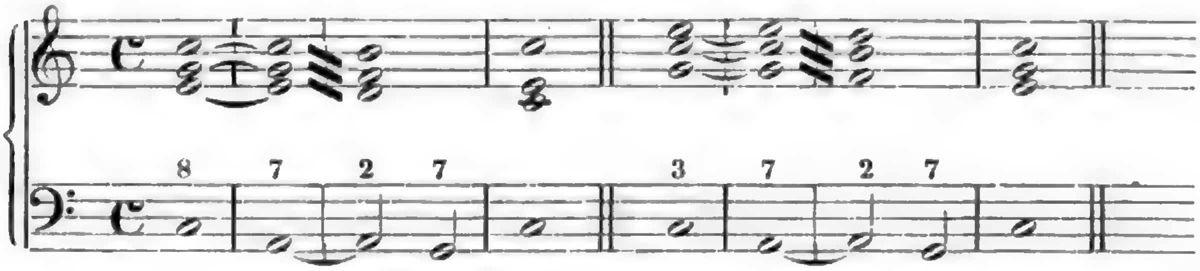
The pupil should observe, that in each progression three tones common to both chords remain stationary, the progressions to the various chords of the seventh being thus fully prepared. In progressions between chords of the seventh connected through the fifth, only two tones remain stationary:



or, beginning with a chord of the fifth and sixth, thus:

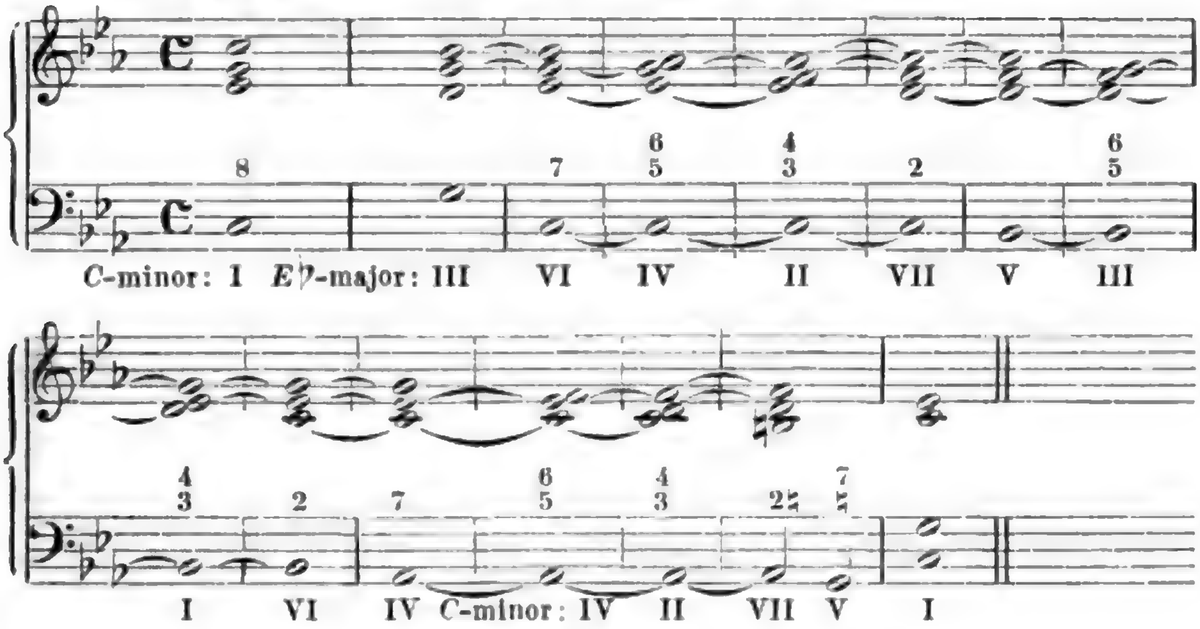


Progressions may be effected between chords of the seventh having only one tone in common, *i. e.* connected through the seventh. Such a progression can however, be brought about only by leading three of the parts in such a manner that they form parallel fourths accompanied by the third, the fourth part being stationary. This stationary tone, the fundamental of the first chord, changes to the seventh in the second chord:

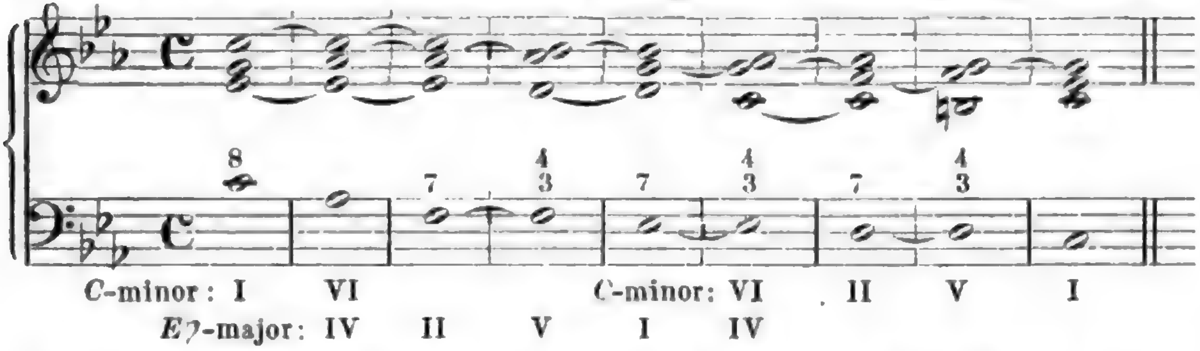


A long succession of chords of the seventh connected through the third or fifth can be made practicable, in minor, only by modulating into the relative major; *e. g.*

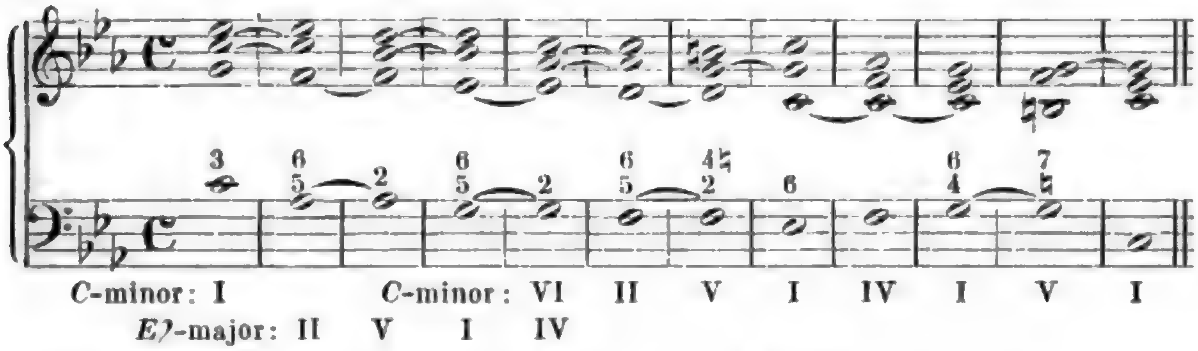
Chords of the seventh connected through the third:



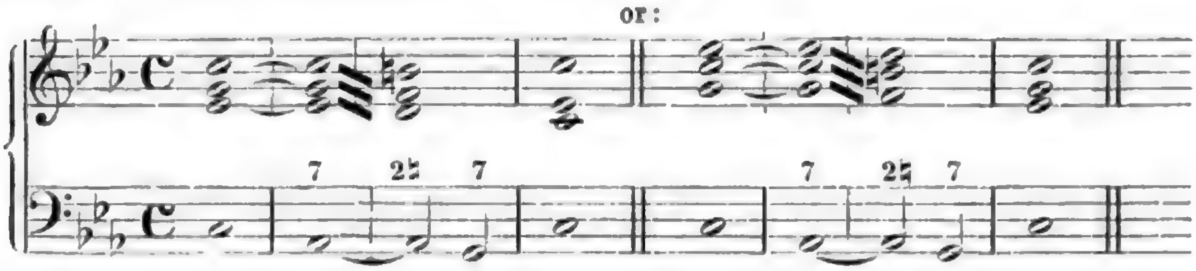
Chords of the seventh connected through the fifth:



Beginning with a chord of the fifth and sixth the progressions appear thus:



Progressions between chords of the seventh having one tone in common, in minor, may also be accomplished, when three parts move in parallel fourths accompanied by the third, as follows:

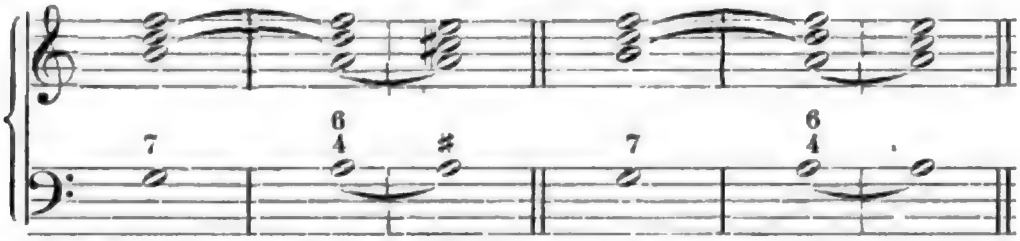


The foregoing explanations will enable the learner to work out exercises with figured bass in the progressions of chords of the seventh and their inversions.

We would repeat here, that progressions of the dominant chord of the seventh to the sixth and fourth degrees were called, by earlier theorists, deceptive closes. This name was later given to all progressions of this chord which did not lead to the tonic.

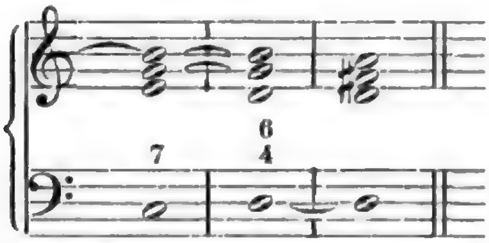
The progressions of the chord of the seventh within the limits of one key have been explained. This chord is likewise well suited for modulation.

In diatonic progression a modulation is brought about when the seventh and fifth remain stationary, the third is led a whole tone downwards, and the fundamental a whole tone upwards, *e. g.*:

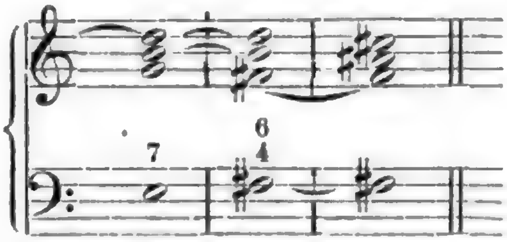


C-major: I *d*-minor: I V *C*-major: I *a*-minor: IV I

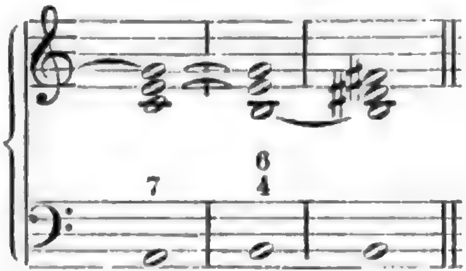
The remaining chords of the seventh may be employed in modulation in the same way; in pure harmony, however, only when the chords of the seventh through the aid of which the modulation is effected are common to both keys, like those on the first, third and sixth degrees in major, *e. g.*:



C-major: I
G-major: IV I V

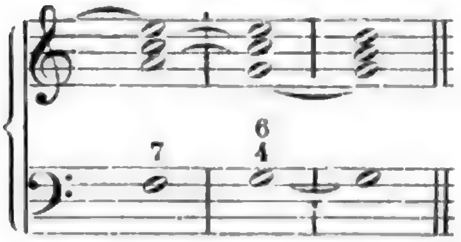


C-major: III
B-minor: IV I V



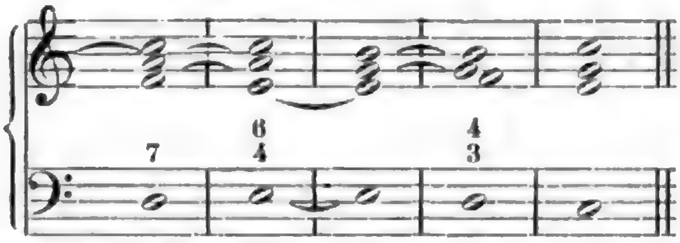
C-major: VI
e-minor: IV I V

The chord of the seventh on the fourth degree in major passes over, in this progression, to the tonic triad of the same key; *e. g.*:



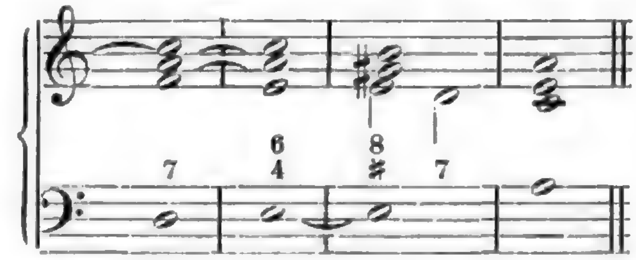
C-major: IV I V

The chord of the seventh on the second degree in major cannot be led to the relative minor; because, taking *C*-major as an illustration, the lower triad *D*/*F*–*a* is not a perfect minor triad; this chord of the seventh must therefore not be confounded with that on the fourth degree in minor. If this chord be led to the chord of the fourth and sixth derived from *a*–*C*–*e*, against which progression no scientific objections can be raised, the dominant chord of *A*-minor cannot follow the last named chord directly, if subtle theoretical distinctions be adhered to; on the contrary, the succession of chords would be as follows:



C-major: II VI III V I

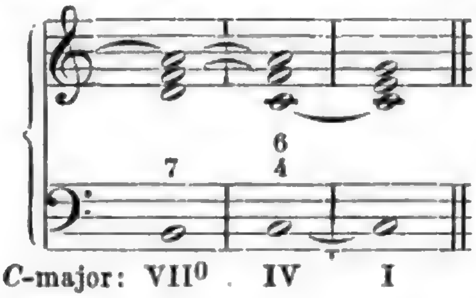
On the piano, however, if a chord of the seventh on the second degree in major pass to one precisely similar in sound in the relative minor key, it then belongs to this minor key, and becomes the chord of the seventh on the fourth degree in minor, composed of a perfect minor triad and a perfect major triad. The example would in this case appear thus:



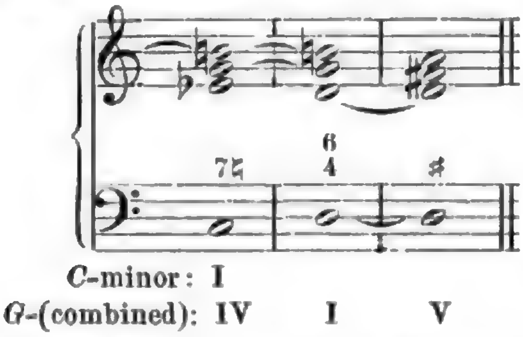
A-minor: IV I V I

The interchanging of two chords between which such a delicate distinction (80 : 81) subsists may be excused (although in certain cases it cannot be justified), because, in instrumental music, such nicety of shading can, in practice, be brought out only in the string-quartette. That the proper treatment of this distinction could not be passed over in silence is made evident by the simple modulations given above, apart from the practical benefit inuring to the composer through an understanding of the distinction, in finding the correct responses to the themes of fugues.

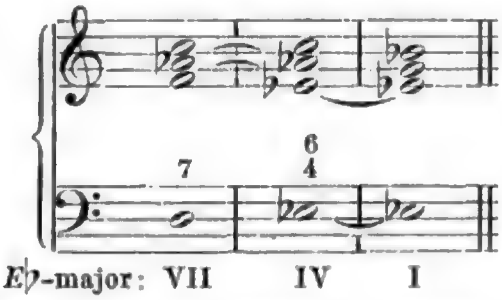
In a similar progression of the chord of the seventh on the seventh degree in major the original key is not departed from:



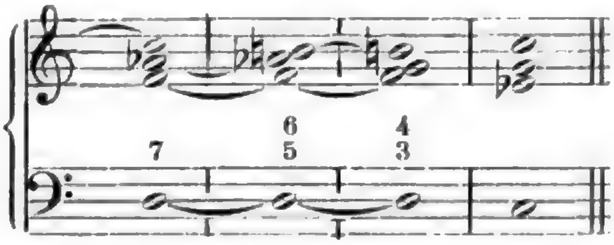
In minor the chord of the seventh on the first degree also belongs to that combined key which begins on the dominant of the minor key. For instance, the chord on the first degree in *C*-minor is the same as that on the fourth degree of the combined key of *G*. The modulation to the latter key would be accomplished thus:



In case the chord of the seventh on the second degree in *C*-minor should be employed in modulating to *E*♭-major, with the aid of the chord of the fourth and sixth derived from the triad on the fourth degree in the latter key

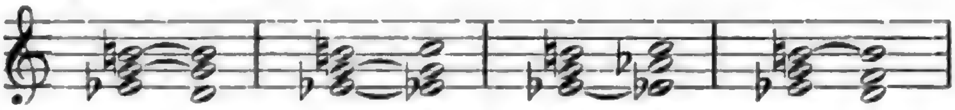


the chord of the seventh on the seventh degree in *E*♭-major would be considered as convertible with that on the second degree in *C*-minor. From a strictly scientific standpoint this modulation cannot be regarded as correct; the same view holds good with respect to the following modulation from the chord of the seventh on the seventh degree in *E*♭-major:

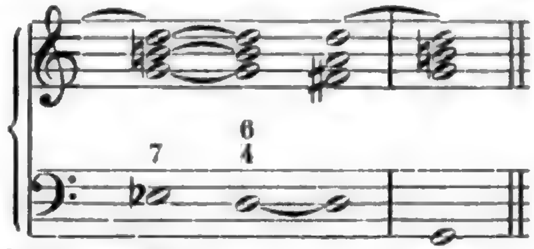


Here the first chord of the seventh is undoubtedly that belonging to the first degree in *C*-minor.

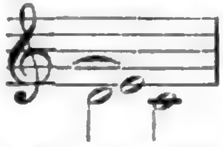
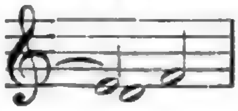
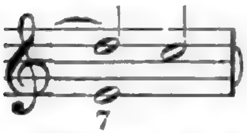
The chord of the seventh on the third degree in minor contains, like that on the first degree, the augmented triad, which makes the following progressions necessary:

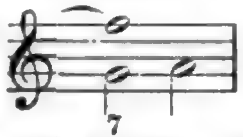


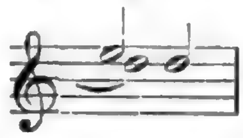
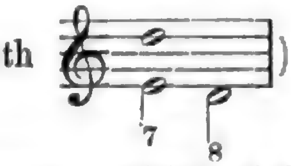
For the modulation of this chord of the seventh only the first progression, as a passing over to the combined key of *G*, has to be taken into consideration, this chord being found on the sixth degree of the latter key. With the chord of the seventh the modulation takes this form:

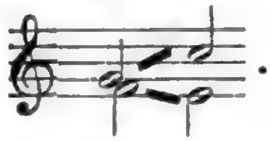


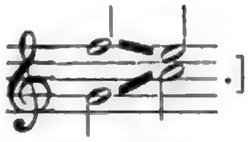
C-minor: III
G-combined: VI I V I

The fundamental tone of the chord of the seventh moving downwards, this progression is incorrect, the proper relation between fundamental and seventh not being observed. Every seventh is the inversion of a second. A second can progress thus:  or thus: . Either the lower tone moves a step downwards, the higher tone remaining stationary (*i. e.*, in the chord of the seventh, the downward resolution 

), or the higher tone moves upwards, the lower remaining stationary (*i. e.* in the chord of the seventh the upward progression of the fundamental, the seventh remaining stationary 

). The higher tone in a second cannot, however, be led downwards in this manner  (which is like leading the fundamental downwards in a chord of the seventh 

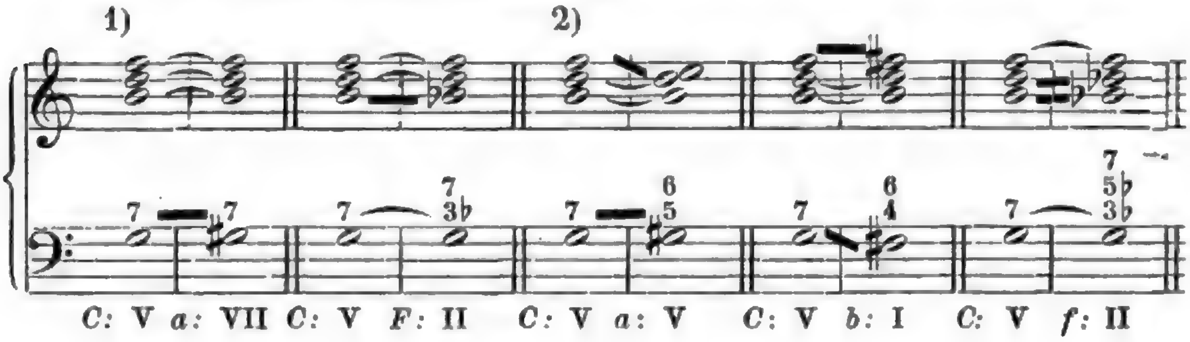
). This could not be called a progression, but rather a mere repetition of the seventh, without sense or object. [A resolution of the second is also often employed in which the second progresses a step downward, and the fundamental, instead of remaining stationary, moves three steps upward, *e. g.*: .

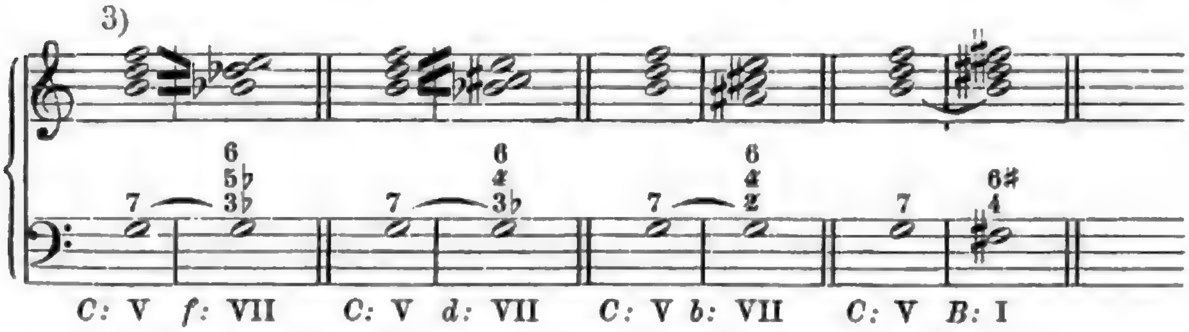
For the seventh the progression would appear thus: .]

Besides these diatonic progressions the chromatic progressions claim our notice. These latter can best be comprehended and systematically explained by an adherence to the principle of the affinity or interconnection of the chords, and by observing the elements which they possess in common.

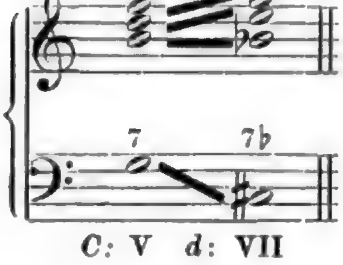
Chromatic progressions can be effected:

1. If three tones remain stationary.
2. If two tones remain stationary.
3. If one tone remains stationary.

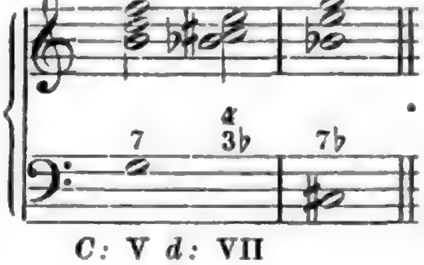




In chromatic progressions in strict composition the parts moving can progress only a step; skips are permissible only in the free style, and such progressions must be based upon the laws of strict composition, *e.g*:



, which is derived from



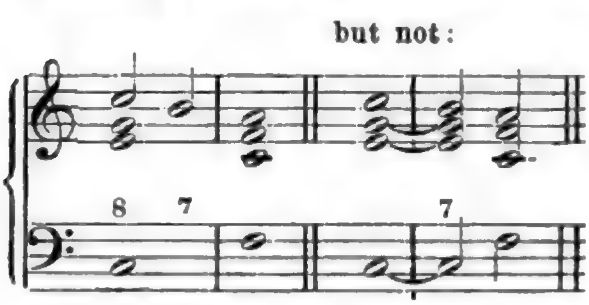
.

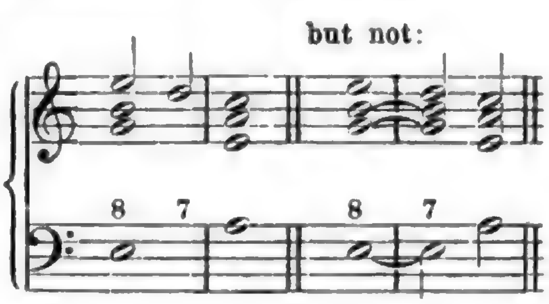
Chromatic progressions are employed in modulating, as explained further on.

All possible chromatic progressions of the chord of the seventh can easily be written after comprehending the principle upon which they are based; we can now proceed to the practical exercises.

Passing sevenths may be employed without harmonic preparation, if they enter in the form of a step from the fundamental, and on the *unaccented* part of the measure (arsis); such chords of the seventh as require preparation cannot, on the contrary, enter free on the *accented* part of the measure (thesis) [see Note XI], even should the seventh enter as a step from the fundamental.

Passing chords of the seventh are, for instance:

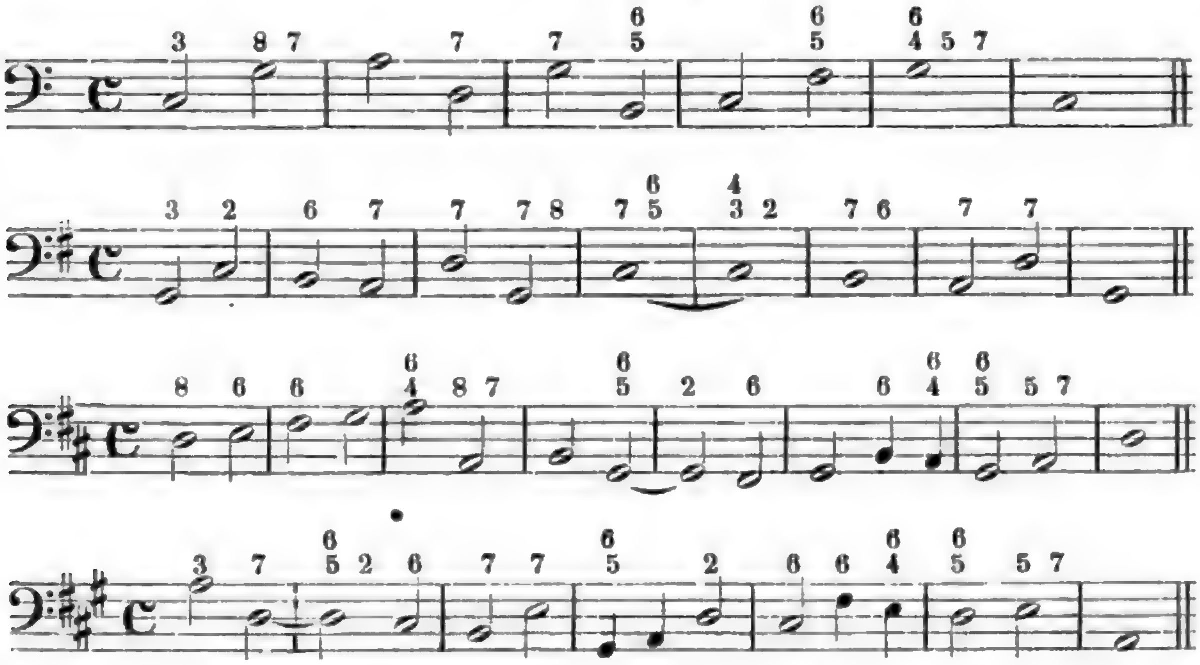


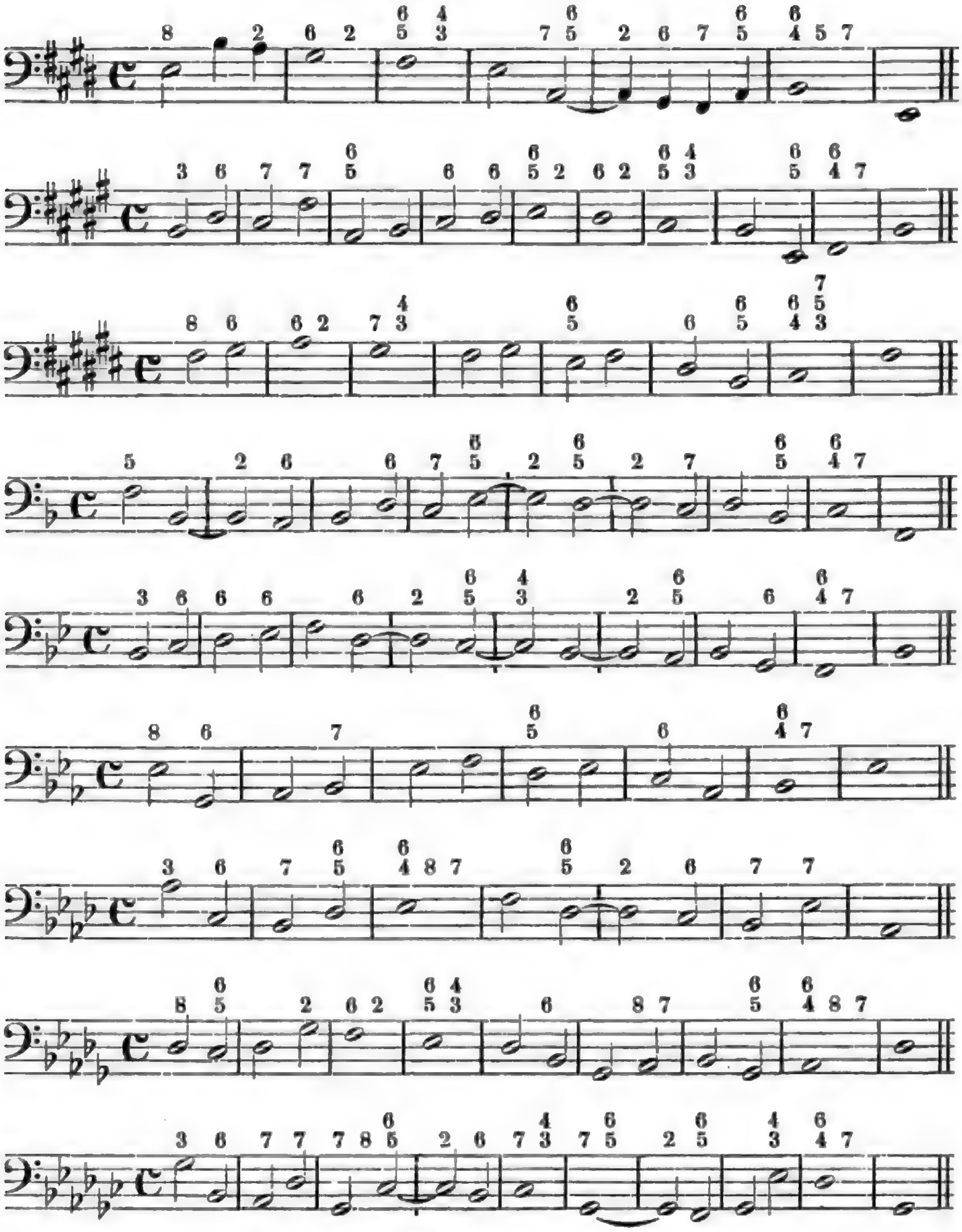


Free composition is emancipated from these rigorous rules. Even where the composer employs the free style in the expression of ideas for which the strict style seems inadequate, he should nevertheless show, through an intelligent treatment of harmonic combinations, that before taking the liberties necessary for the accomplishment of his purpose he had obtained a thorough knowledge of the laws of pure harmony.

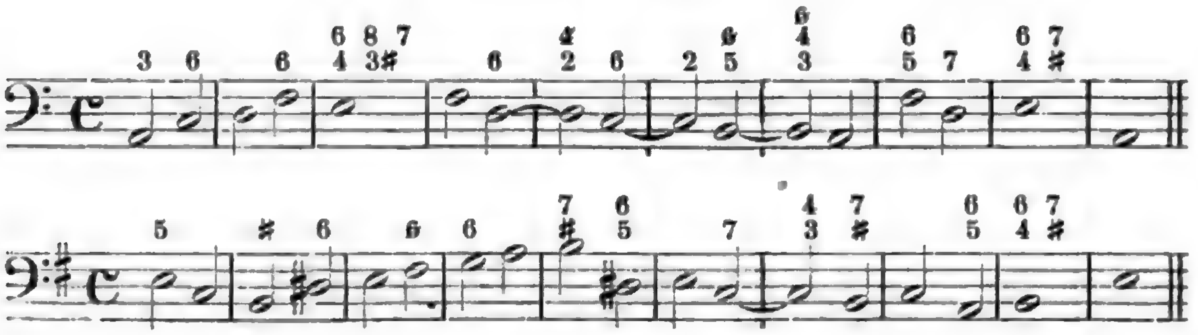
After the pupil has written the various chords of the seventh, together with their inversions, in all the keys, the following exercises may be worked out.

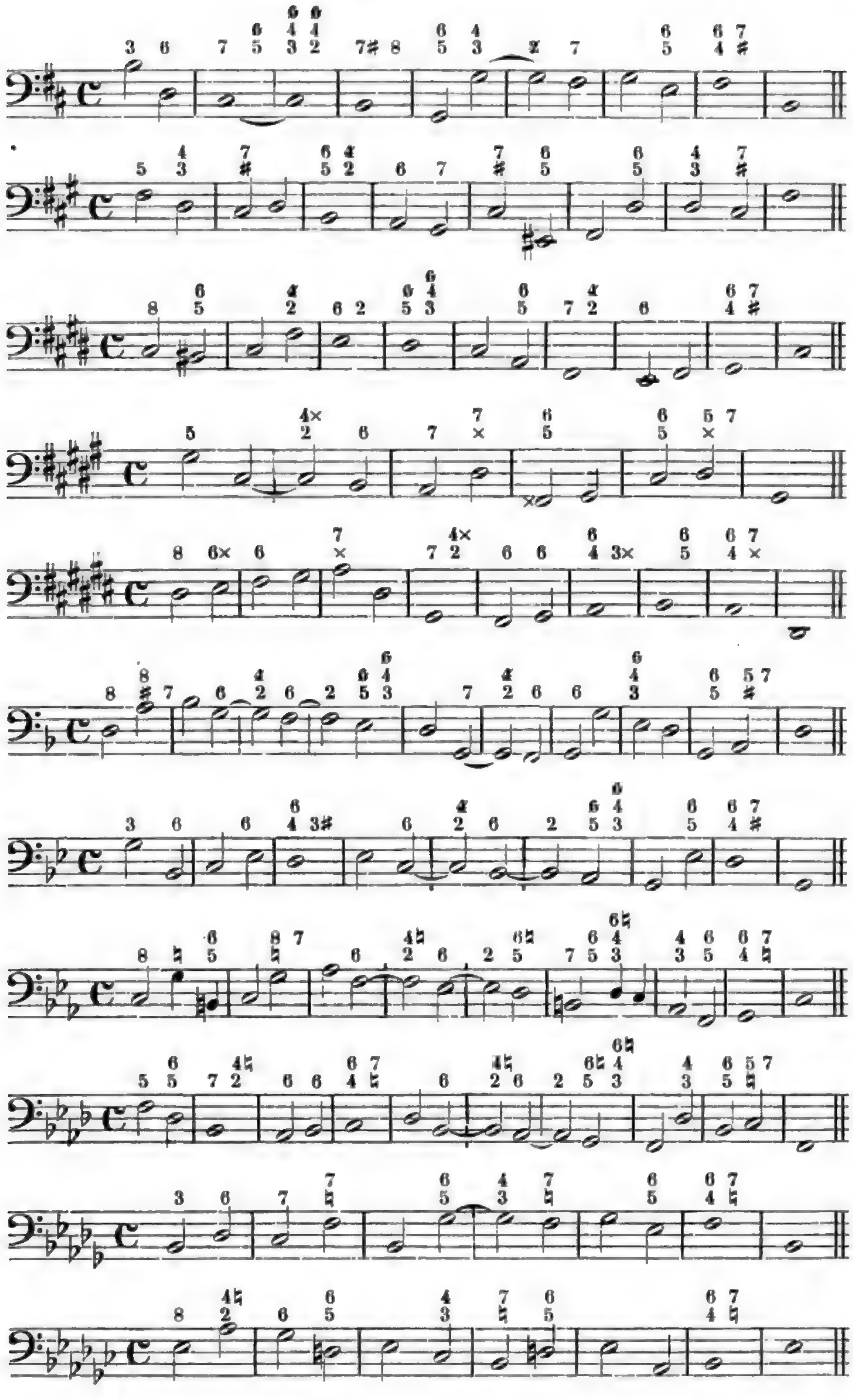
Exercises in Major.



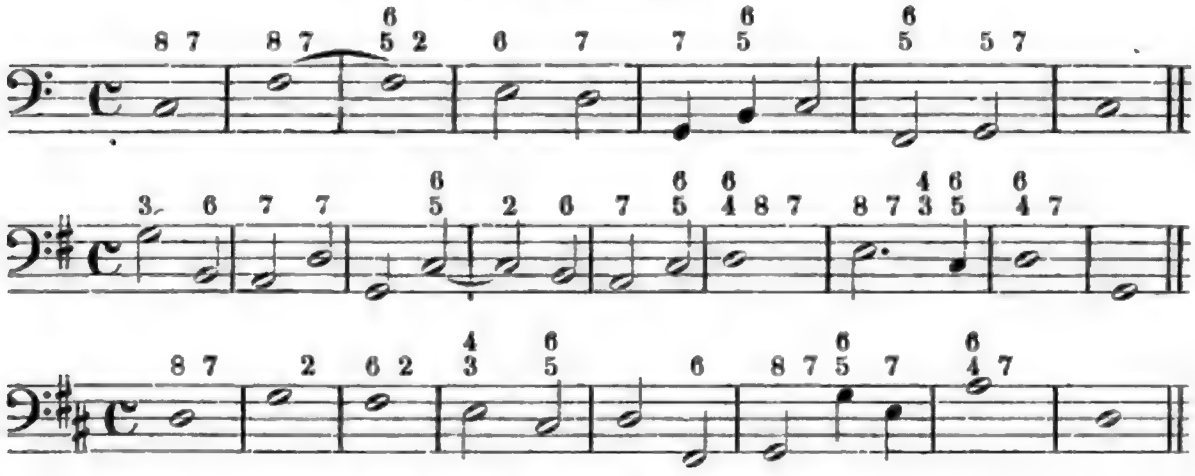


Exercises in Minor.

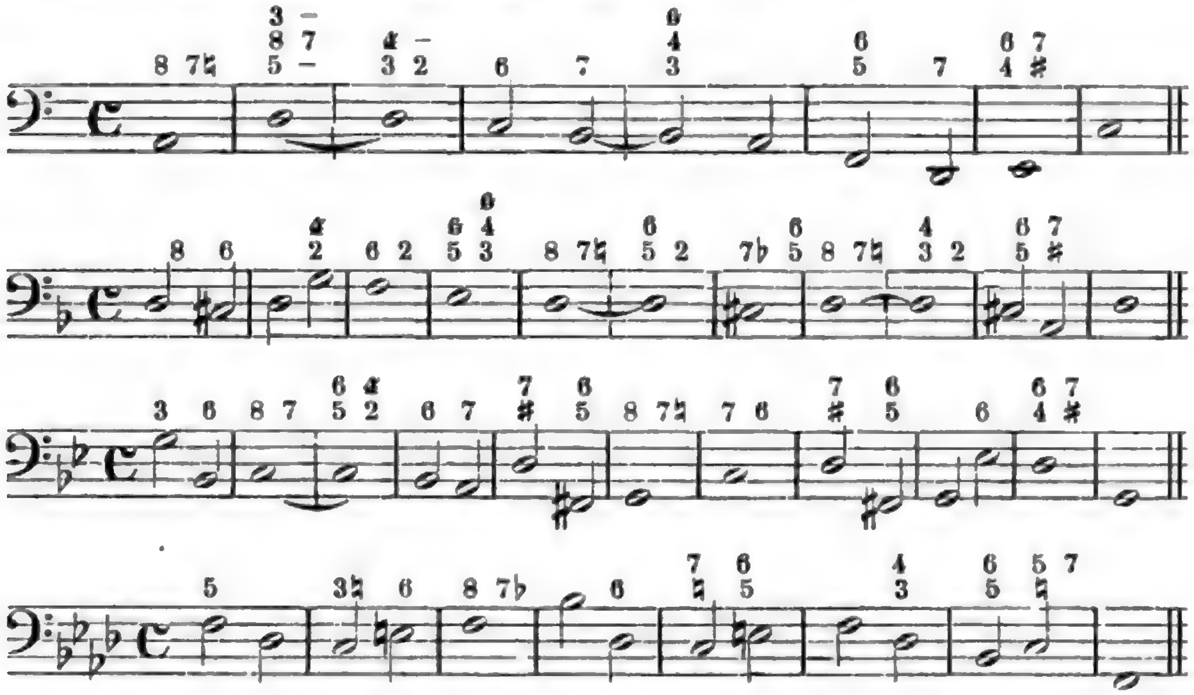




Exercises with passing sevenths.

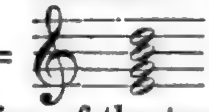
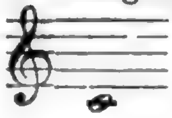
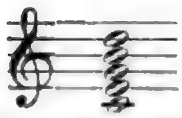
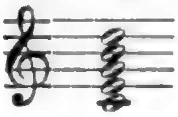


In minor the melodic scale may also be employed.

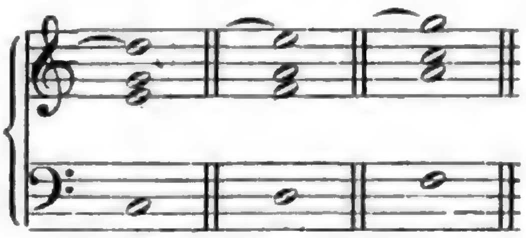


CHAPTER XI.

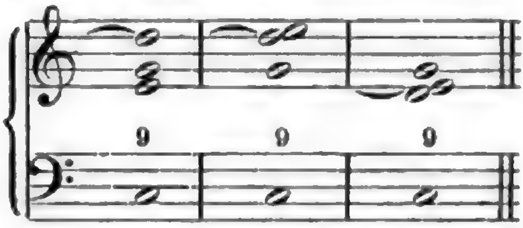
Chords of the Ninth.

A chord of the ninth is formed, when to any chord of the seventh a tone is added which lies a third lower than the fundamental tone of the chord of the seventh, and in the same key as that chord. The chord of the seventh on the third degree in *C*-major, *e-G-b-D* = , may be changed to a chord of the ninth through the addition of the tone *C* = , which lies a third below the fundamental of the above chord. The entire chord of the ninth would then appear thus: . It is composed of five tones: fundamental, third, fifth, seventh and ninth, one of which must be dropped in four-part harmony. The tone most frequently left out is the seventh, or, what amounts to the same thing, the fifth in the original chord of the seventh, which forms, with the added third, the chord of the ninth. In this chord , designated by the figures $\begin{smallmatrix}9\\7\\5\\3\end{smallmatrix}$, or, as abbreviated, 9, the tone *b* is generally left out. The chord of the ninth, as it occurs in four-part composition, may therefore be presented as follows.

When a chord of the seventh, from which the fifth has been dropped, is sounded together with a tone lying a third below its fundamental, the entire harmony is called a chord of the ninth; *e. g.*:

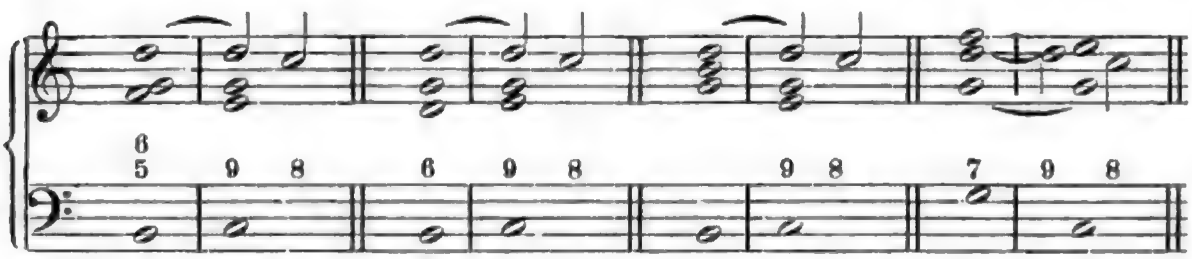


The tones of the chord of the seventh above the fundamental of the chord of the ninth may be arranged at pleasure; the first chord may, for instance, assume the following positions:

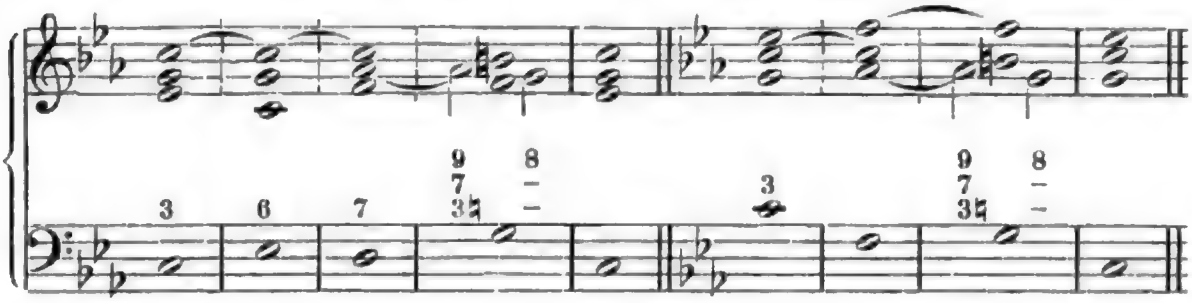


The ligatures signify that the seventh in the chord of the seventh (= the ninth in the entire chord) must be prepared.

The bass-note can enter only from below, or from the fifth above, *e. g.*:



The cases in which the fifth in the chord of the ninth (= third in the chord of the seventh) is dropped likewise belong to strict composition, the place of the tone dropped being taken by the seventh in the chord of the ninth (= fifth in the chord of the seventh), as shown below:

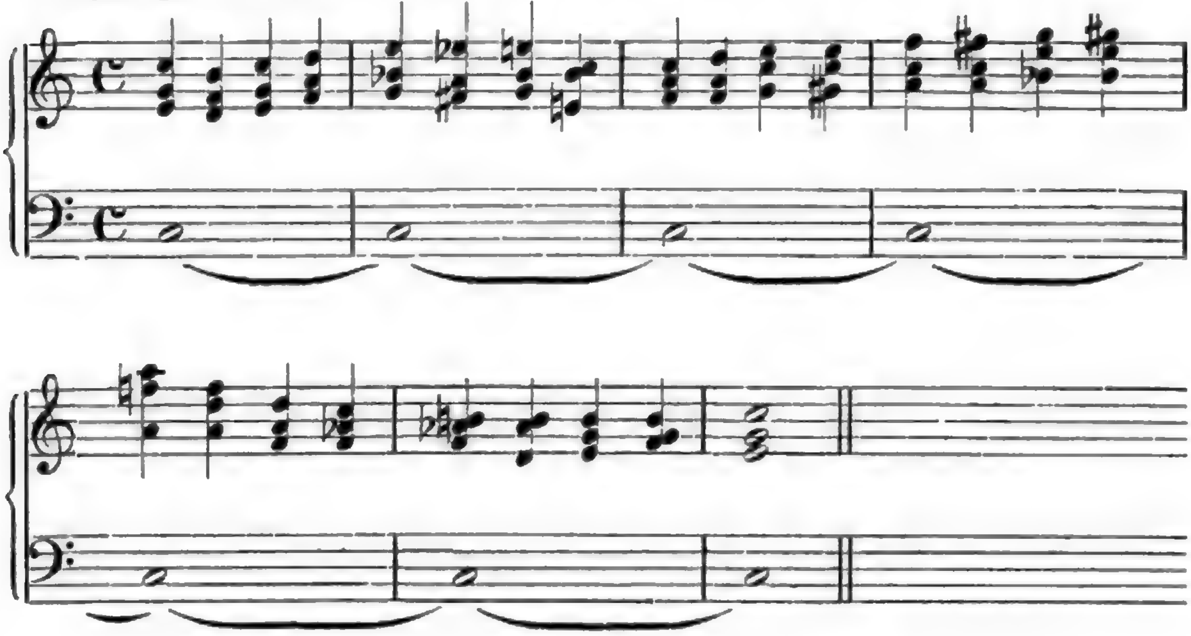


Such an employment of the chord of the ninth immediately before the dominant chord of the seventh is eminently suitable.

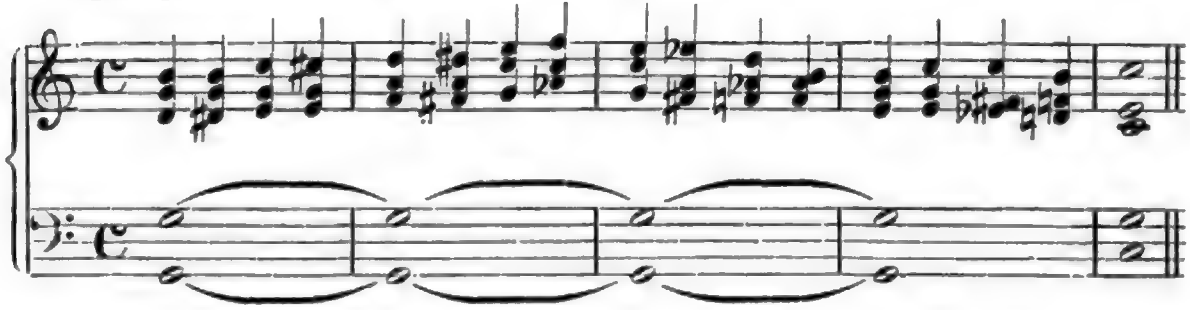
Hauptmann's simple definition of the chord of the ninth, as *a chord of the seventh over an organ-point,* is certainly far more sensible than the unscientific explanations of various other writers.

An organ-point is a stationary tone, above which a succession of chords in infinite variety may be sounded, the organ-point serving as connecting link. Either the prime or the fifth, or both together, may be used as organ-point.

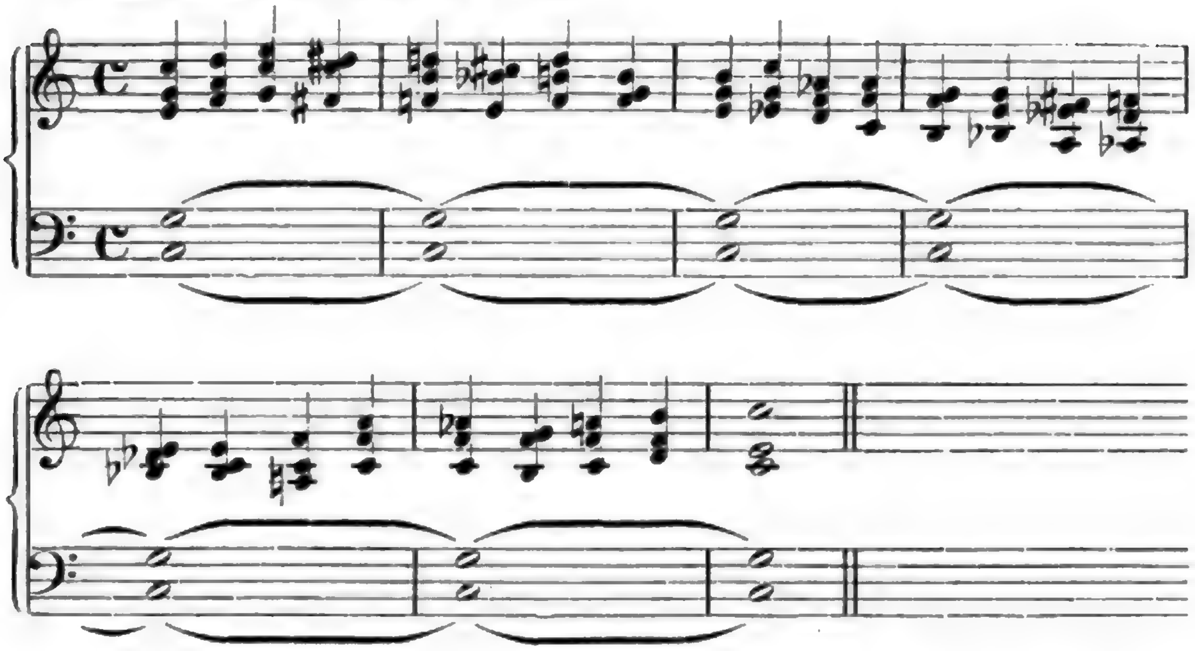
Organ-point on the Prime.



Organ-point on the Fifth.

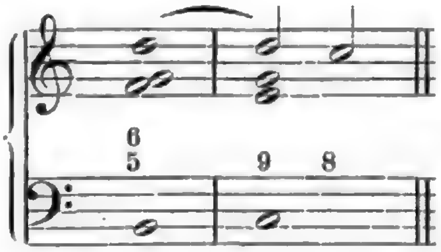


Organ-point on Prime and Fifth.

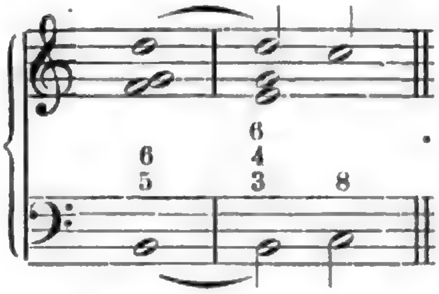


If the nature of the chord of the ninth be explained by the aid of the organ-point, the definition is to be understood as meaning that the chord of the ninth is an organ-point entering together with a chord of the seventh, of which latter the fifth is dropped.

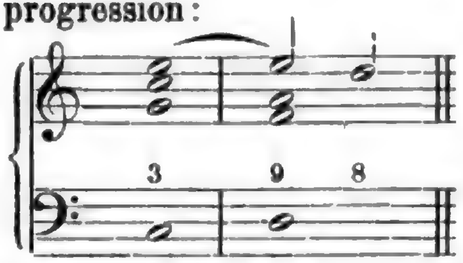
The chord of the ninth might also, with reason, be regarded as resulting from a progression between two chords of the seventh, and naturally and rationally be explained as arising from a resolution of a chord of the third, fourth and sixth, whose basis (the fifth in the chord of the seventh) moves a step upwards prior to the resolution of the whole chord. For instance, this progression:



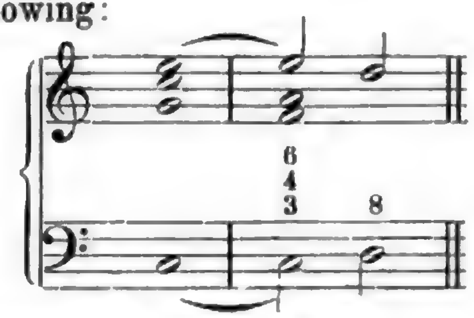
may be considered as derived from the following:



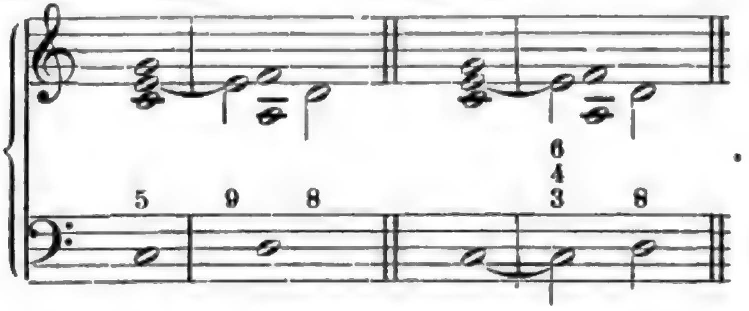
The strong attraction to the fundamental of the succeeding triad probably led to the omission of the chord of the third, fourth and sixth. In the same manner this progression:



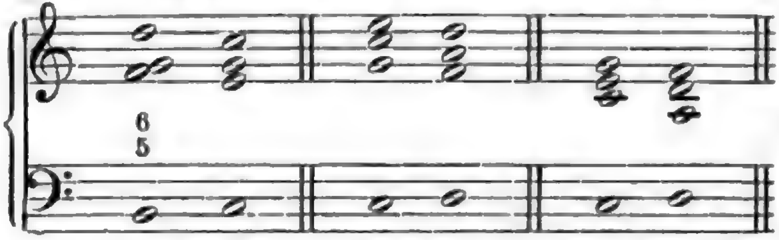
is based upon the following:



or, with a different arrangement of the parts:



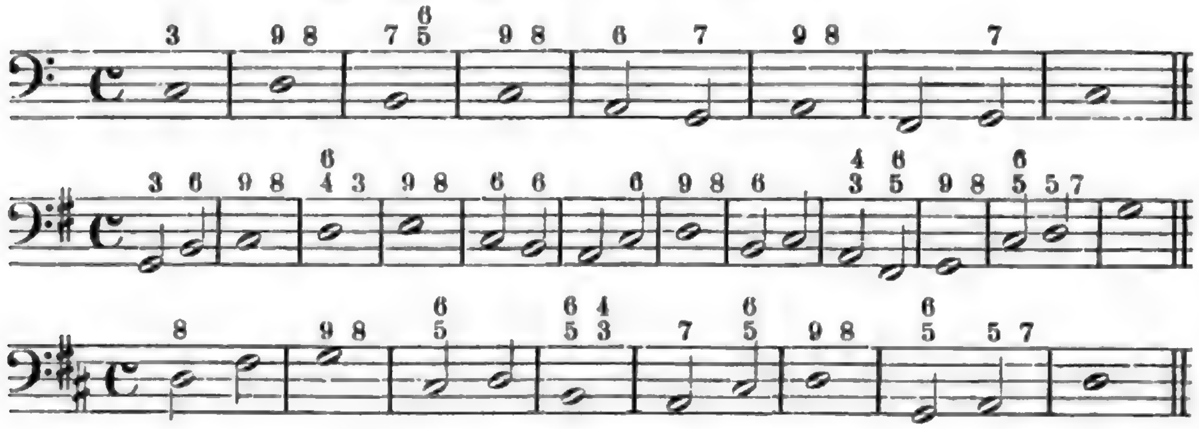
Instead of permitting such direct progressions as these

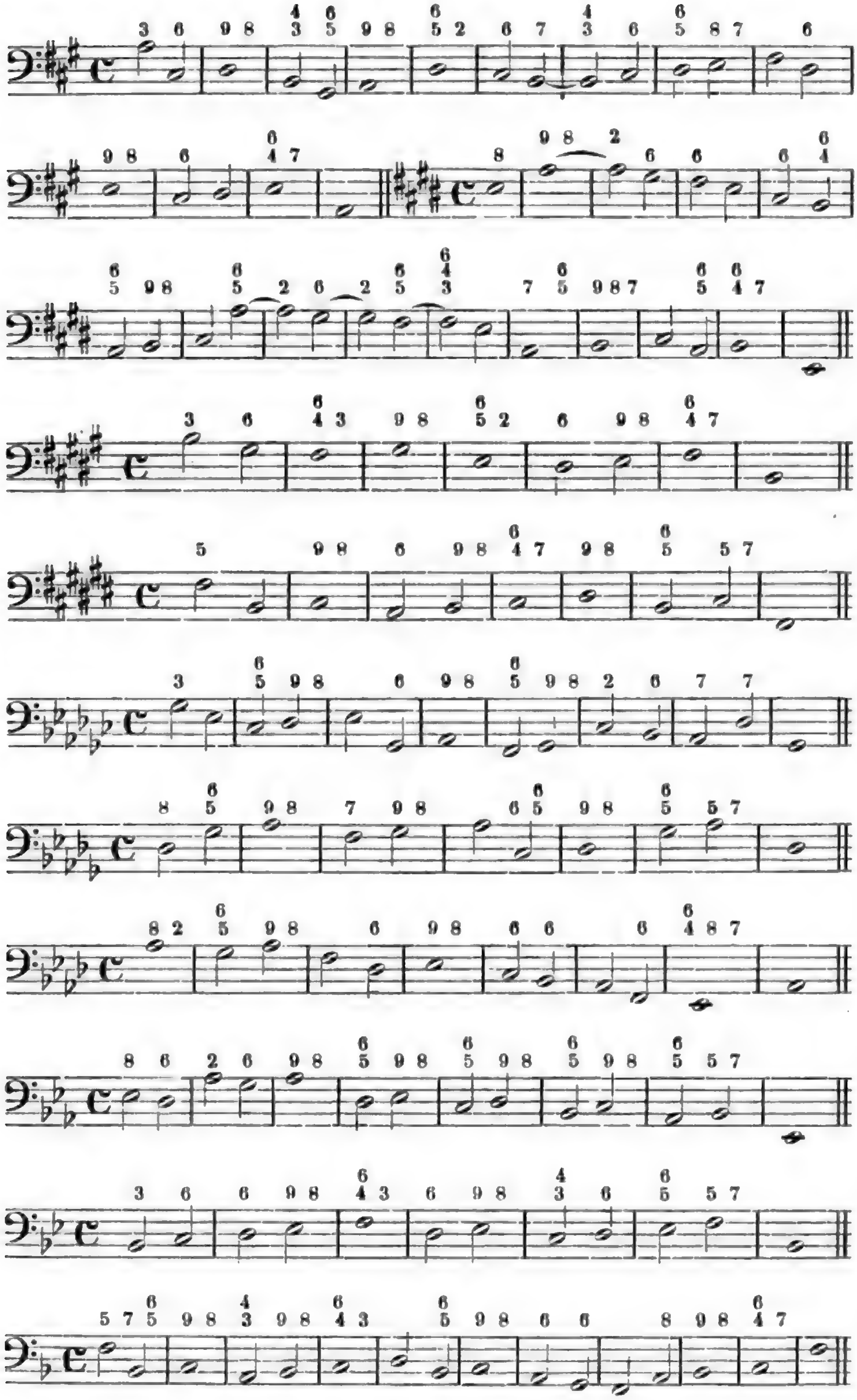


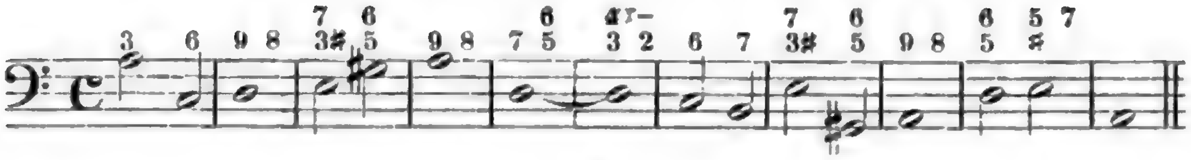
etc.,

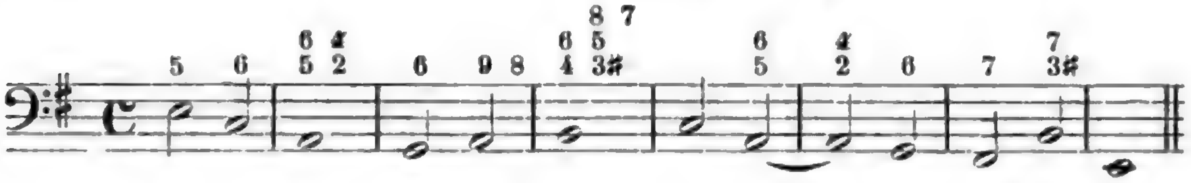
the seventh or second is therefore employed to form a *suspension*, *i. e.* as a dissonant tone retarding the direct progression to a consonance. For this reason the chord of the ninth may also be employed as a suspension.

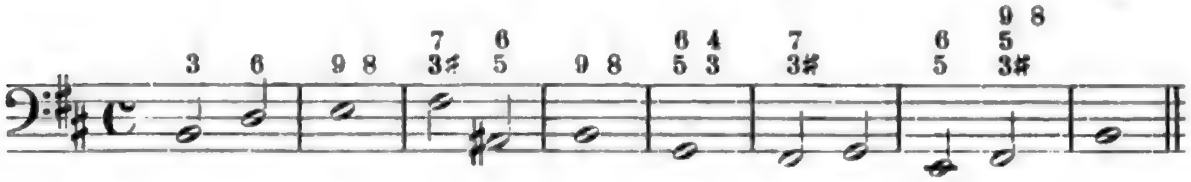
Exercises with the Chord of the Ninth.

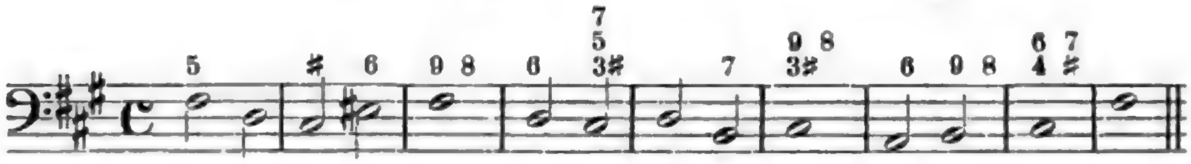


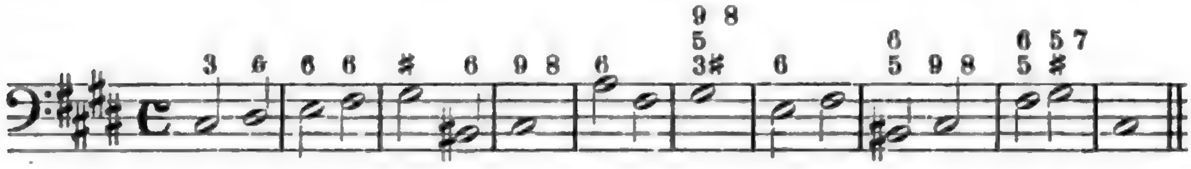

3 6 9 8 4 3 6 5 9 8 6 5 2 6 7 4 3 6 6 5 8 7 6
9 8 6 6 4 7 8 9 8 2 6 6 6 6 4
6 5 9 8 6 5 2 6 2 6 5 6 4 3 7 6 5 9 8 7 6 5 6 4 7
3 6 6 4 3 9 8 6 5 2 6 9 8 6 4 7
5 9 8 6 9 8 6 4 7 9 8 6 5 5 7
3 6 5 9 8 6 9 8 6 5 9 8 2 6 7 7
8 6 5 9 8 7 9 8 6 6 5 9 8 6 5 5 7
8 2 6 5 9 8 6 9 8 6 6 6 6 4 8 7
8 6 2 6 9 8 6 5 9 8 6 5 9 8 6 5 9 8 6 5 5 7
3 6 6 9 8 6 4 3 6 9 8 4 3 6 6 5 5 7
5 7 6 5 9 8 4 3 9 8 6 4 3 6 5 9 8 6 6 8 9 8 6 4 7

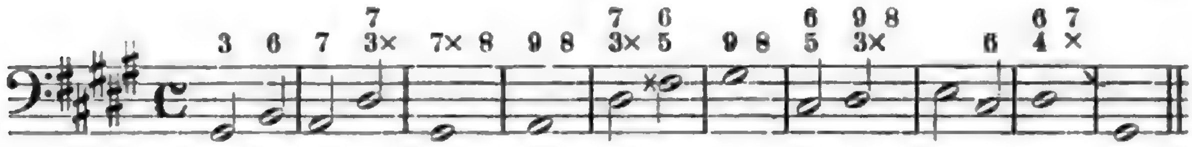

7 6 6 4♯– 7 6 6 5 7
3 6 9 8 3♯ 5 9 8 7 5 3 2 6 7 3♯ 5 9 8 5 ♯

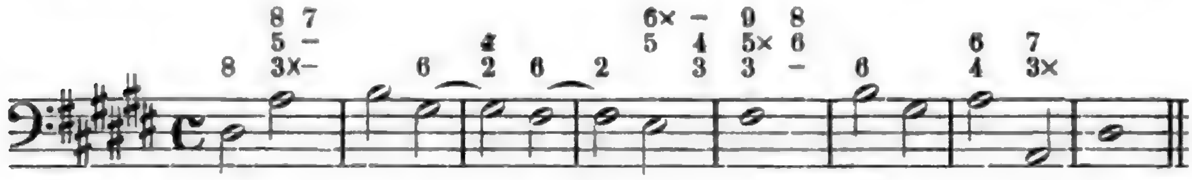

8 7
6 ♯ 6 5 6 ♯ 7
5 6 5 2 6 9 8 4 3♯ 5 2 6 7 3♯

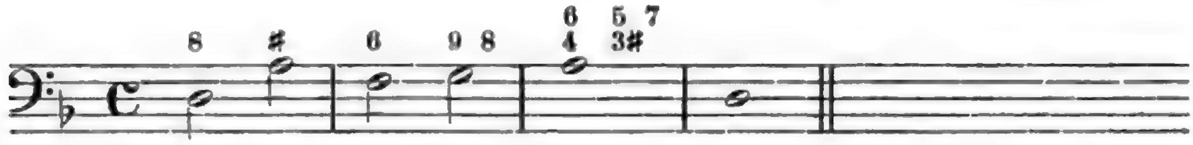

9 8
7 6 6 4 7 6 5
3 6 9 8 3♯ 5 9 8 5 3 3♯ 5 3♯

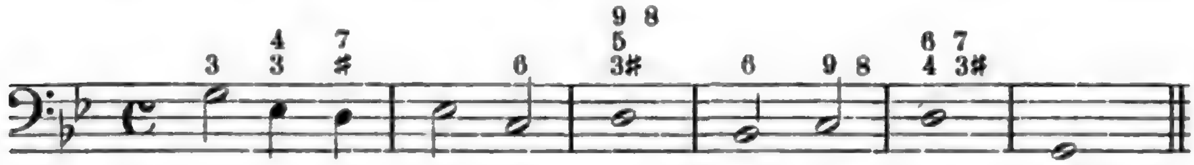

7
5 9 8 6 7
5 ♯ 6 9 8 6 3♯ 7 3♯ 6 9 8 4 ♯

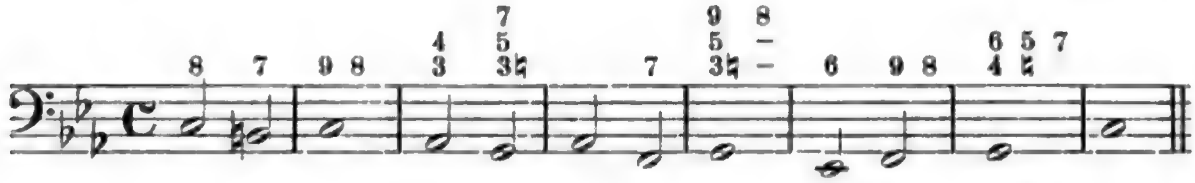

9 8
5 6 6 5 7
3 6 6 6 ♯ 6 9 8 6 3♯ 6 5 9 8 5 ♯

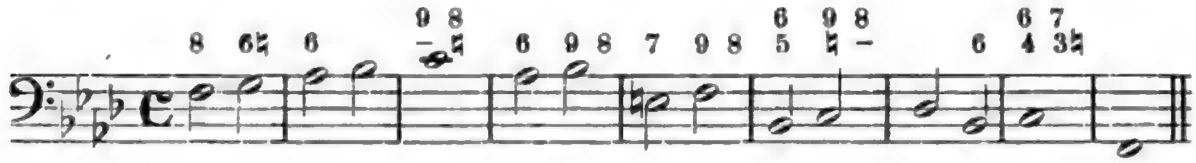

7 7 6 6 9 8 6 7
3 6 7 3× 7× 8 9 8 3× 5 9 8 5 3× 6 4 ×

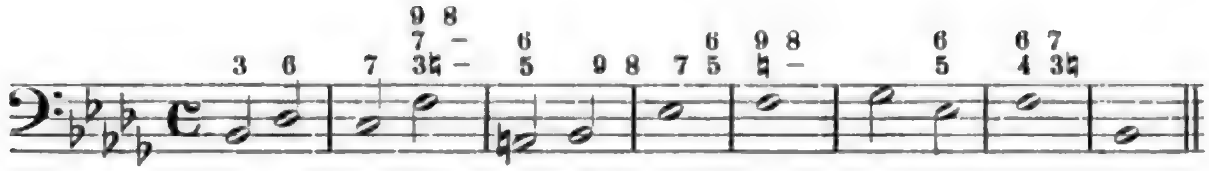

8 7 6× – 9 8
5 – ♯ 5 4 5× 6 6 7
8 3×– 6 2 6 2 3 3 – 6 4 3×

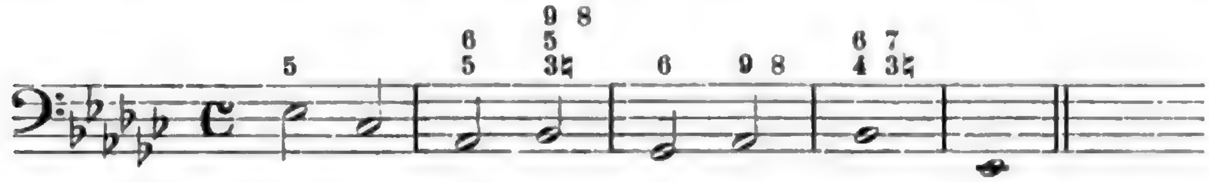

6 5 7
8 ♯ 6 9 8 4 3♯


9 8
4 7 5 6 7
3 3 ♯ 6 3♯ 6 9 8 4 3♯


7 9 8
4 5 5 – 6 5 7
8 7 9 8 3 3♮ 7 3♮ – 6 9 8 4 ♮

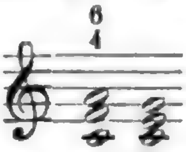
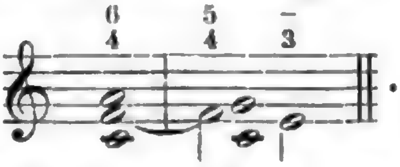

9 8 6 9 8 6 7
8 6♮ 6 – ♮ 6 9 8 7 9 8 5 ♮ – 6 4 3♮



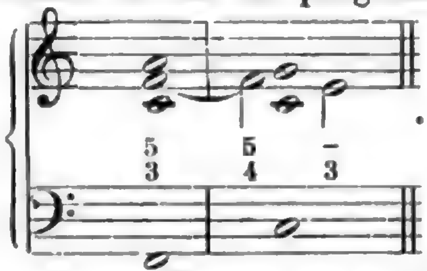


CHAPTER XII.

Suspensions.

A suspension is the dissonance caused by either a seventh or a second, and occurs immediately before a chord which could have entered without the suspension. Suspensions may occur before triads or chords of the seventh. Taking for example this progression between triads , the direct progression to the triad *C-e-G* may be suspended through *F*, which note would then form a second with *G*: .

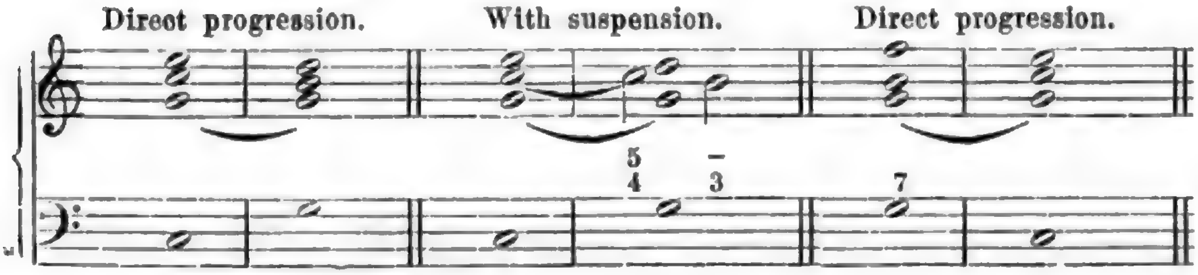
If the fundamentals be added as bass the progressions appear thus:

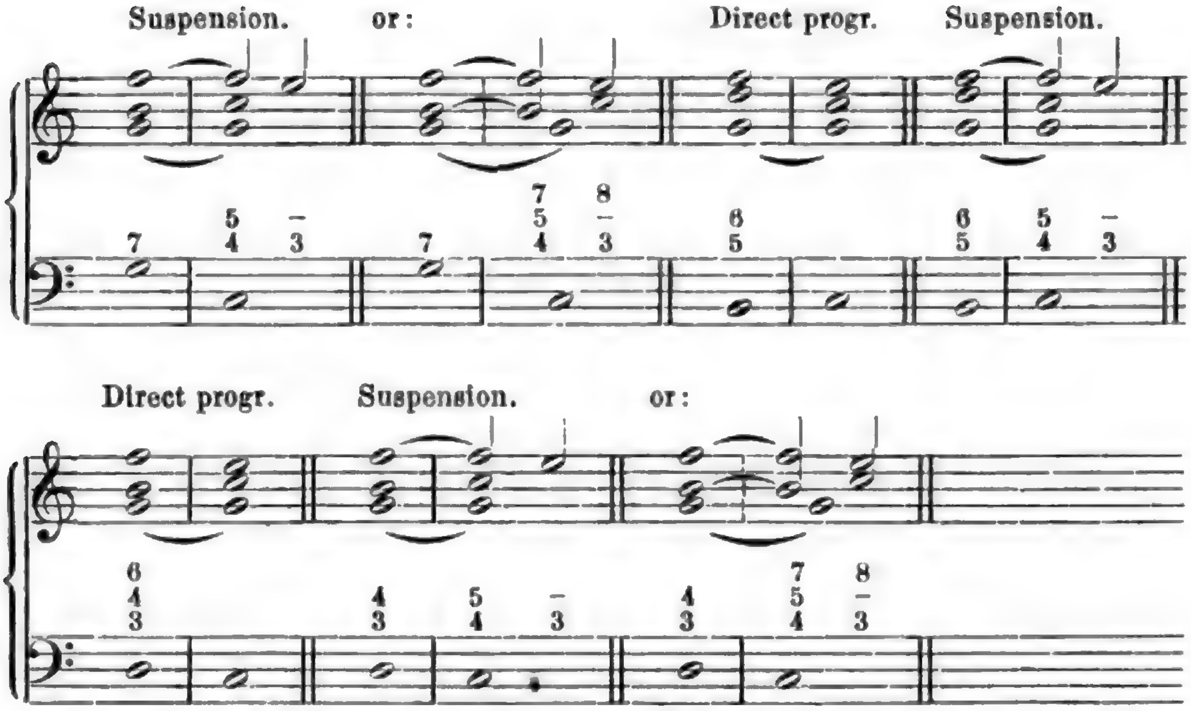


A few examples will suffice as illustrations of such progressions.

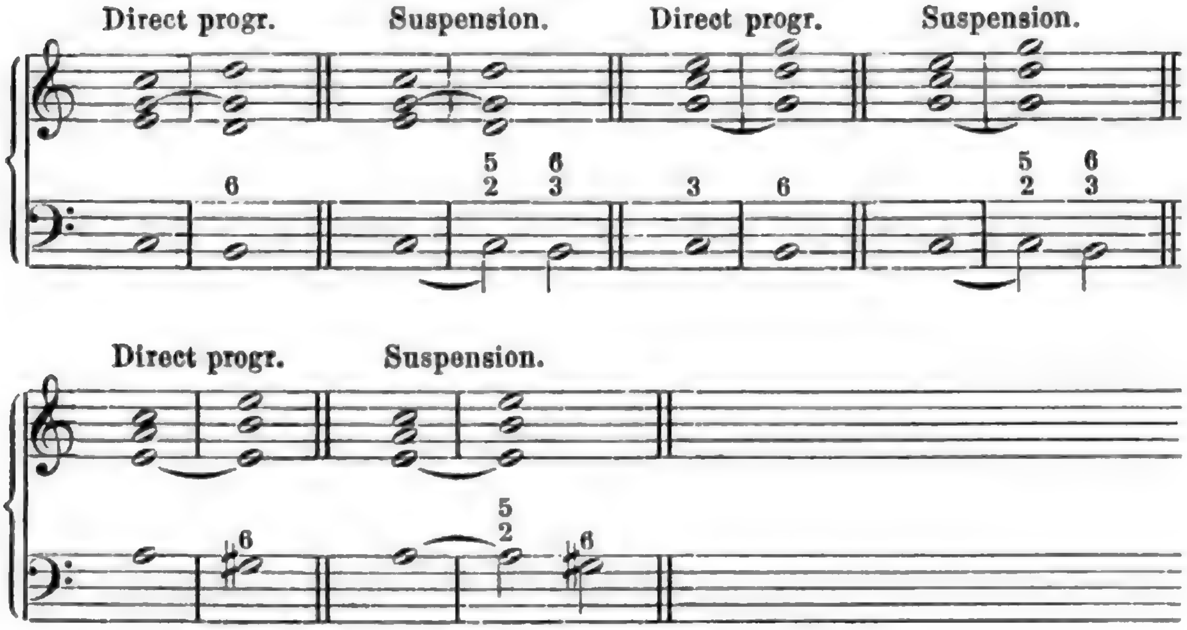
I. Suspensions before triads.

1. *Before the triad in its fundamental position.*

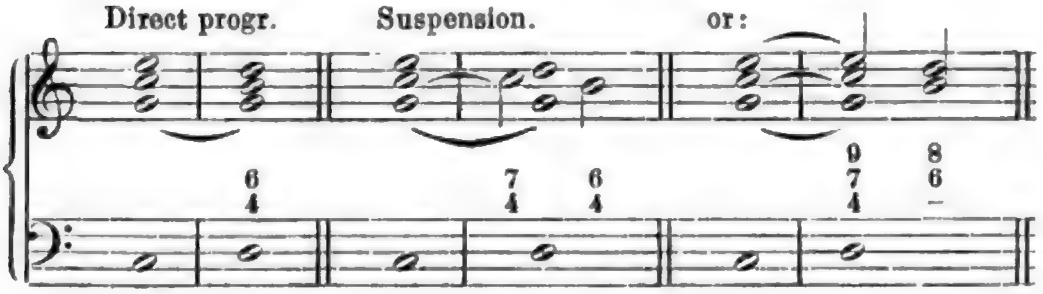




2. *Before the chord of the sixth.*

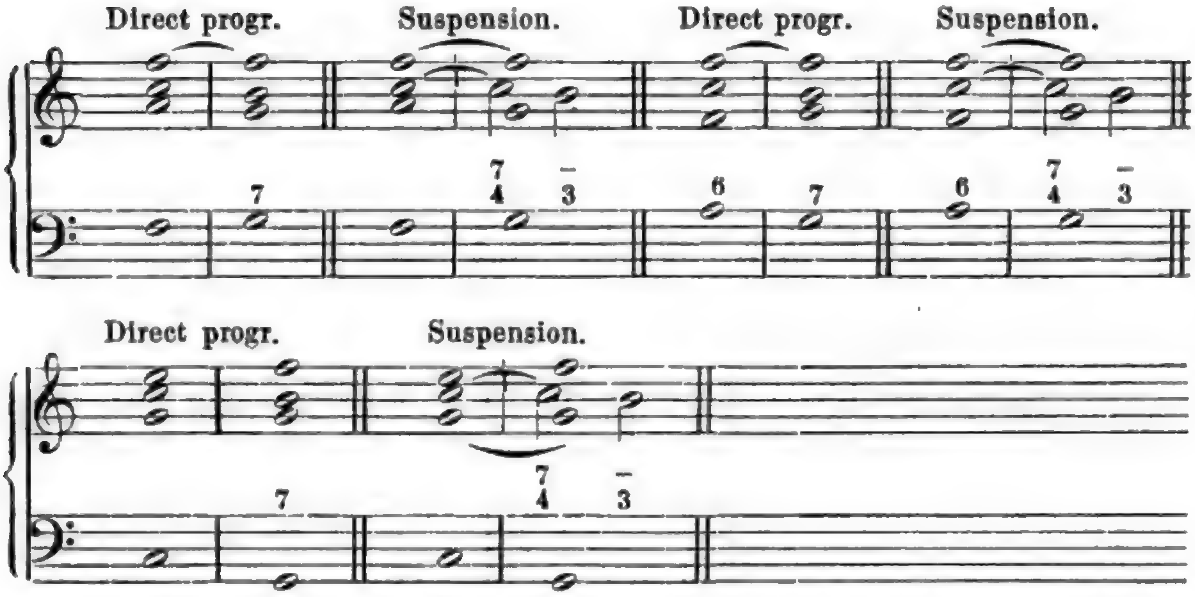


3. *Before the chord of the fourth and sixth.*

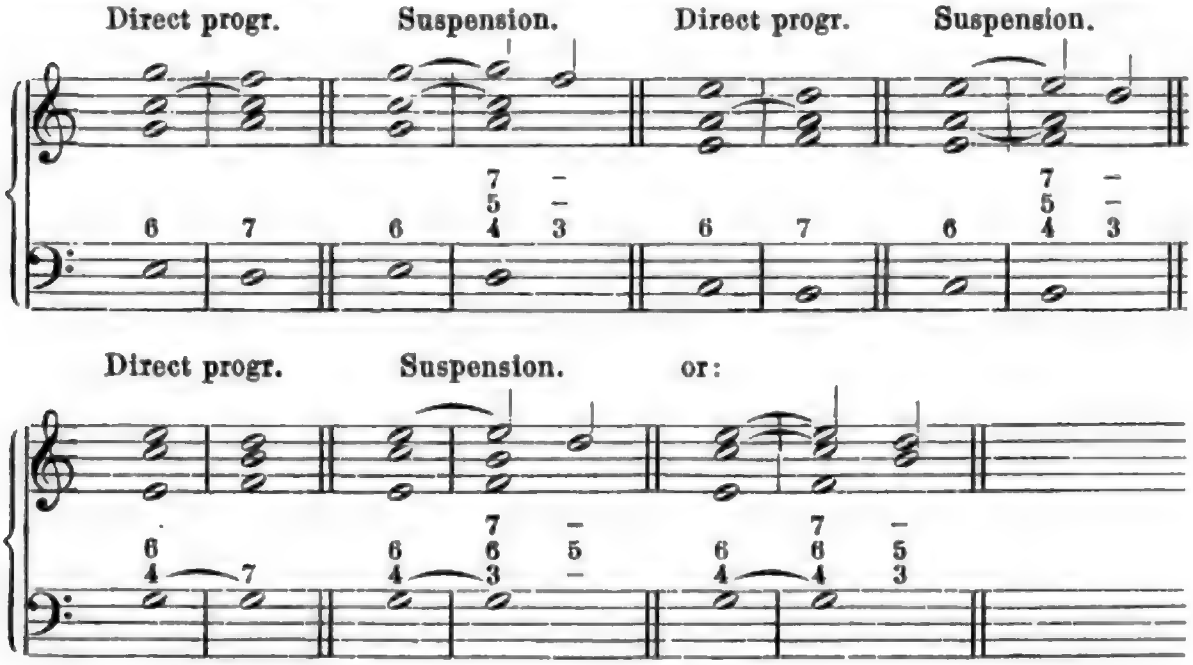


II. Suspensions before Chords of the seventh.

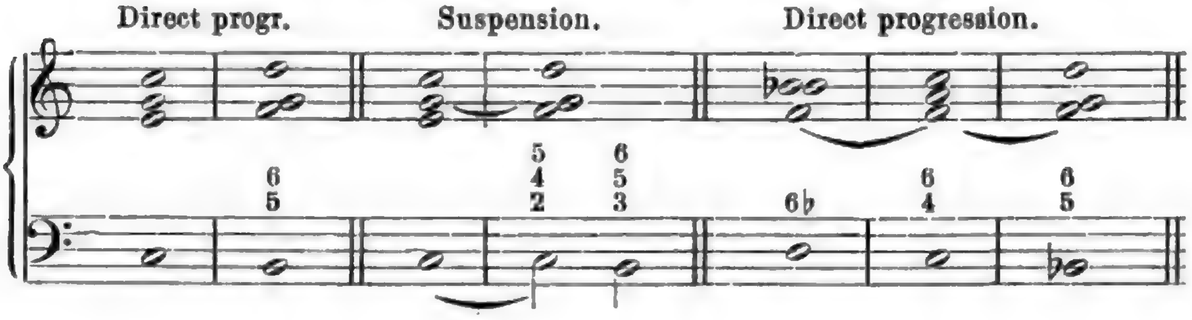
1. *Before the chord in its fundamental position.*

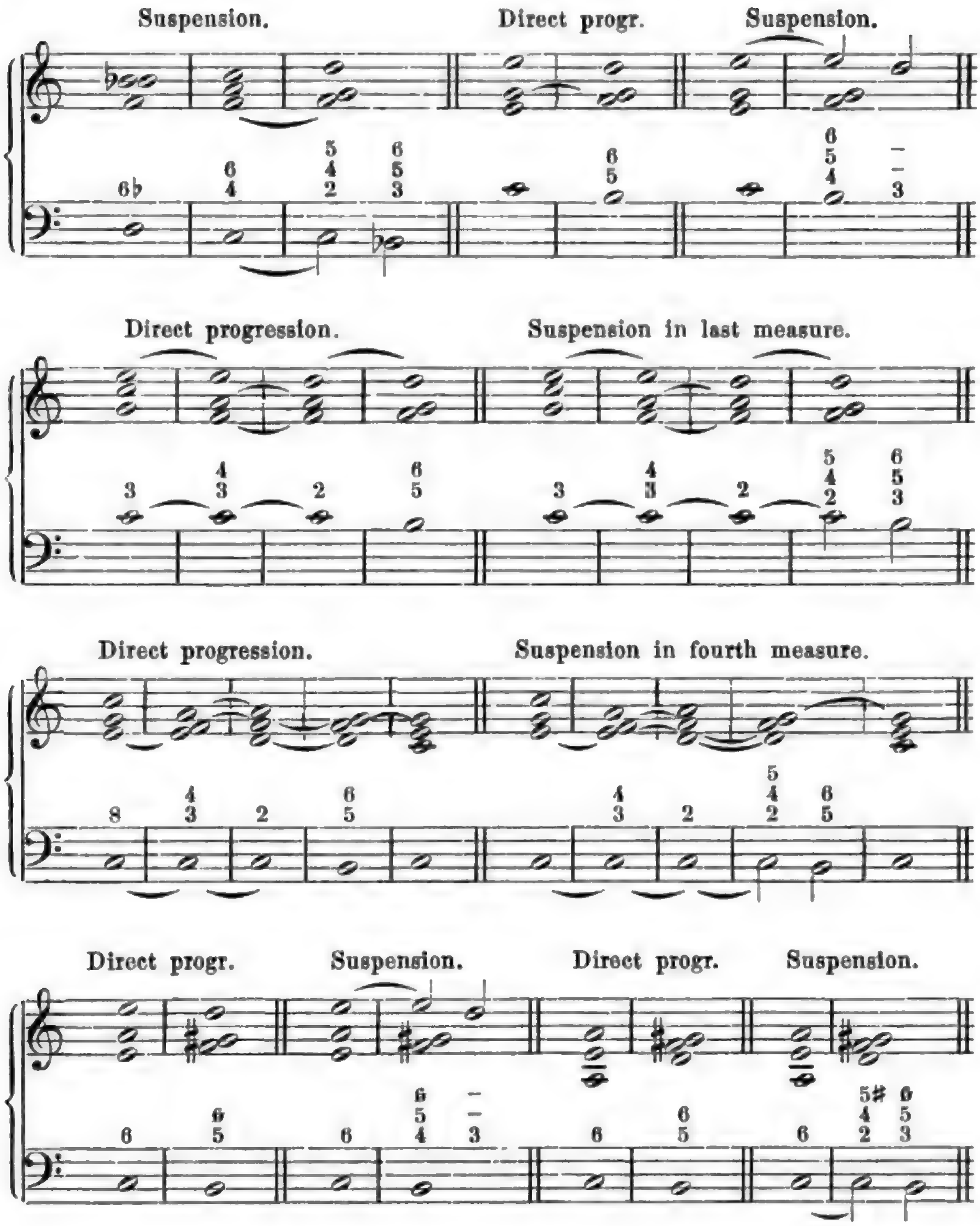


This last form can be used only with dominant chords of the seventh, whose seventh does not need preparation if the fundamental be present.



2. *Suspensions before the chord of the third, fifth and sixth.*



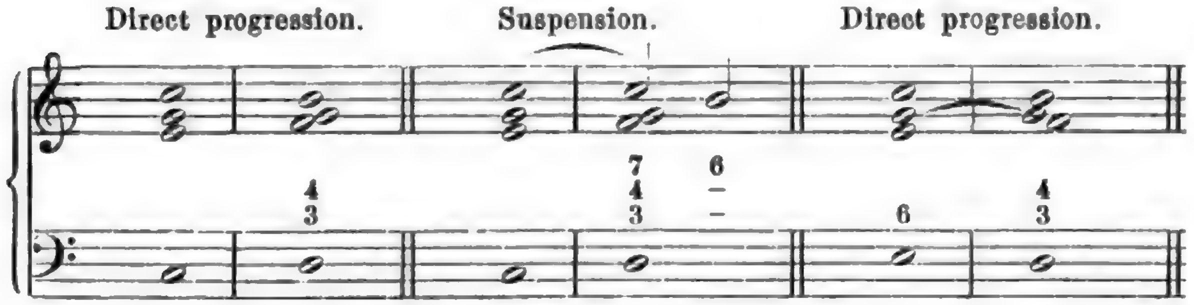

Suspension. Direct progr. Suspension.

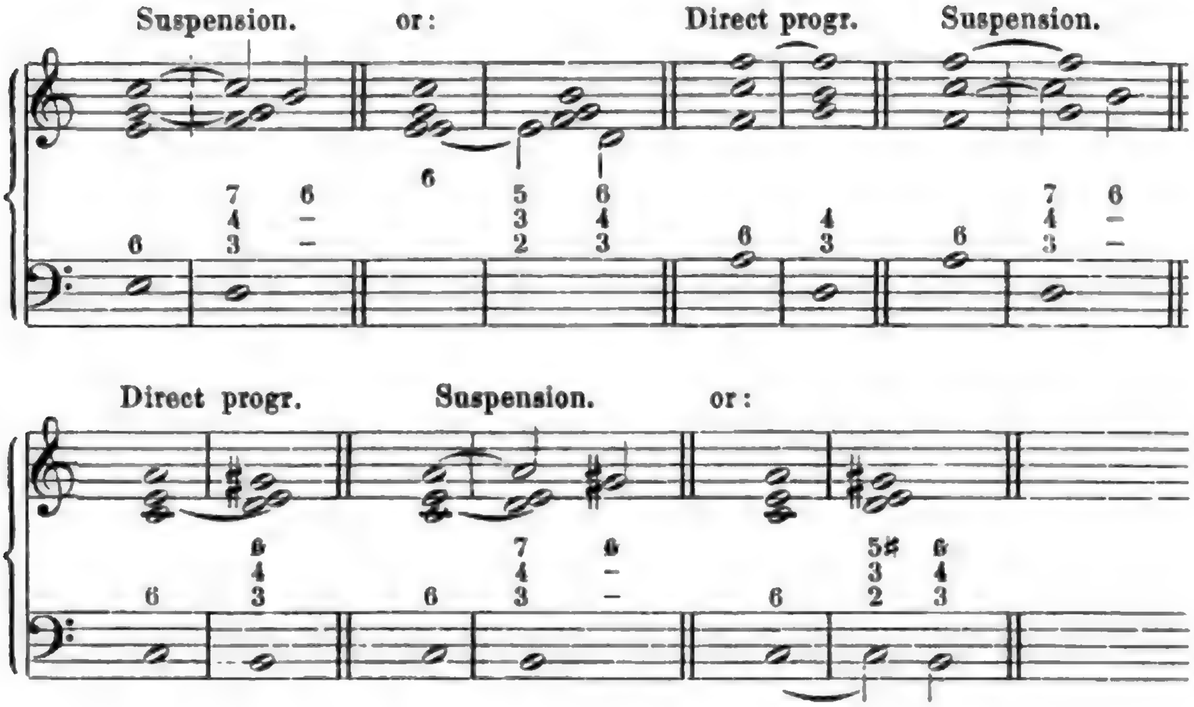
Direct progression. Suspension in last measure.

Direct progression. Suspension in fourth measure.

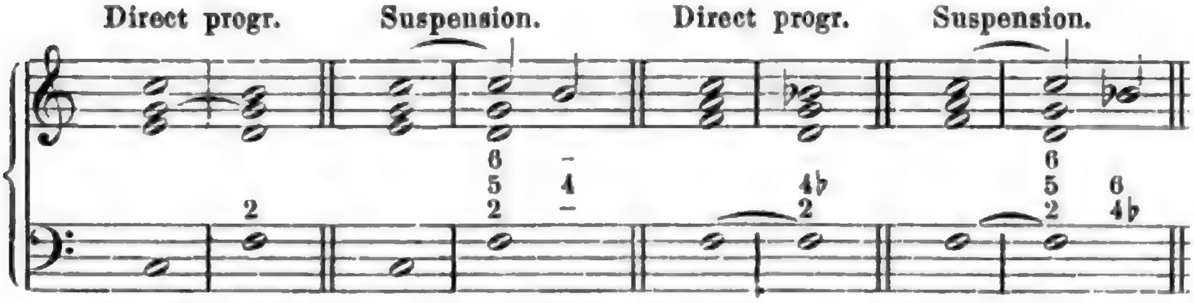
Direct progr. Suspension. Direct progr. Suspension.

3. *Suspensions before the chord of the third, fourth and sixth.*

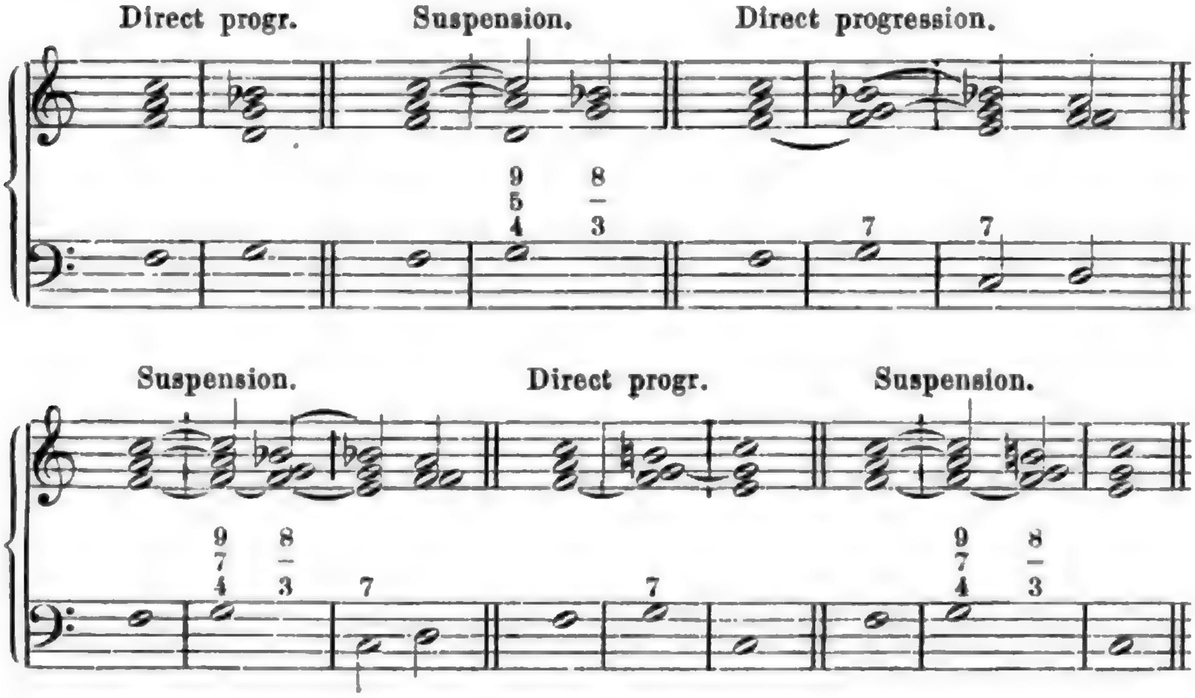

Direct progression. Suspension. Direct progression.



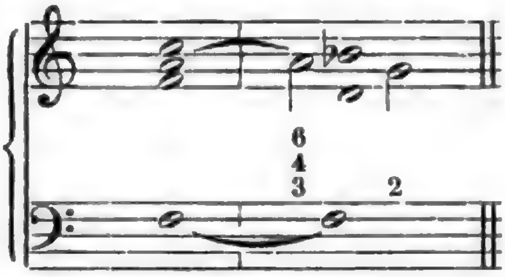
4. *Suspension before the chord of the second.*



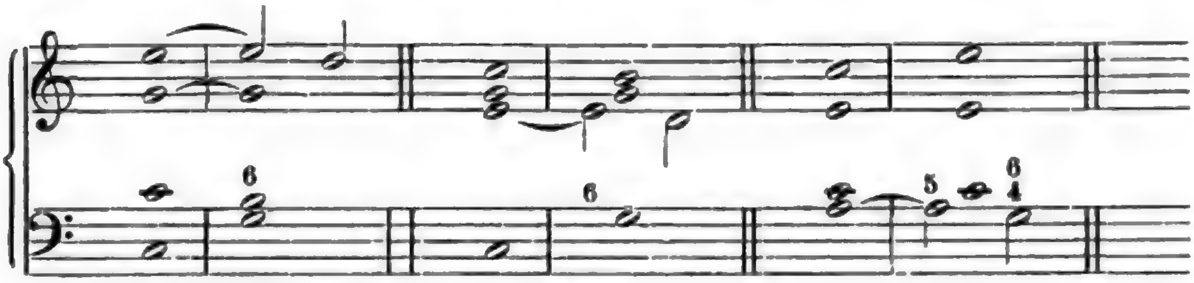
5. *Suspensions with ninths.*



Such successions of chords of the seventh as this:

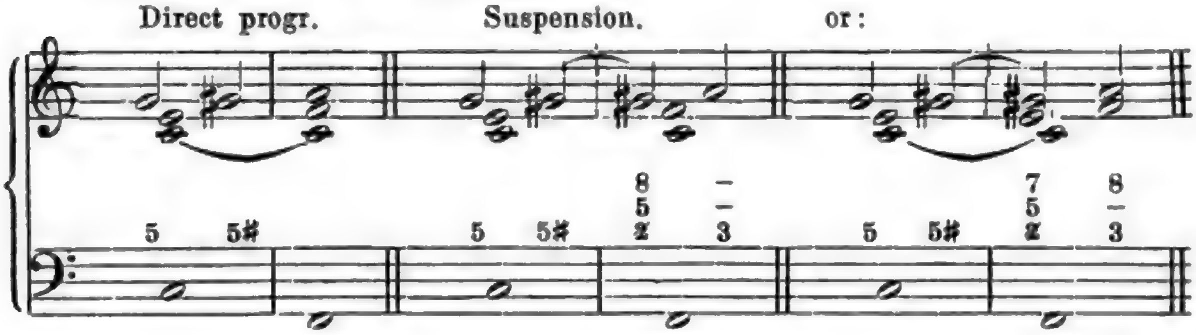


are, of course, not to be regarded as suspensions, they being entirely independent chords; for although the chord of the third, fourth and sixth precedes the chord of the second, it is nevertheless not a suspension of the latter. It is a gross error to give consonant triads as examples of suspensions; because progressions like these:



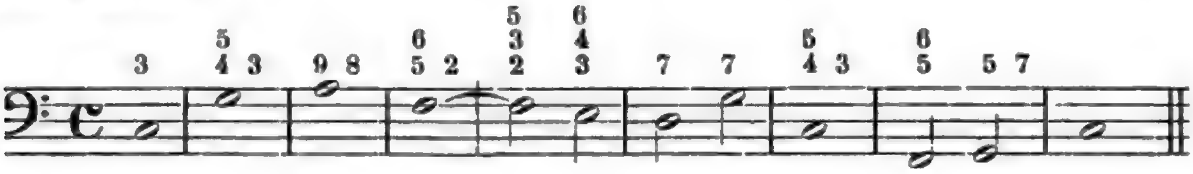
are simply progressions between triads, and no true suspensions.

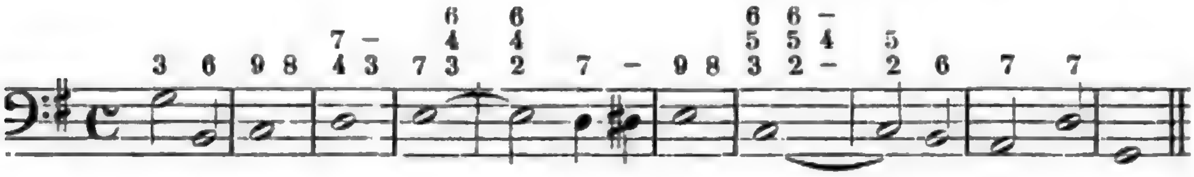
The augmented triad is, on the other hand, to be employed in suspensions having an upward resolution:

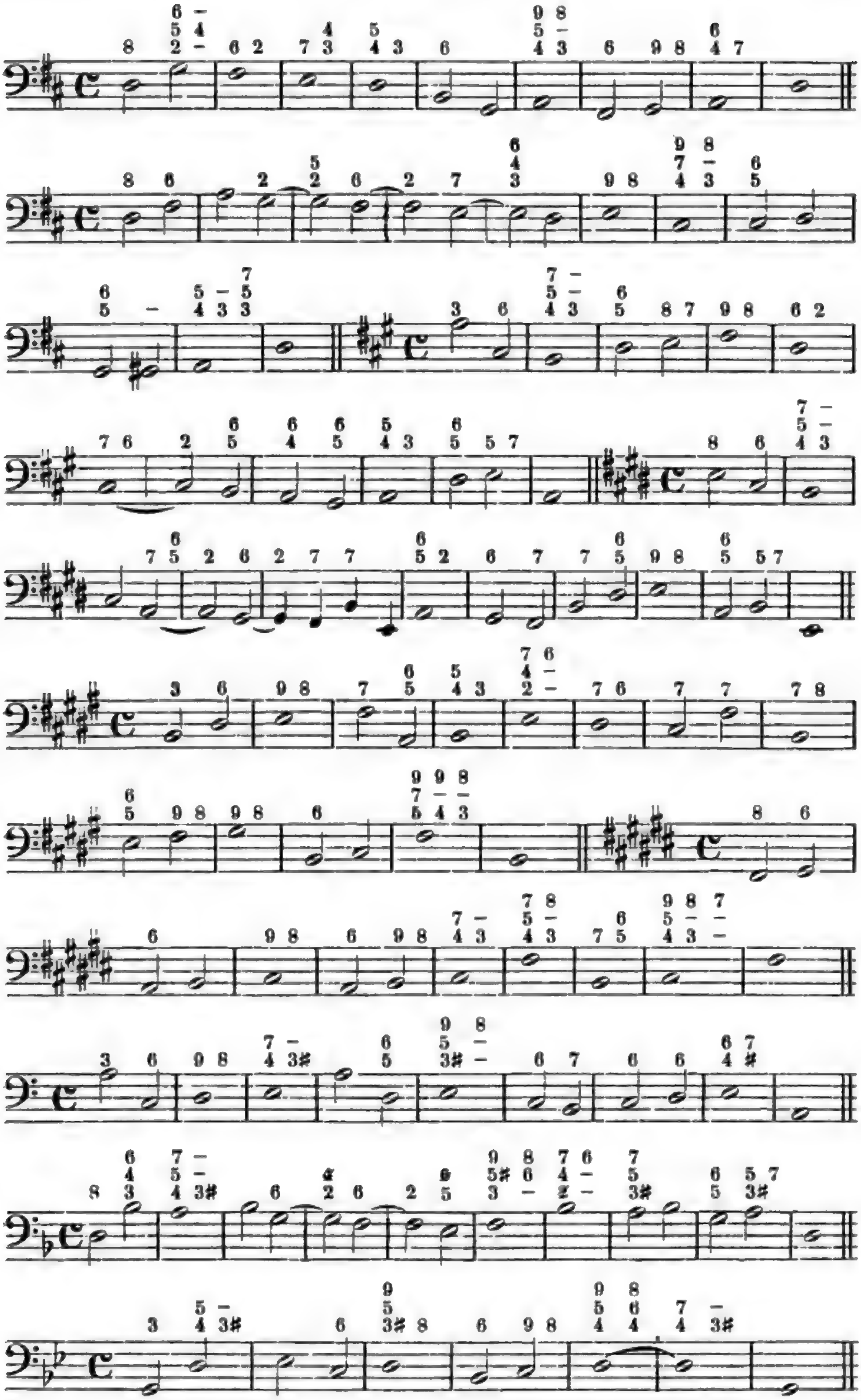


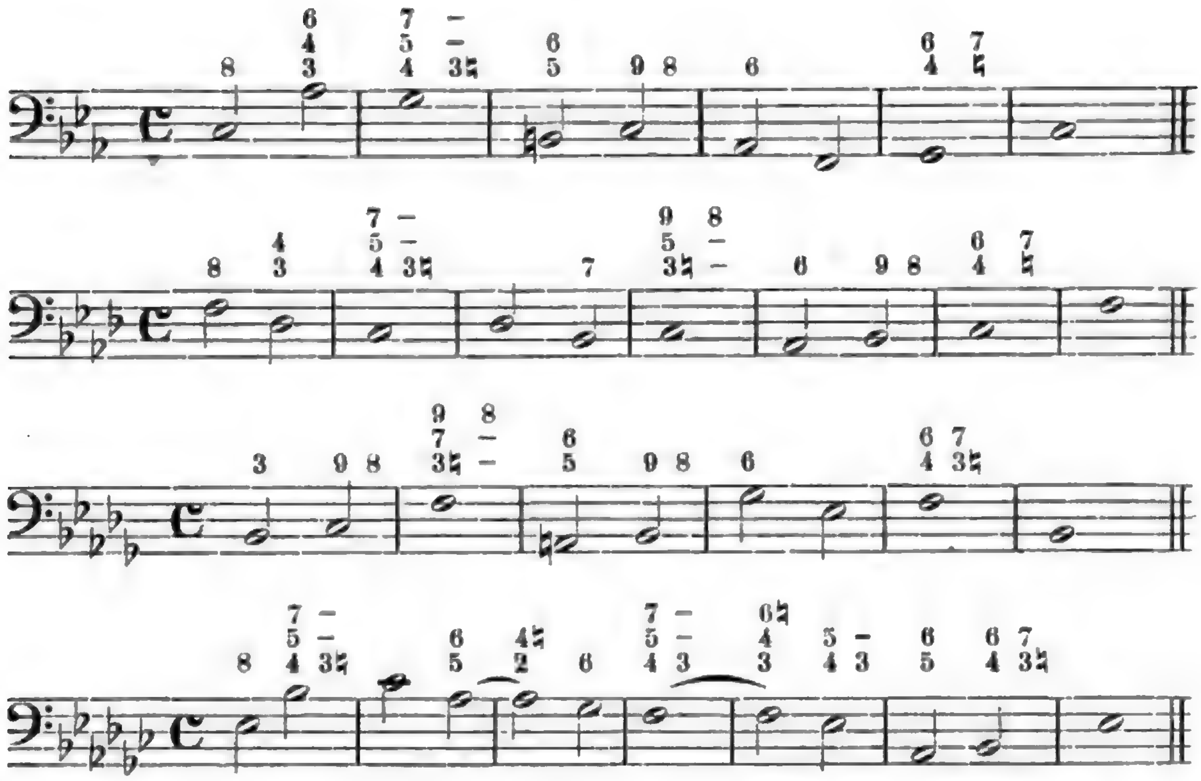
Here the suspension is prepared through the augmented fifth, which resolves itself upwards.

The following exercises will aid in impressing the foregoing rules on the memory.









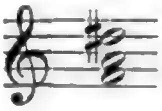
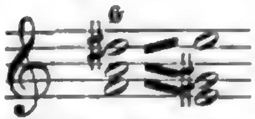
Suspensions with an upward resolution may also be employed, when augmented intervals are formed, in resolutions of the so-called "augmented chords", the names of which, as given in various manuals of harmony, are: "the augmented chord of the sixth; augmented chord of the fifth and sixth; augmented chord of the third, fourth and sixth; and augmented chord of the second, fourth and sixth".

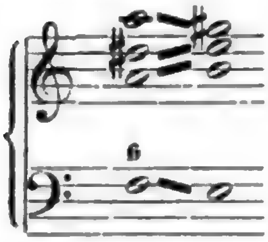
CHAPTER XIII.

The Augmented Chords.

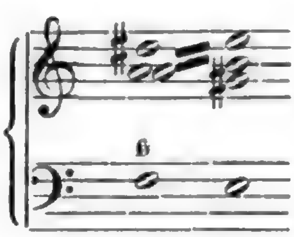
The explanations usually offered of the derivation of these chords can be regarded neither as scientific nor as practical. I have invariably found that even the most talented and industrious scholars, who had only studied the definitions given by Richter, and had not made themselves acquainted with HAUPTMANN's logical demonstration, were without a thorough and intelligent comprehension of their character. In order to exhibit the chords in such a manner that the pupil may, previous to their demonstration, impress their structure upon his memory, a few preliminary remarks are necessary. The augmented chord of the sixth, the augmented chord of the fifth and sixth, and the augmented chord of the third, fourth and sixth lie on the sixth degree of the harmonic minor scale, and are resolved to the dominant.

1. *The augmented chord of the sixth.*

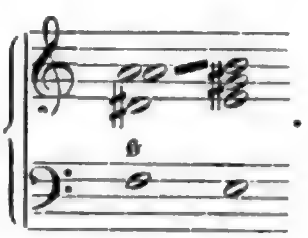
This chord has a major third and augmented sixth; *e. g.* the sixth degree in *A*-minor is *f*; and the augmented chord of the sixth, having a major third and augmented sixth, would be ; as resolved to the dominant . If used in four-part harmony the third, as consonant with the lower tone, should be doubled, giving the following resolutions:

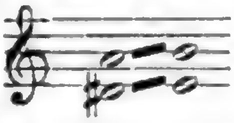
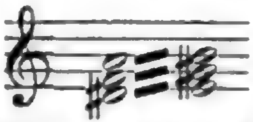
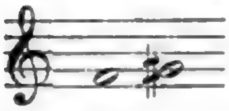
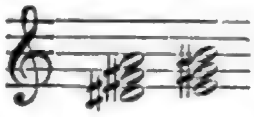


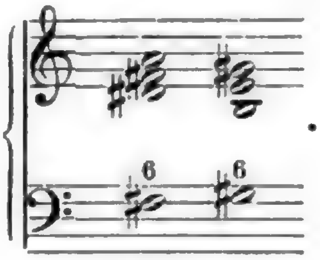
or:



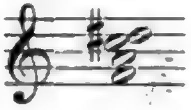
but not:

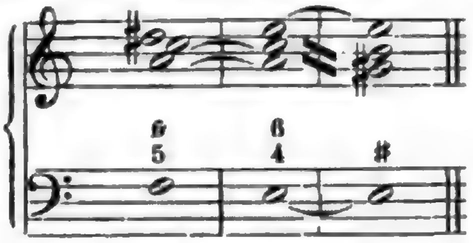


The last progression appears of doubtful correctness, not only on account of the parallel fifths  but also because the chord in its fundamental position could not progress thus: . Here we find, besides the parallel fifths, the unmelodic augmented second , even leaving out of the question the fact that these indirectly connected triads cannot, any more than these , make such a progression, although in this last example, the augmented second not occurring, the triads may be led in contrary motion:



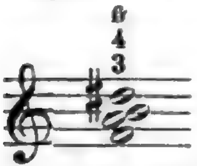
2. *The augmented chord of the fifth and sixth.*

This chord is based upon the sixth degree in minor, and has a major third, perfect fifth, and augmented sixth. Its form, in *A*-minor, would be . It passes in its resolution through the chord of the fourth and sixth derived from the tonic triad to the dominant triad:

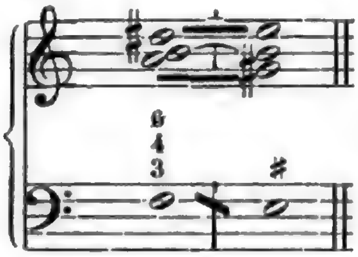


3. *The augmented chord of the third, fourth and sixth.*

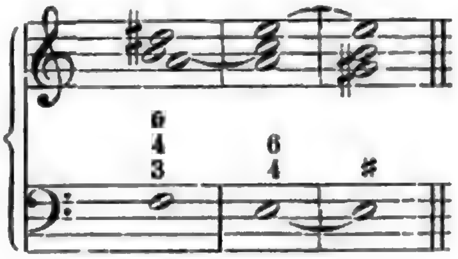
This chord, which also lies upon the sixth degree, has a major third, augmented fourth, and augmented sixth, and appears, in *A*-minor, thus:



. It may be resolved either to the dominant triad directly, or indirectly through the chord of the sixth and fourth derived from the tonic triad:



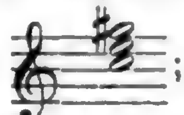
or:



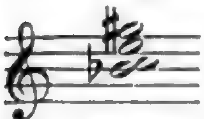
Finally the

4. *Augmented chord of the second, fourth and sixth*

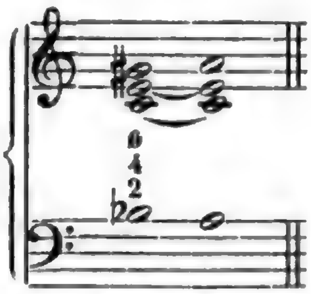
is to be noticed. It contains the augmented triad in minor, and, below the fundamental tone of this triad, a major second. The augmented triad in *A*-minor is *c*-*E*-*g*♯ =



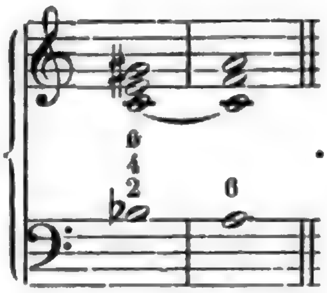
; *c* is the fundamental, and *b*♭ the major second below the fundamental; the entire chord would therefore be



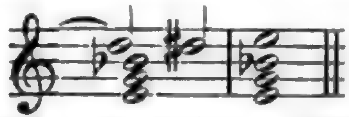
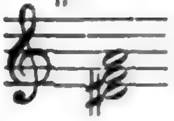
. Its resolution is dependent upon that of the augmented triad, which exercises a predominating influence, and would consequently be either to the chord of the tonic, or to the triad on the sixth degree in minor (in *A*-minor *f*-*A*-*c*):

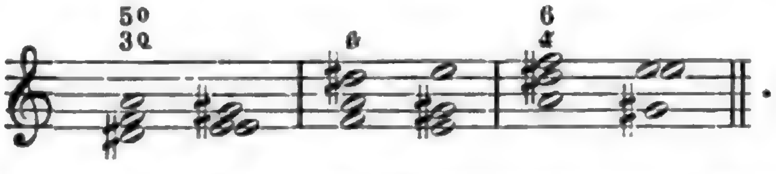


or:

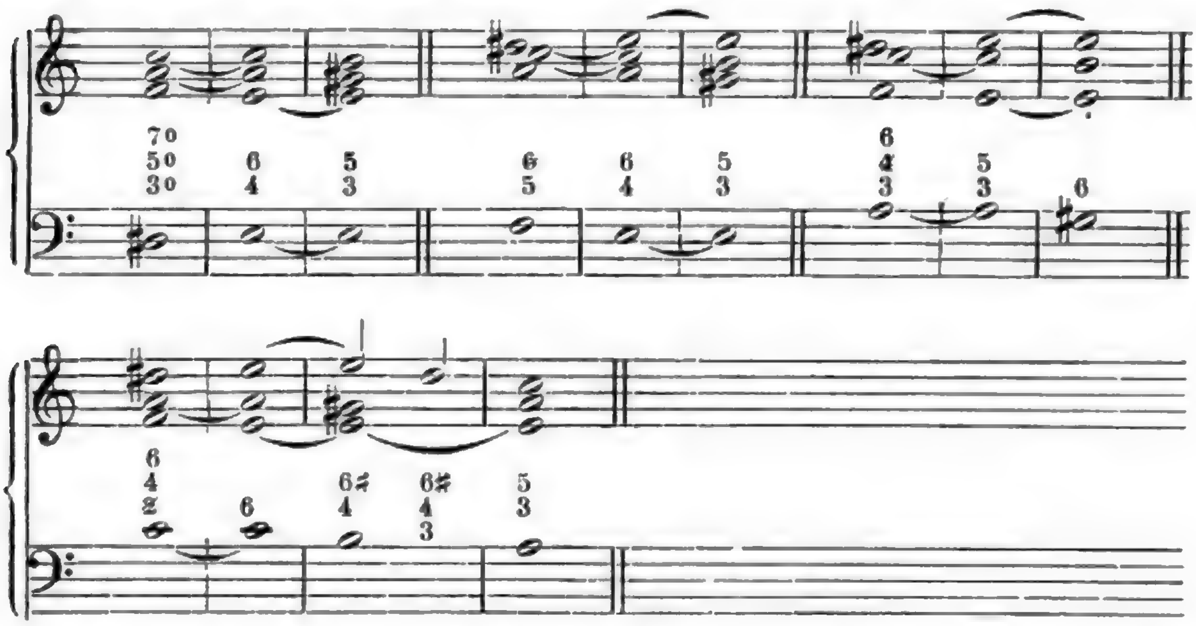


These chords are sometimes named "altered chords", *i. e.* chords formed by means of chromatic changes.

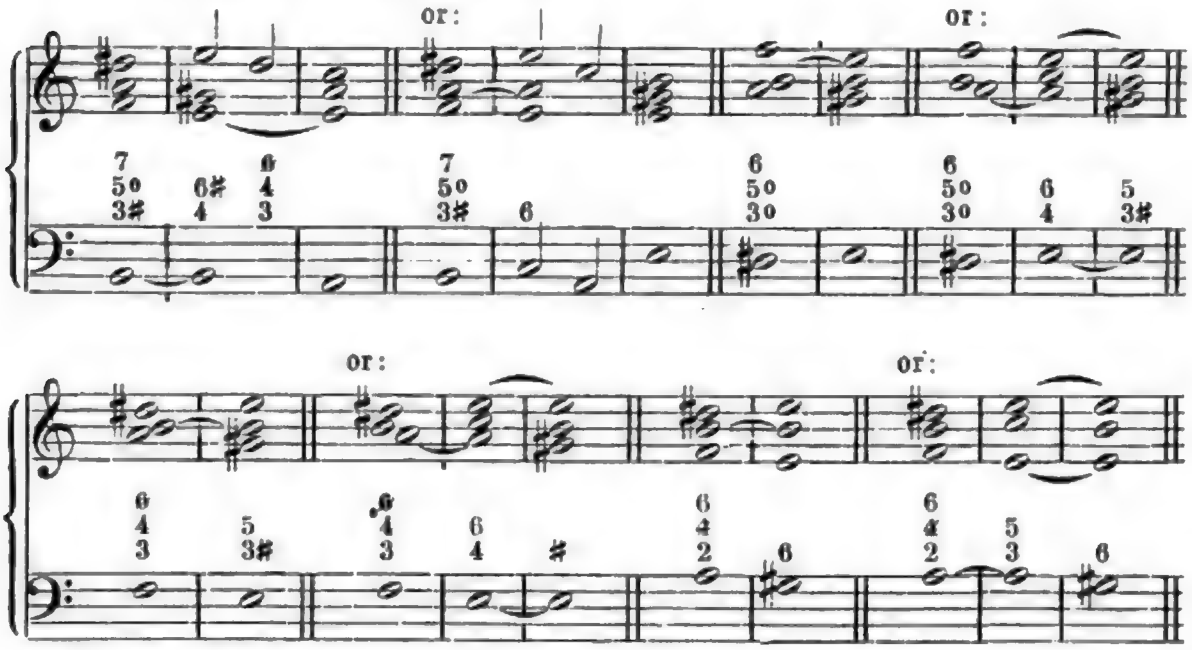
Tones raised by chromatic signs do not, however, occur in the formation of elementary chords. Chromatic tones which do not assist in forming chords, can be regarded merely as passing notes, and cannot lead to "altered chords", which are, in reality, non-existent in harmony. The following example shows a chromatic passing note , the minor seventh passing through *d*♯ in its upward progression. This *d*♯ on the arsis does not, however, alter the fundamental harmony, and cannot be considered as metrically and harmonically equal to the constituent elements of the chord. The only scientifically correct demonstration of the formation of the augmented chords is that of **Moritz Hauptmann**, which is based upon an extension of the formula for the modes. This may be explained, in popular language, as follows. The formula for the minor mode is extended, on the dominant side, as far as the third of the dominant fifth. *E. g.*, the formula for *A*-minor is *D-f-A-c-E-g*♯*-B*. If we desire to extend the formula by adding a new triad based on *B*, this triad would be *B-d*♯*-F*♯. [**Hauptmann**'s theory is, that this extension of the minor formula or "tone system" can be accomplished on the dominant side only by the addition of major triads, the dominant triad being major; and on the sub-dominant side by adding minor triads, the sub-dominant triad being minor. The formula so extended he calls the "overlapping" or "overreaching" system.] By this procedure, however, the key of *A*-minor is given up for the key of the dominant, the combined key of *E*. Remaining in *A*-minor we extend the formula only to *d*♯, the third of the dominant fifth, thus giving up the fundamental of the sub-dominant. The formula would then be /*f-A-c-E-g*♯*-B-d*♯/. By setting *d*♯ next to *f*, the sub-dominant third, the triad *d*♯/*f-A*: , is formed, which contains the third of the dominant fifth, the sub-dominant third, and the sub-dominant fifth (= the fundamental of the tonic triad). This triad belongs to the formula or mode as extended on the dominant side, and is generally employed as a chord of the sixth or so-called "augmented chord of the sixth", although it also occurs in other positions, *e. g.*:

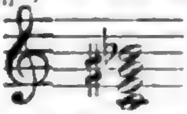
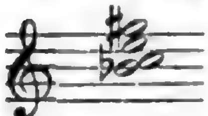


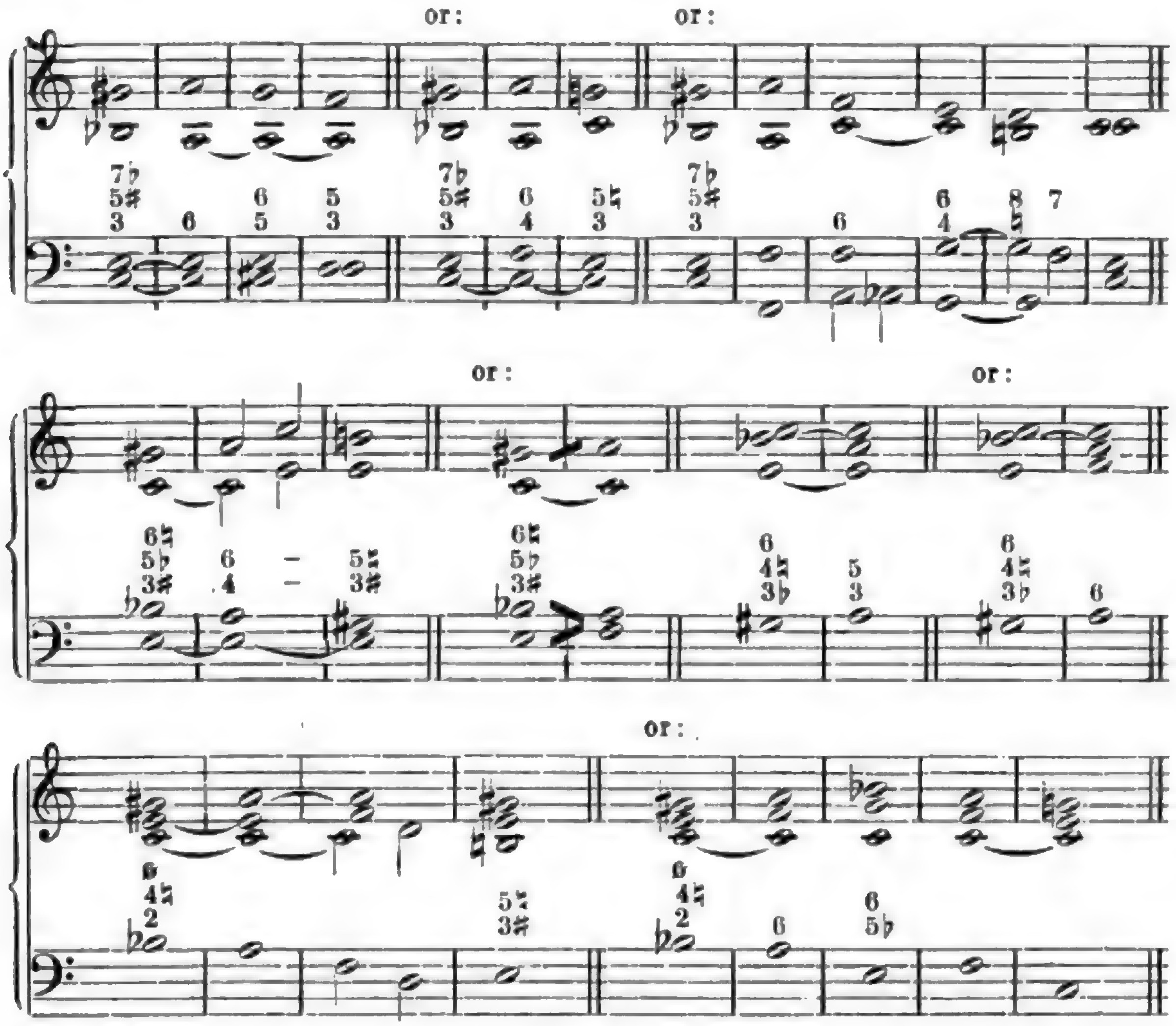
The formula *d*♯/*f-A-c-E-g*♯*-B-d*♯/ likewise contains the chord *d*♯/*f-A-c*, the first inversion of which is the so-called "augmented chord of the fifth and sixth". This chord, although usually met with in the above position, may also take other forms, as follows:



By setting the dominant fifth and its third, as given in the above formula, next to the third and fifth of the sub-dominant, we obtain the fundamental form of that chord, (*B̄-d♯/f-A*) whose second inversion or third position is the so-called "augmented chord of the third, fourth and sixth". This chord is most naturally employed in the given position, but may, like the others, take various forms.



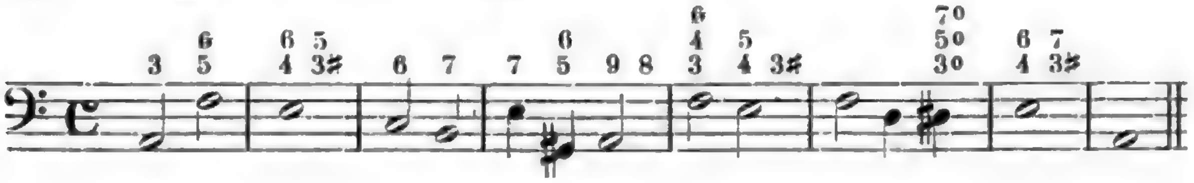
We obtain the fundamental position of the so-called "augmented chord of the second" by extending the minor formula *D-f-A-c-E-g♯-B* on the sub-dominant side, giving up the dominant fifth. The triad formed below *D* would be *G-b♭-D*. After adding this triad, thus abandoning the minor mode, the formula would appear as *G-b♭-D-f-A-c-E*. If however, the formula be extended downwards only to *b♭*, *b* alone is given up: */b♭-D-f-A-c-E-g♯/*; *b♭* set next to *g♯* gives the fundamental form of the chord *c-E-g♯/b♭*:  ; the third inversion or fourth position of which is the so-called "augmented chord of the second, fourth and sixth" . This chord can also be employed in other positions, as follows:

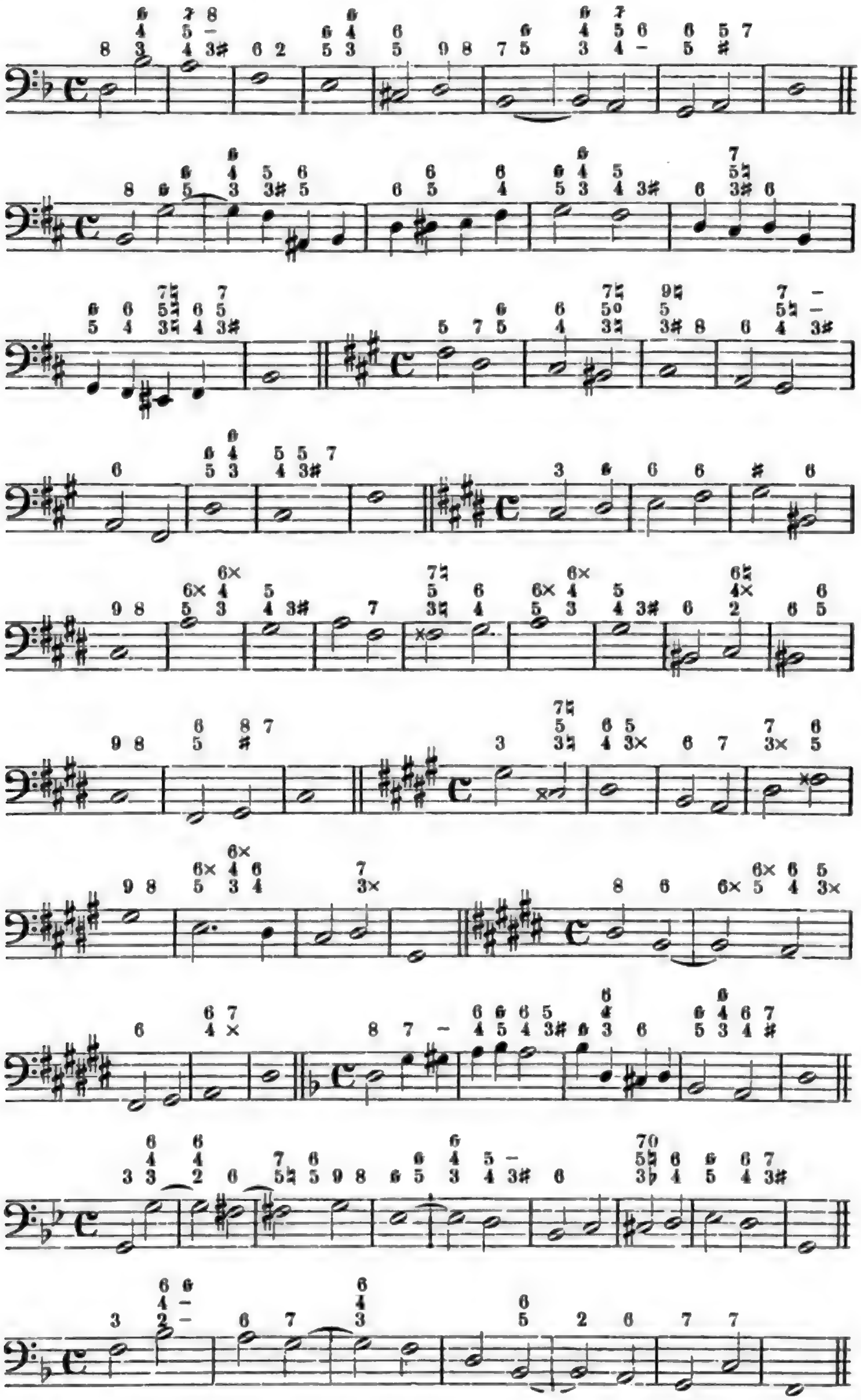


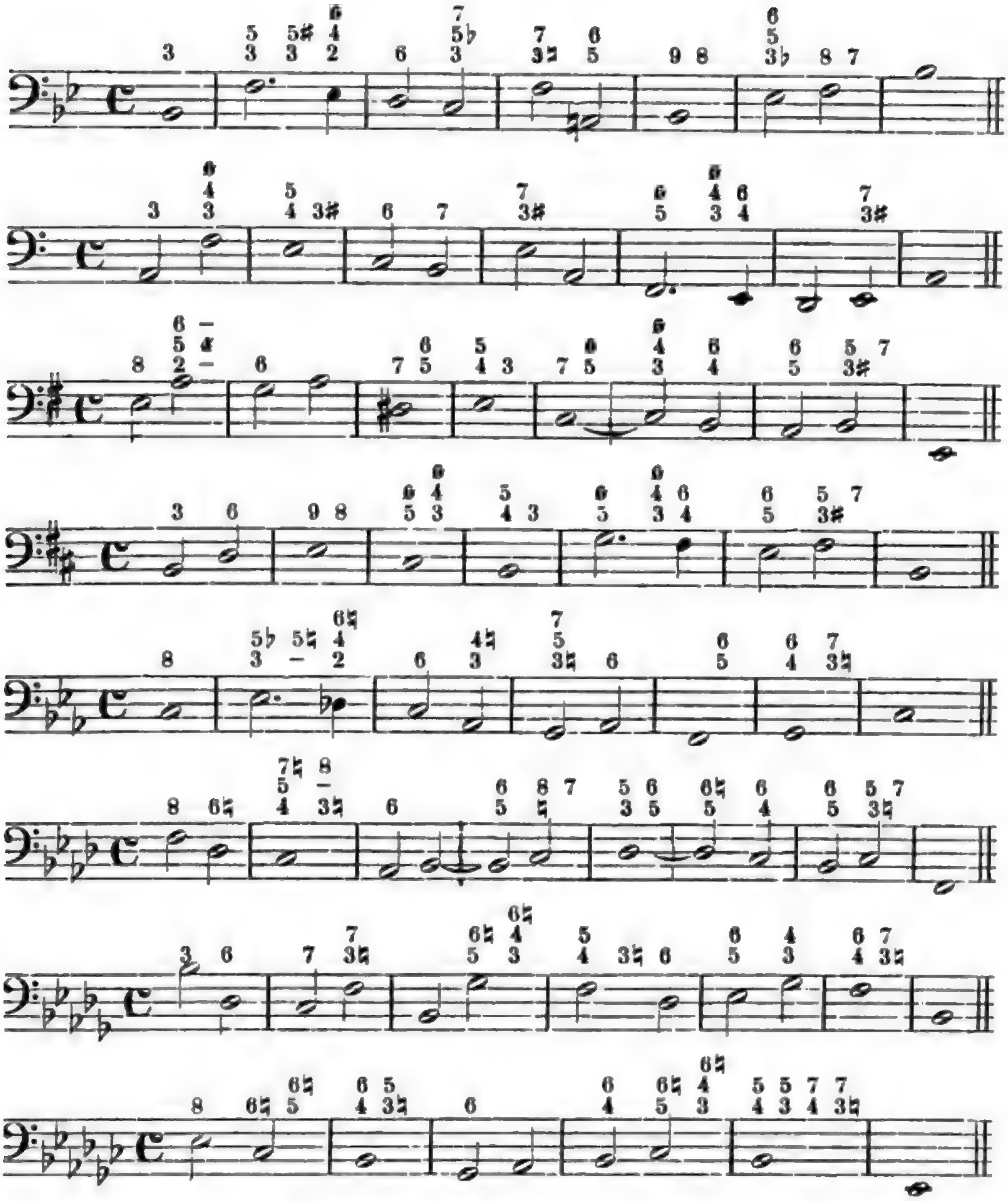
The following exercises are for practice in the

"*Chords of the 'extended' modes*".

In the thorough-bass figuring here given a nought (o) is frequently placed beside a figure, to indicate that the interval is a diminished one, and to prevent mistakes on the part of the pupil. A well-trained musician naturally needs no such helps.







It is apparent from the foregoing elucidation that "altered chords" have no actual existence, and that the so-called "augmented chords" are not formed by merely raising the tones through chromatic signs, but are the product of an organic development. HAUPTMANN's axiom, that "every tone in music has a definite place in a chord belonging to a definite key", must be taken literally. Through the employment of these last-mentioned chords a wider field is opened to the art of modulation, which can be thoroughly comprehended only by applying HAUPTMANN's principle. The augmented chords, together with the chords of the seventh, are pre-eminently adapted for use in modulation.

CHAPTER XIV.

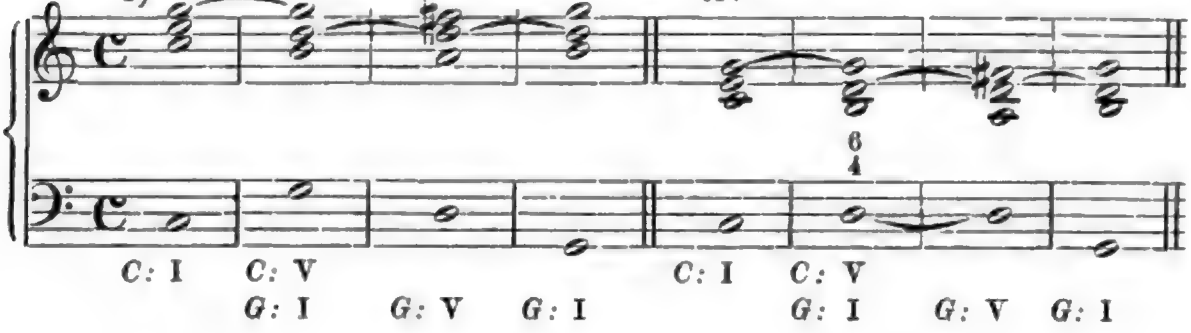
Modulation.

The affinity between the chords is recognizable chiefly through the tones which they possess in common. Such tones are especially helpful in effecting modulations from one key to another, the various families of tones being thus brought into closer connection. With triads alone the most manifold modulations, based upon this principle, may be accomplished without in the least infringing upon the laws of correct progression. The composer's power of combination finds here an opportunity for expansion in the most diverse directions, in all of which the end desired may be attained if the above principle be strictly kept in view. The following examples will suffice, without detailed explanation, to convince the learner of the correctness of these statements. The ligatures between the tones common to successive chords will aid in comprehending the theory of interconnection, which is of the utmost importance in modulating as well as in simple harmonic combinations.

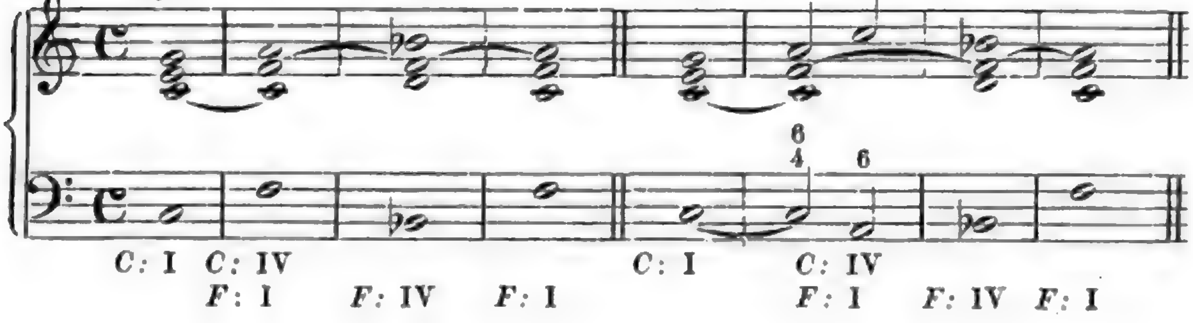
Exercises in Modulation.

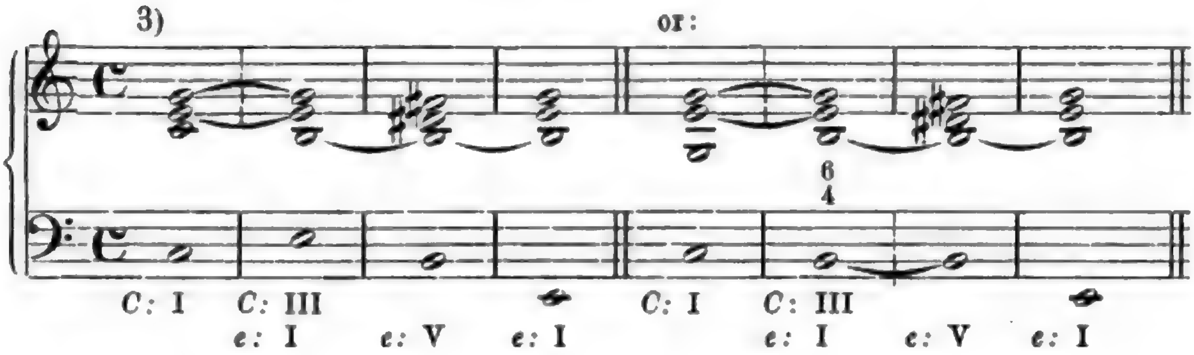
Taking the key of *C*-major as point of departure, the keys most closely connected with it are: *G*-major, *F*-major, *E*-minor, *A*-minor and *C*-minor. Modulation is effected chiefly through the various harmonic significance of the chords. If the triads alone be employed, the above-named keys are to be chosen as those proper for our purpose, as shown in the following examples, in which the major keys are indicated by capitals, and the minor keys by small letters.

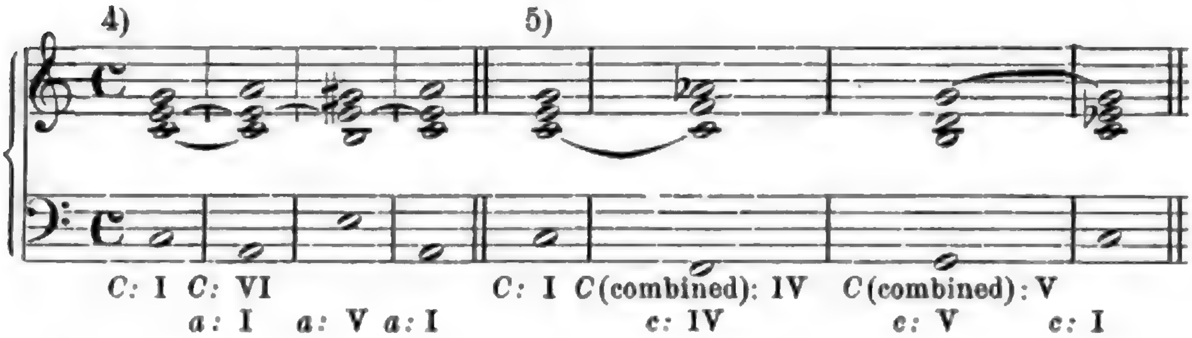
1)



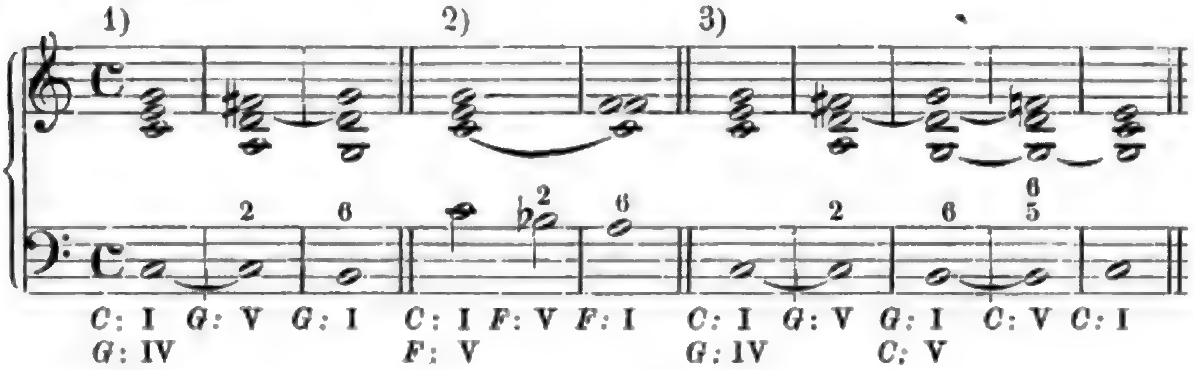
2)

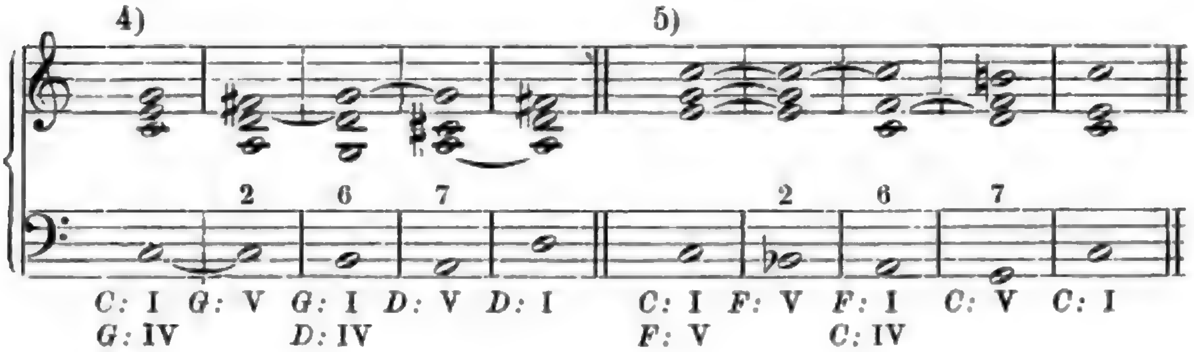


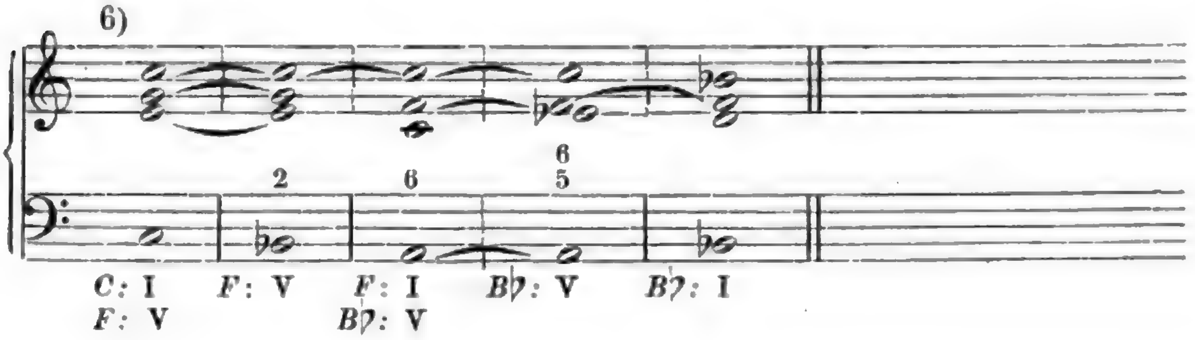


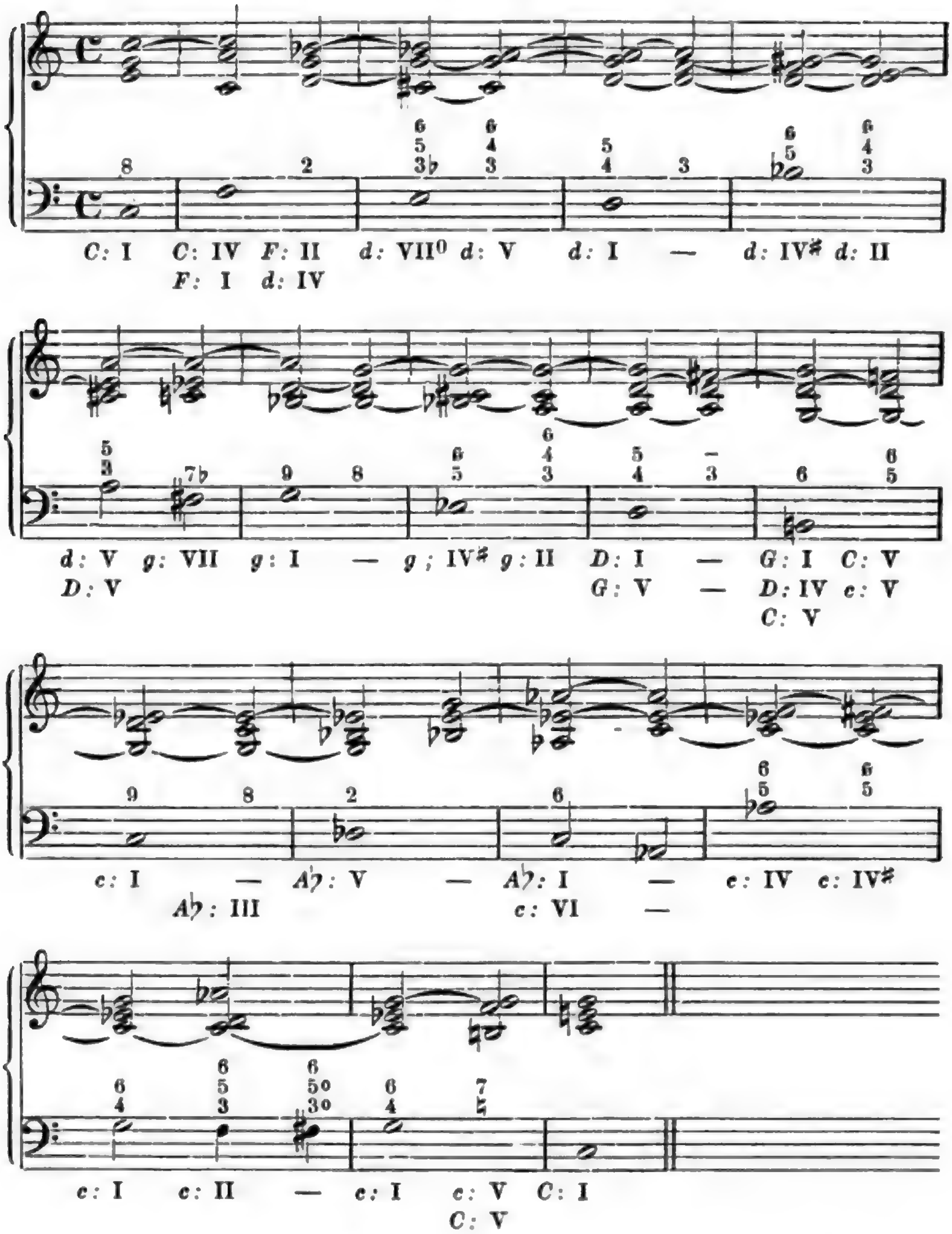


By using the chords of the seventh, particularly the dominant chord of the seventh, in modulating, the keys appear in still closer affinity, as shown below.





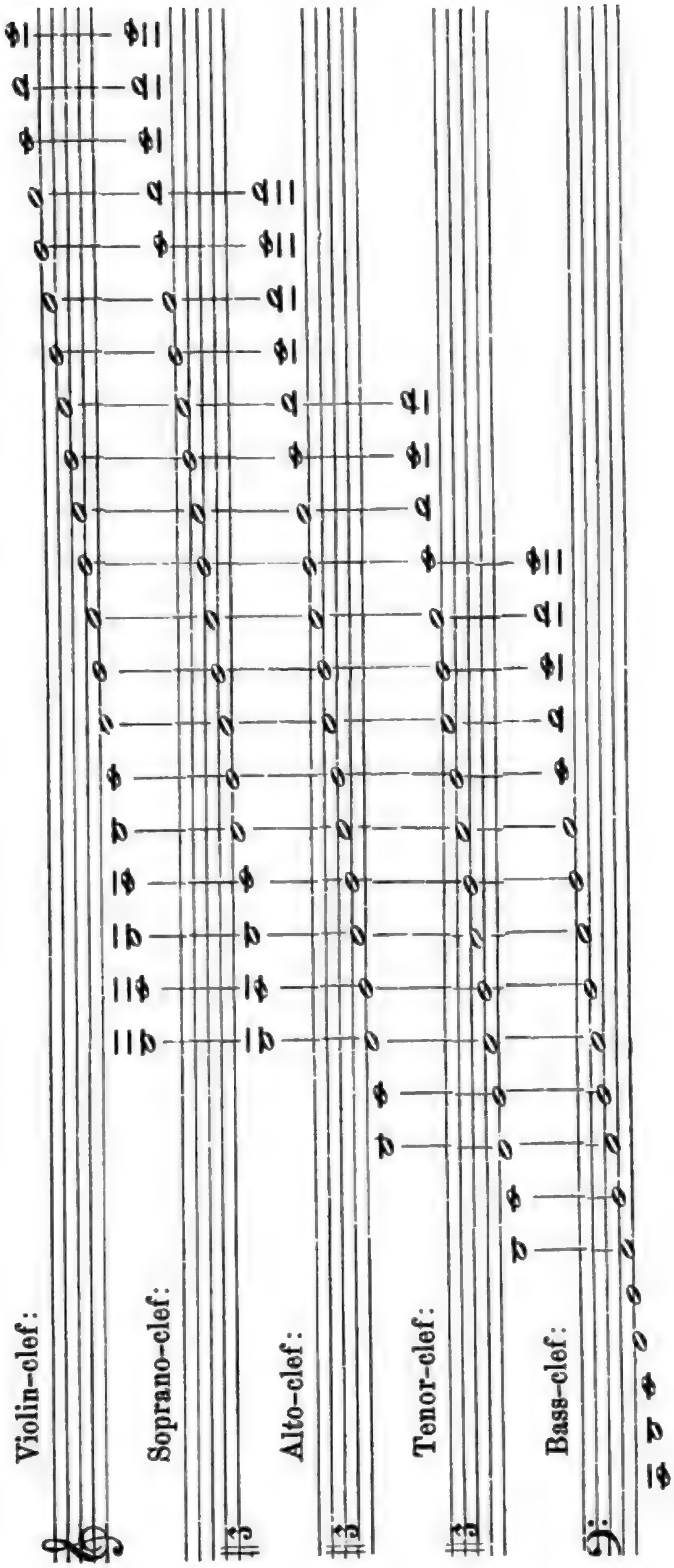




The pupil, after studying these examples, will be able to work out the following exercises.

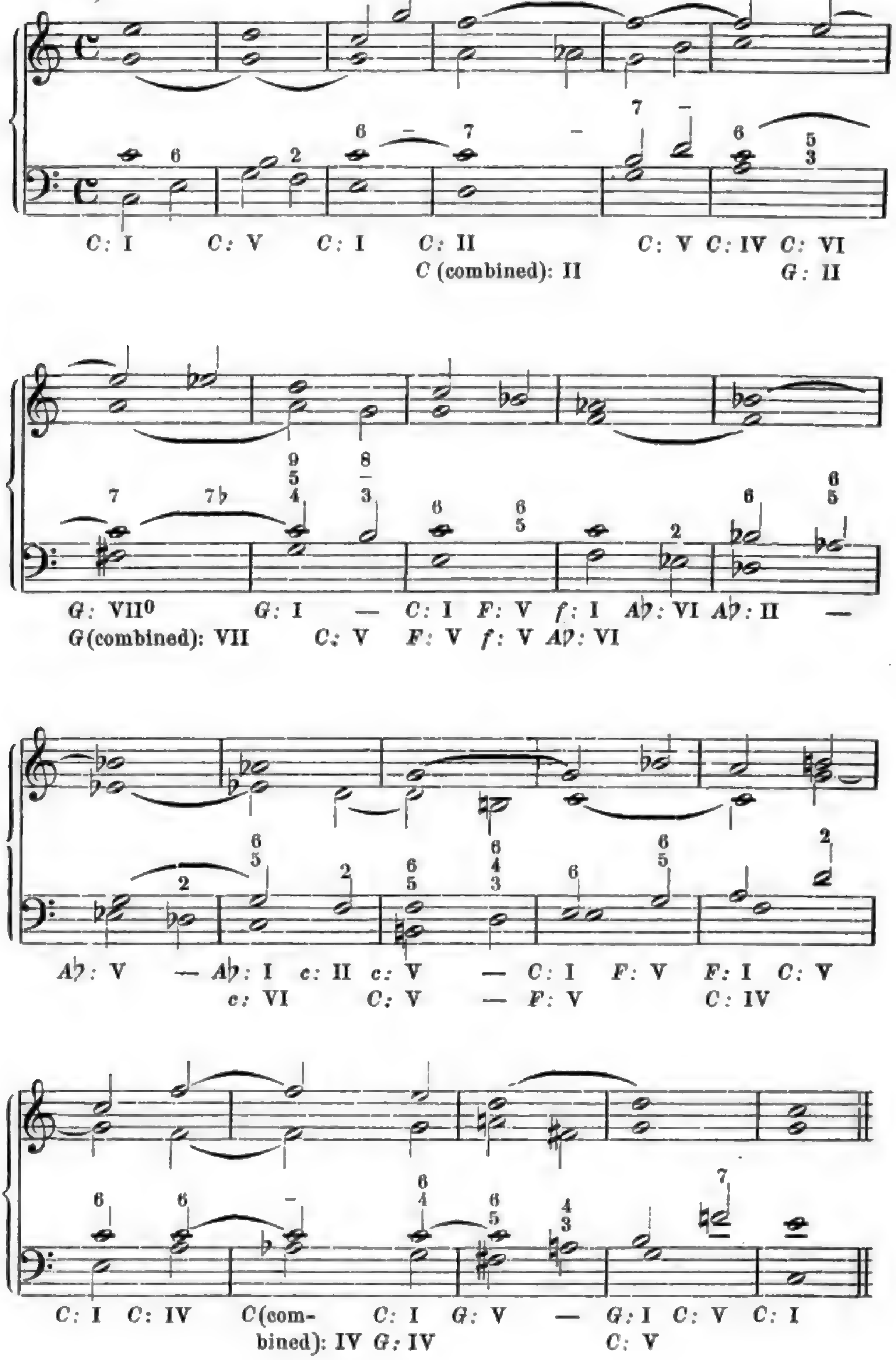
In order to learn the use of the various clefs, all exercises should be written, like the example in modulation given below, first in open position, on two staves, and then copied onto four staves, using four different clefs.

These four clefs may be learned by examining the following comparative table.

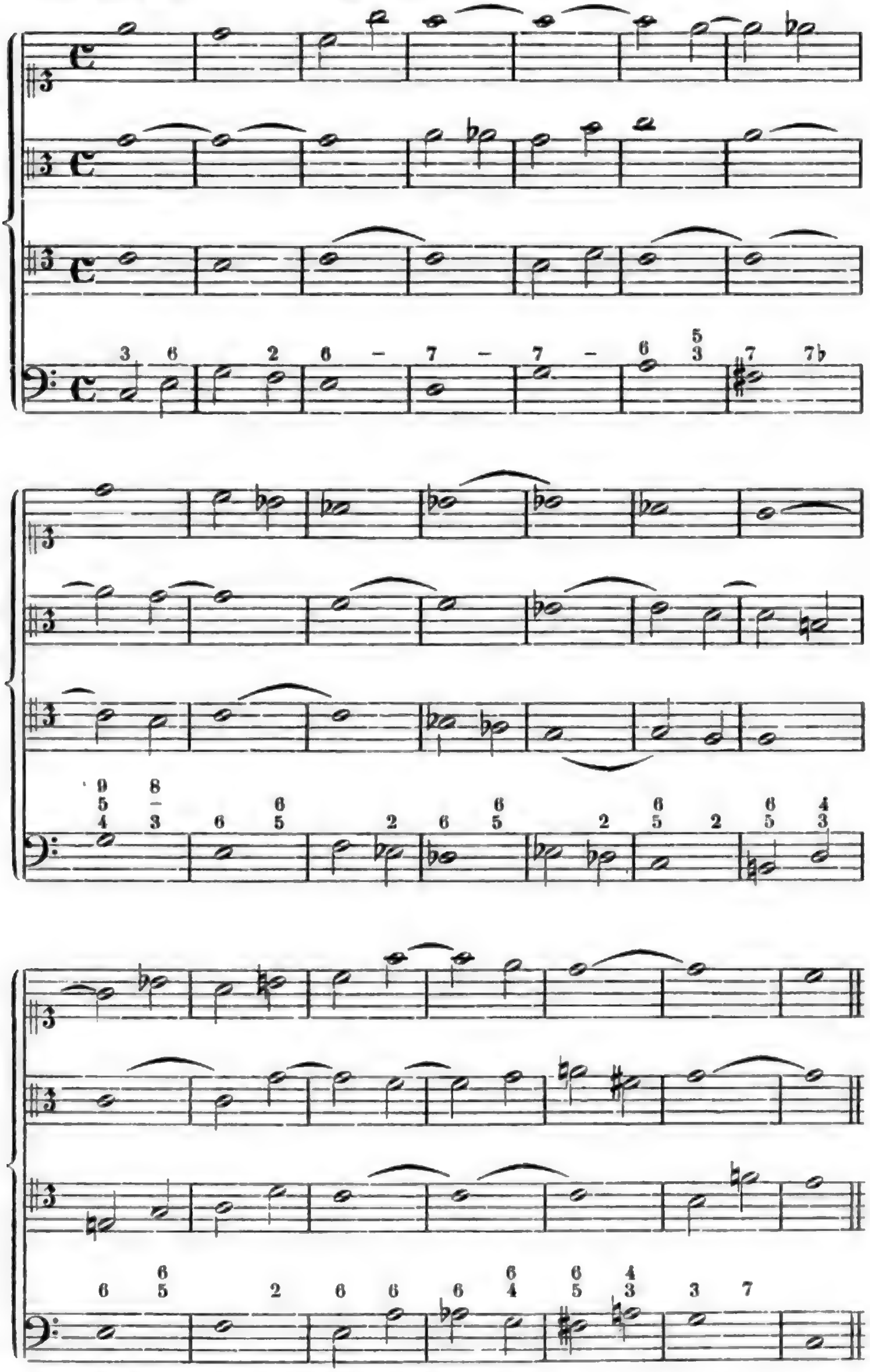

Violin-clef:
Soprano-clef:
Alto-clef:
Tenor-clef:
Bass-clef:

Examples in Modulation.

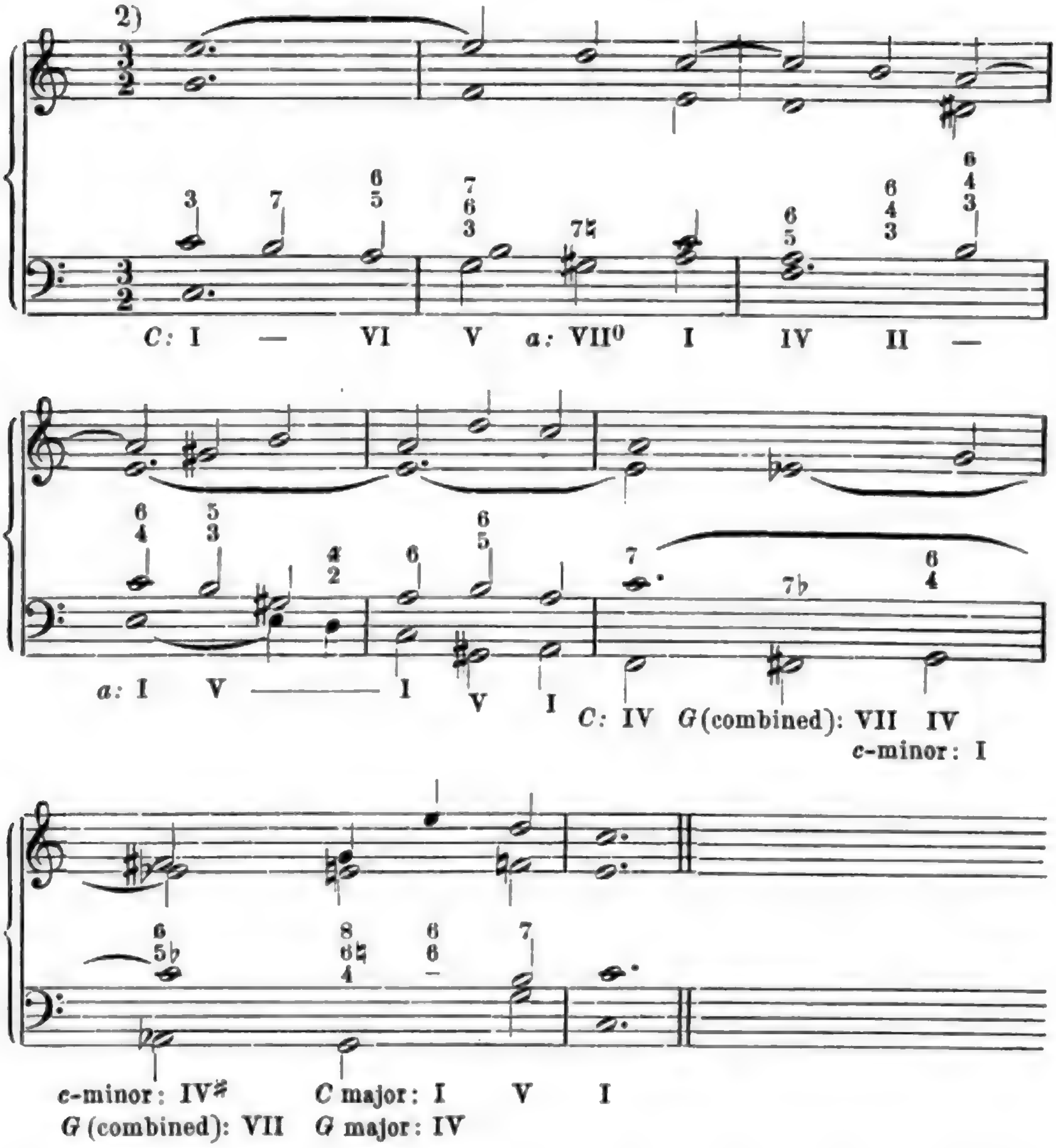
1)


C: I C: V C: I C: II C: V C: IV C: VI
C (combined): II G: II
G: VII⁰ G: I — C: I F: V f: I A♭: VI A♭: II —
G (combined): VII C: V F: V f: V A♭: VI
A♭: V — A♭: I c: II c: V — C: I F: V F: I C: V
c: VI C: V — F: V C: IV
C: I C: IV C (com-bined): IV C: I G: V — G: I C: V C: I
G: IV C: V

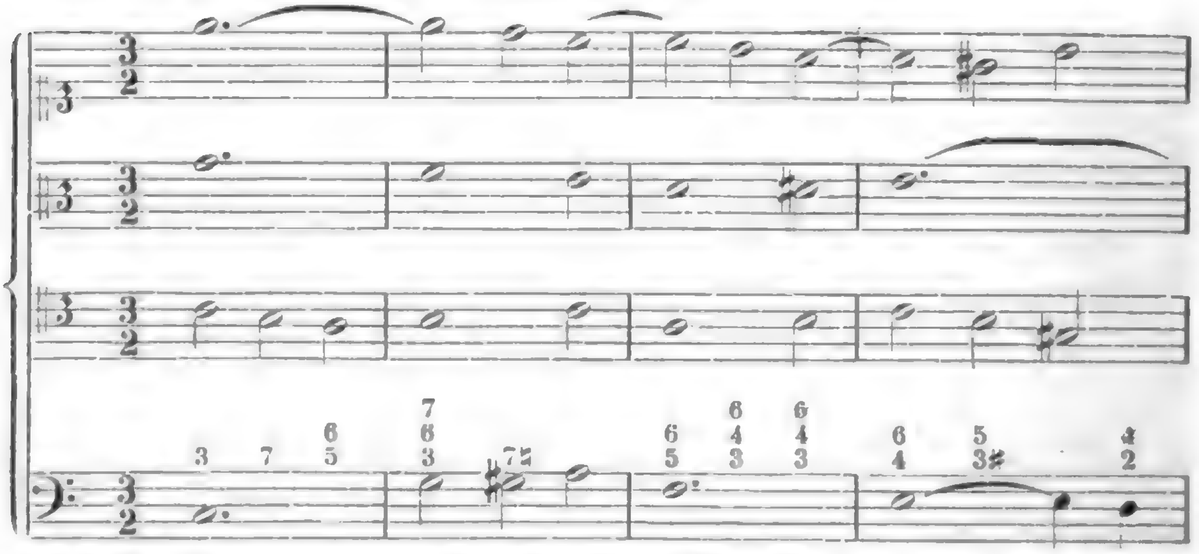
The same on four staves, using four different clefs:

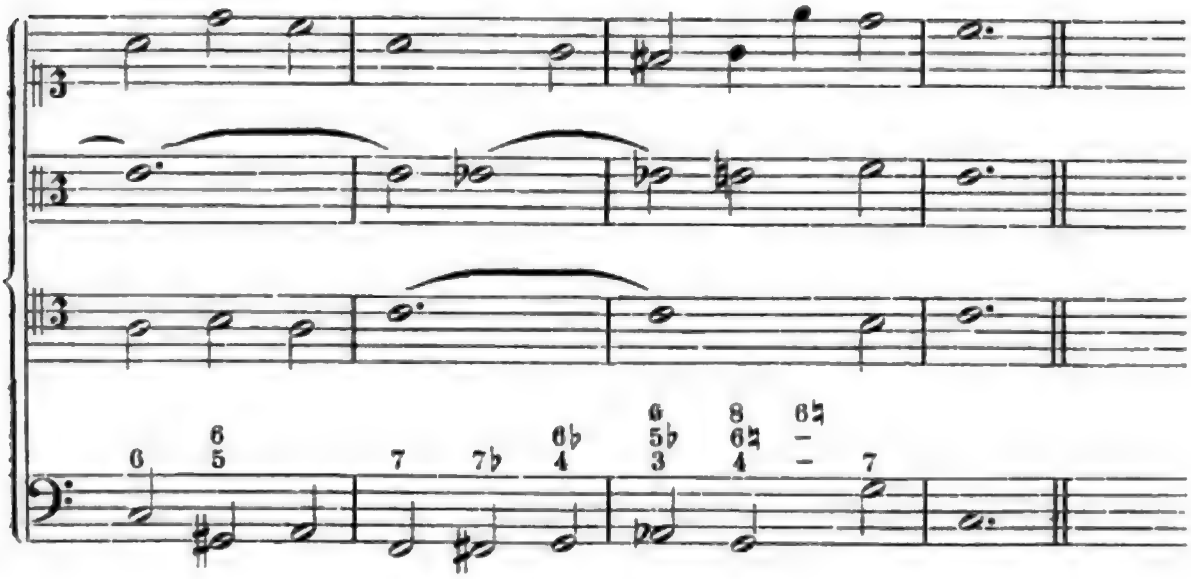


2)


C: I — VI V a: VII⁰ I IV II —
a: I V — I V I C: IV G(combined): VII IV
c-minor: I
c-minor: IV♯ C major: I V I
G (combined): VII G major: IV

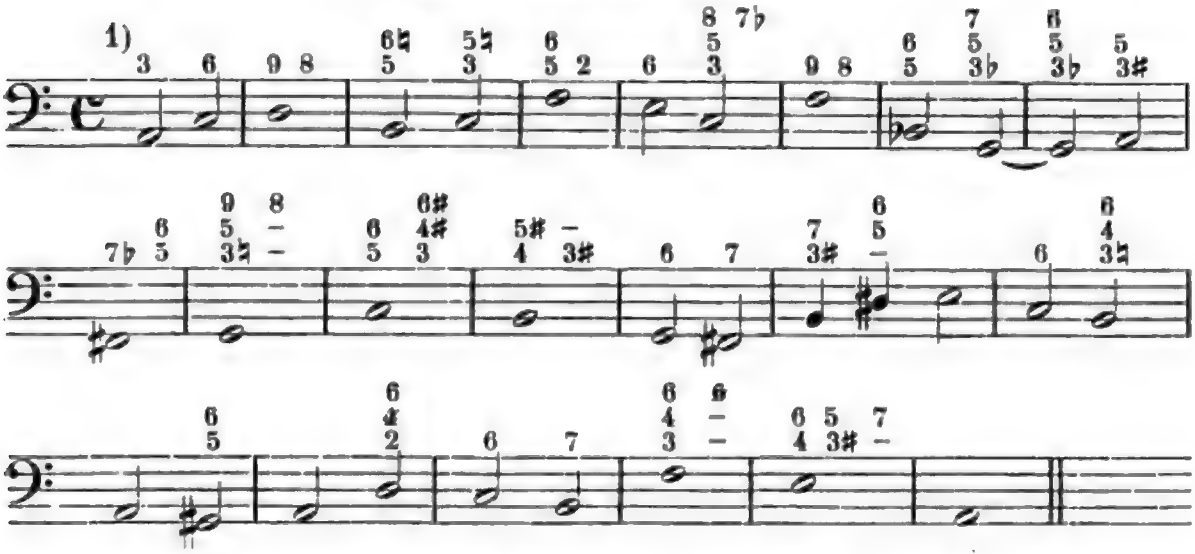
On four staves, using four different clefs:



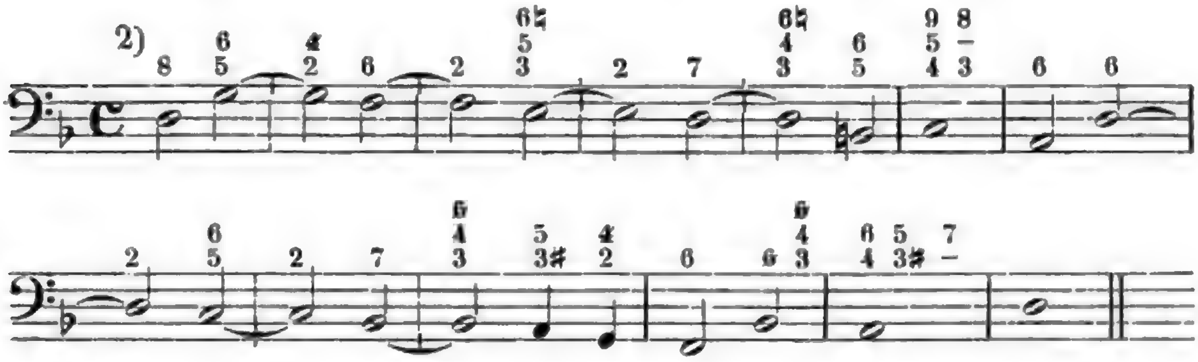


Exercises to be worked out.

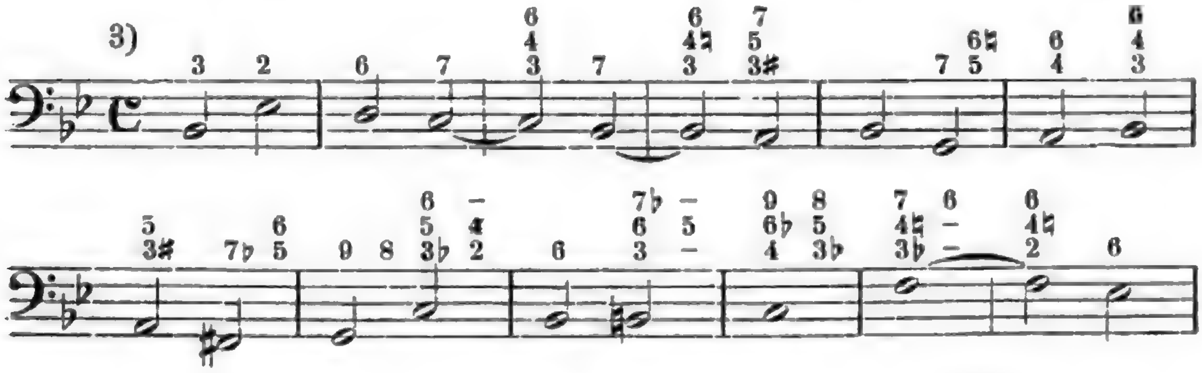
1)

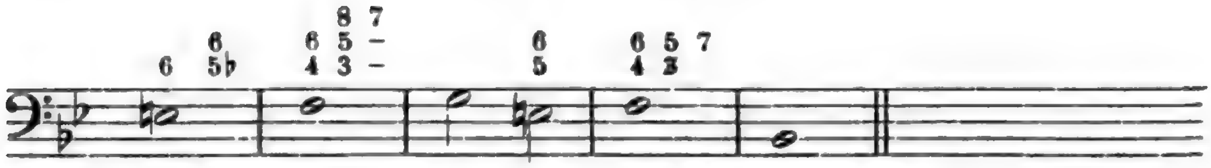


2)

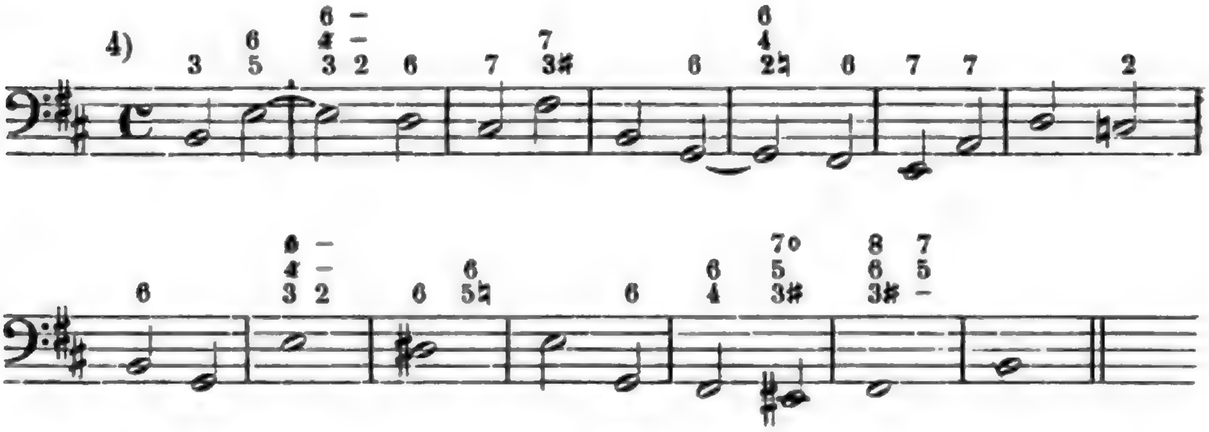


3)

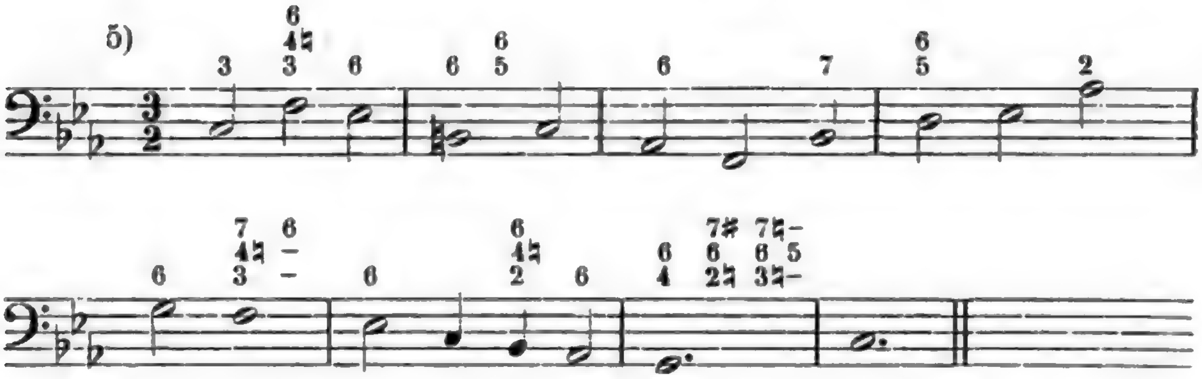




4)

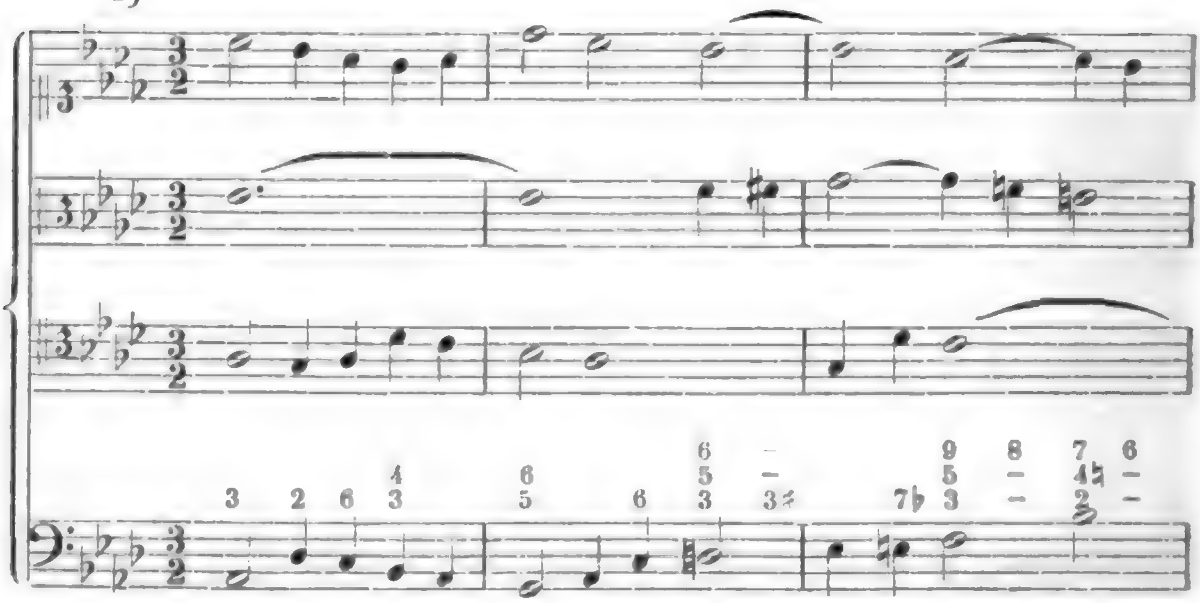


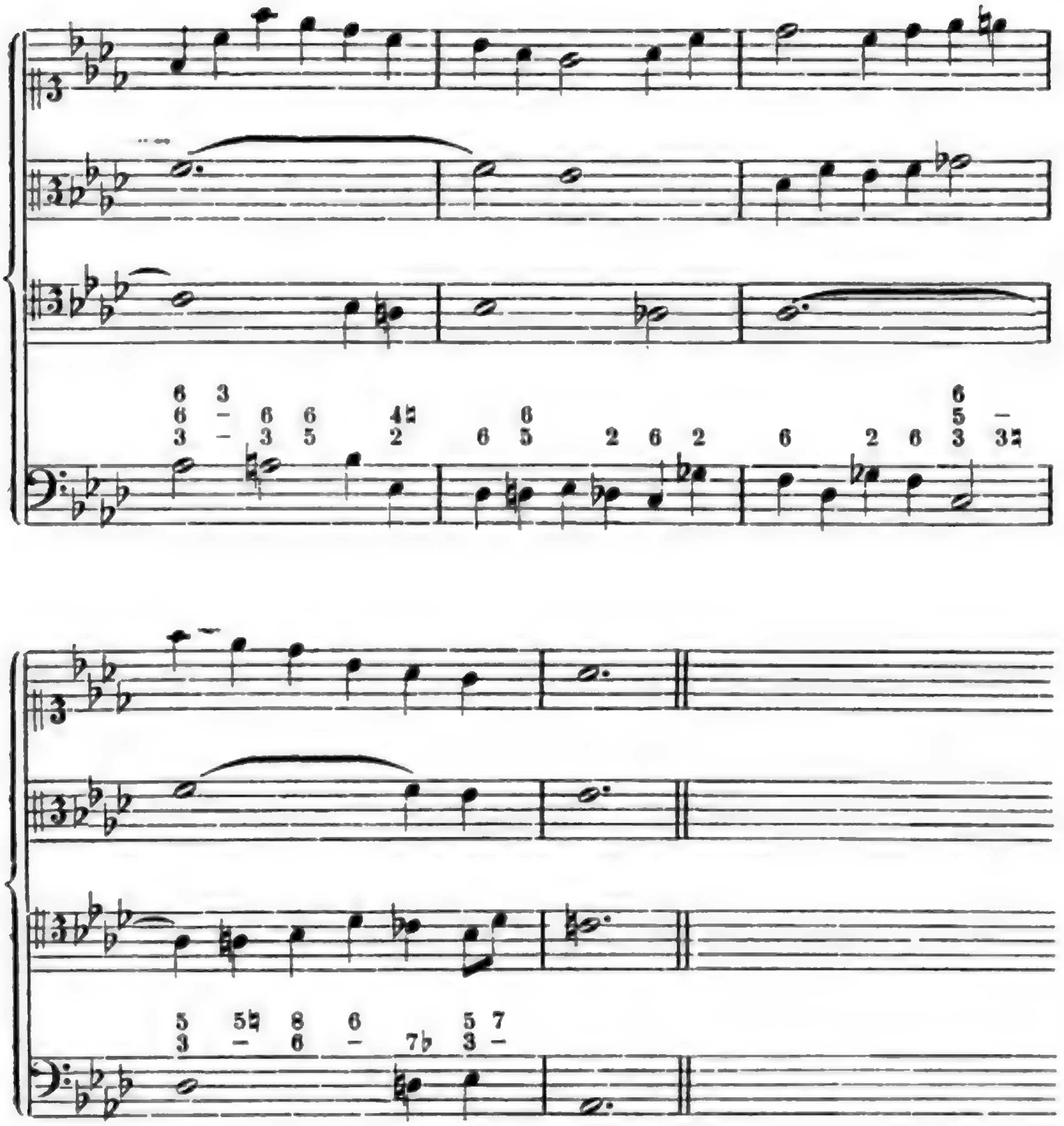
5)



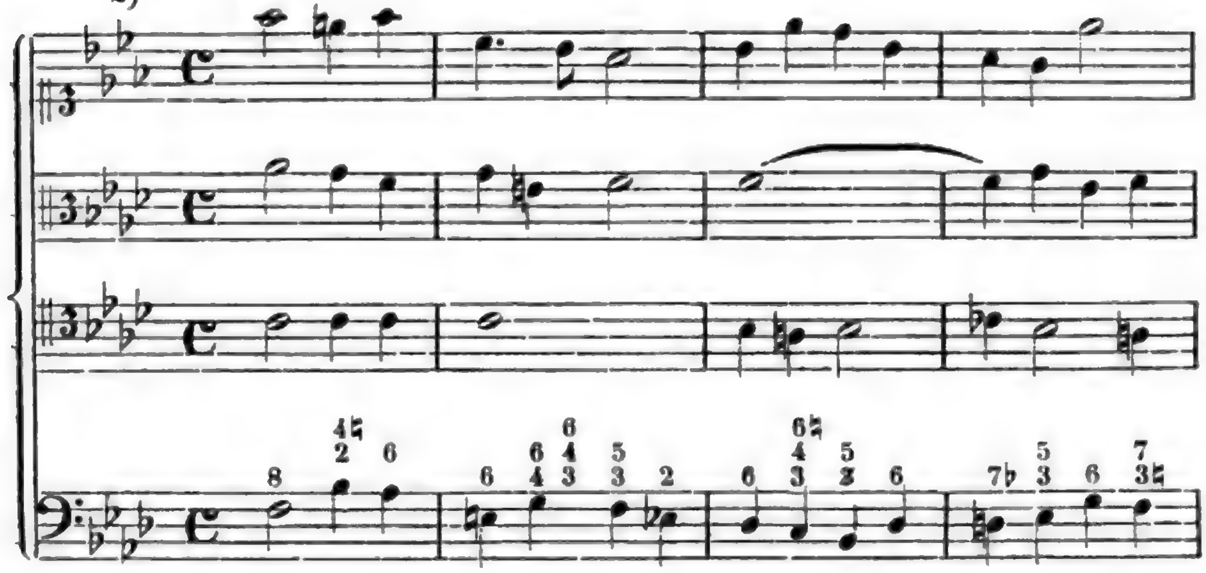
After working out these exercises on four staves, the pupil may attempt to compose little pieces, each of which should be eight measures in length. The following short pieces will serve as an introduction to such efforts in independent composition.

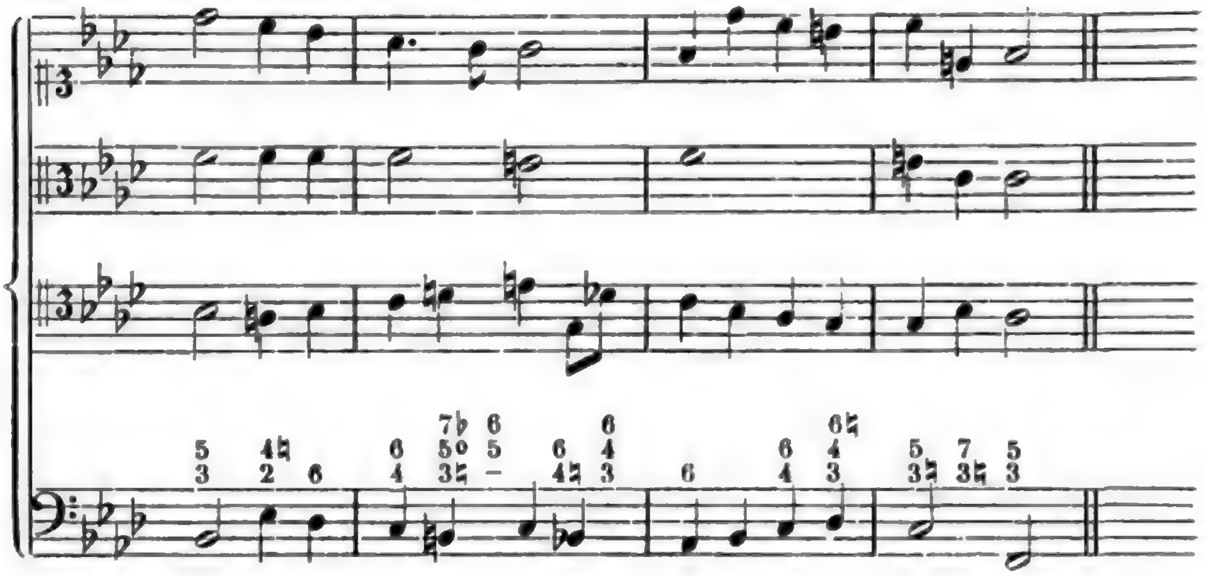
1)



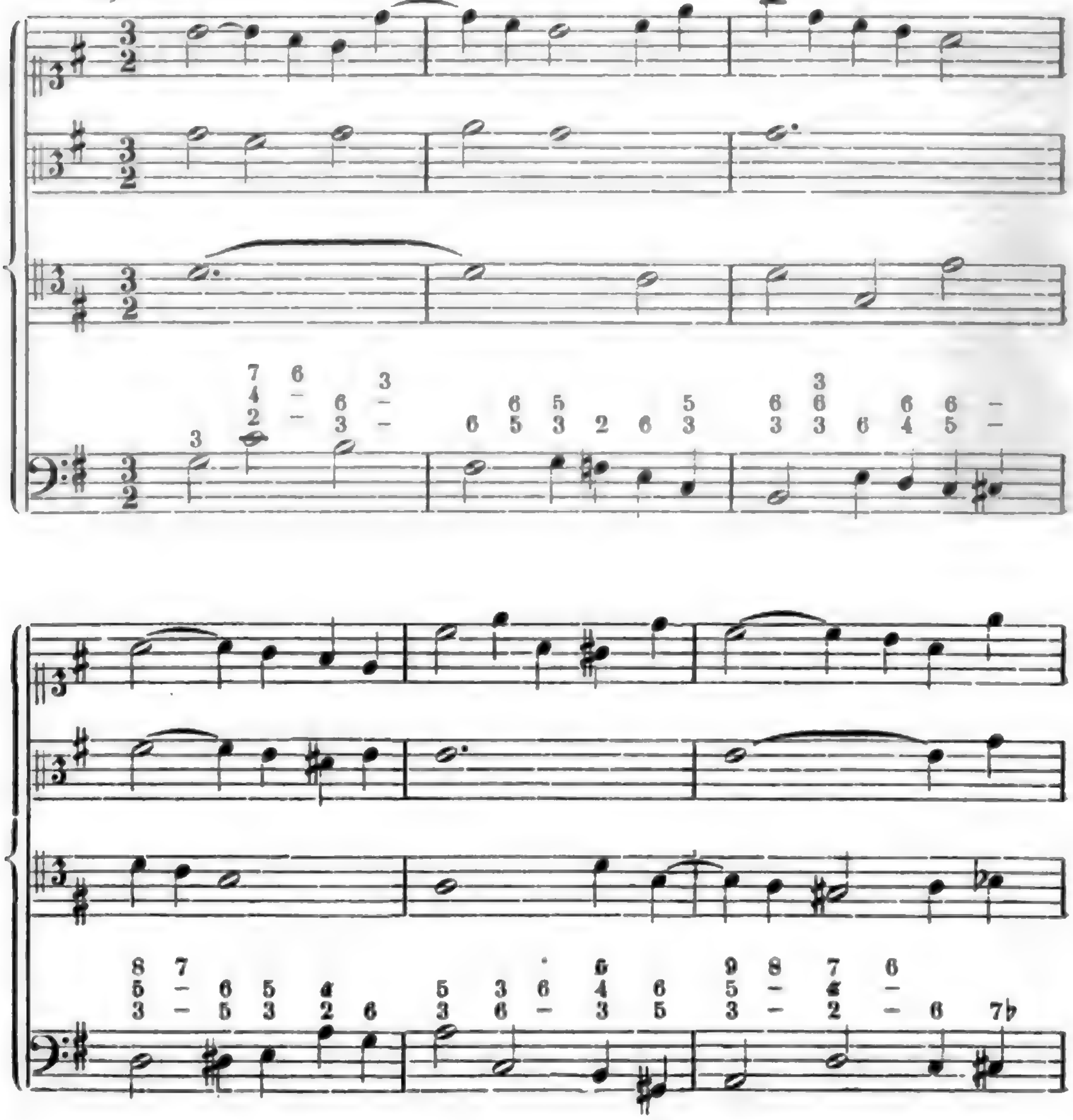


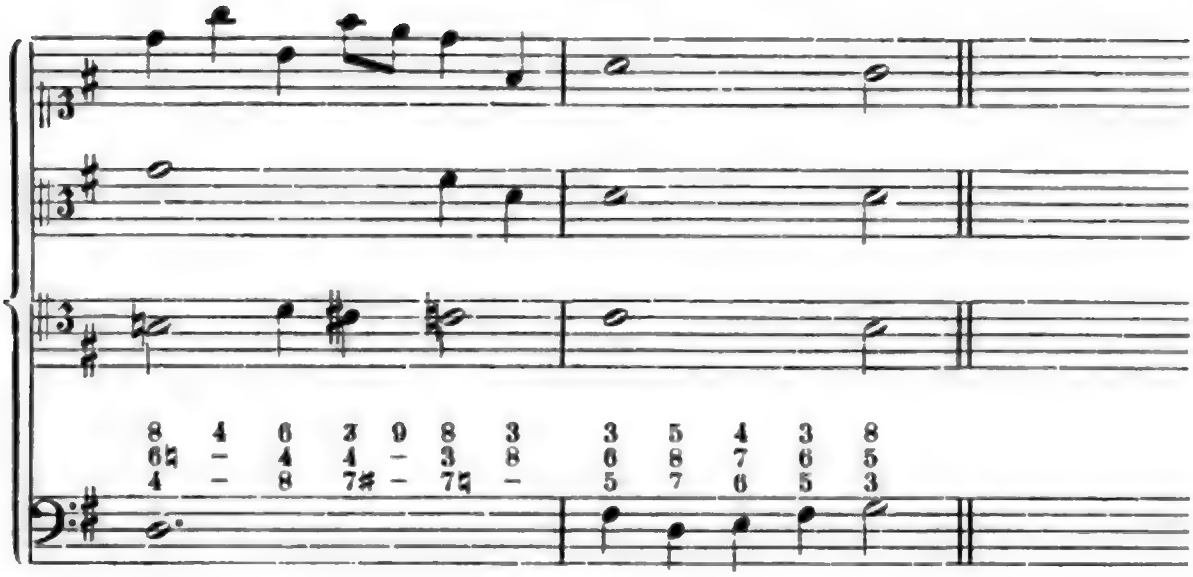
2)





3)





CHAPTER XV.

Fifths and Octaves.

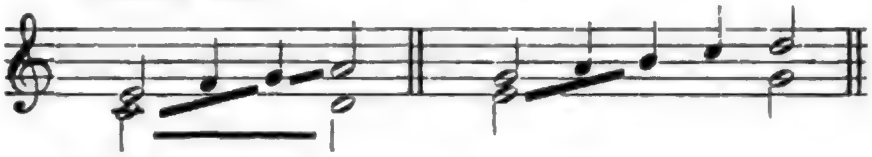
Although at the time of Sebastian Bach the laws of the interconnection of the chords were not so distinctly set forth as at present, we nevertheless find, among the practical rules of his son Carl Philipp Emanuel Bach, author of the uncommonly valuable work entitled "Essay on the true method of playing the piano" [Versuch über die wahre Art, das Clavier zu spielen (Berlin, second edition, 1759)] a foreshadowing of the theory of interconnection, an adherence to which renders the rule "to avoid parallel fifths and octaves" superfluous. An acquaintance with the views generally received in the Bach period, as to the treatment of fifths in polyphonic composition, can hardly fail to be of service to the practical musician.

Parallel octaves are less exhaustively treated of than parallel fifths, the occurrence of the former in four-part harmony changing it, by the simple doubling of one part, to three-part harmony; parallel fifths, on the other hand, doing violence to our musical sense through their unprepared entrance and contradictory character. For their avoidance "contrary motion" is principally recommended in the Bach period, particularly for an accompanist playing from a figured bass.

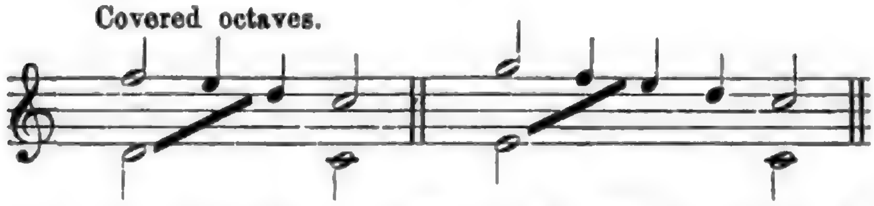
"Contrary motion", says C. Ph. E. Bach, "is in general the best and safest for an accompanist, more especially in the case of these chords (*i. e.* the triads); by this means 'open' and 'covered' fifths and octaves are avoided".

"Covered fifths and octaves occur between two parts progressing in parallel motion, if, by interpolating the notes skipped by either part, parallel fifths or octaves are formed by such notes." *E. g.*

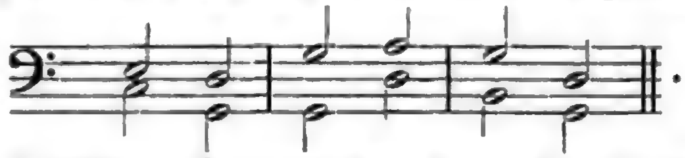
Covered fifths.

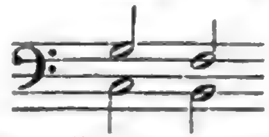
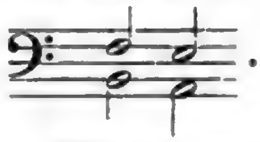
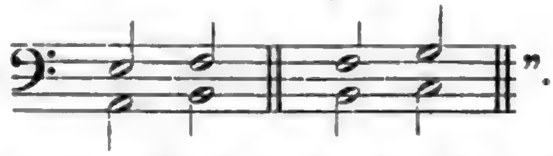


Covered octaves.



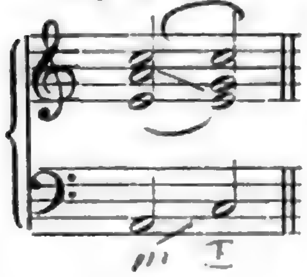
"They are more allowable (according to C. Ph. E. Bach) between the middle parts, or between these and the bass, than between soprano and bass, because in the latter case the laws of strict composition and pure vocal music must be more scrupulously observed .... This progression is impure, and has therefore a bad effect in vocal music."

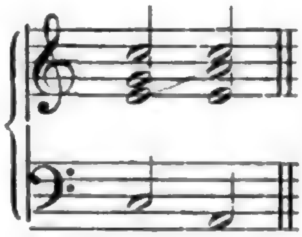
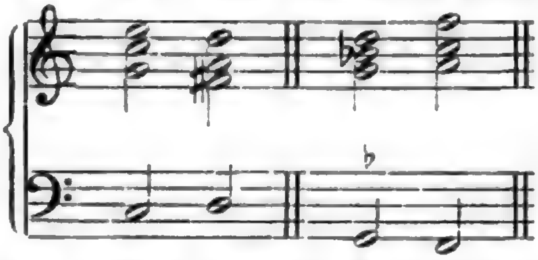
"The following covered fifths may be considered as permissible between the outer parts: .
Two open fifths of different character may succeed each other."

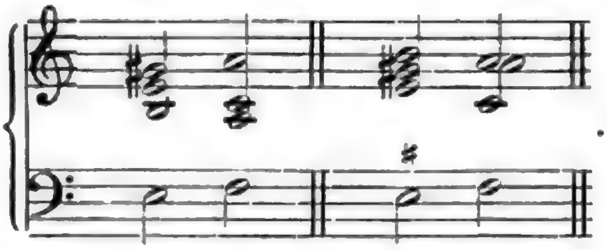
In downward progression a diminished fifth may follow a perfect fifth between any two parts, *e. g.* . (In the Bach period a diminished fifth was called a "false" [falsche] fifth.) "But the progression from a diminished fifth to a perfect fifth is permitted only in case of necessity, and scarcely ever between the outer parts, *e. g.* .
The ascending progression from a perfect to a diminished fifth is better than from a diminished to a perfect fifth, the natural progression of the diminished fifth being downwards: ".

To avoid parallel fifths between triads, Bach gives rules for leading the parts according as the bass moves upward or downward, which rules lead to the same result as does an observance of the laws of interconnection. The latter, however, present the essence of the matter more clearly, and prevent mistakes with greater certainty, than such rules, which, as in the case of closes, where covered octaves frequently and quite lawfully occur, do not always appear adequate or pertinent.

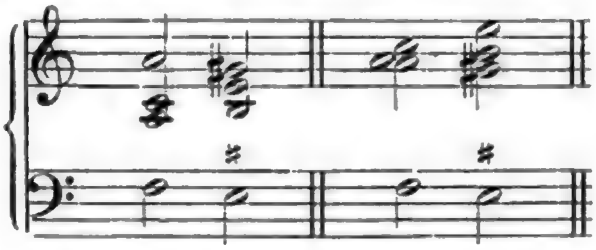
Bach gives, for instance, the following rule for triads connected through the third: "when the bass progresses two steps upward, the intervals belonging to the first tone are held, only the fifth being taken anew:"

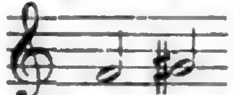


"and when it progresses two steps downward, the octave only need be changed:"  . "If the bass progress one step upward or downward, the remaining parts are to be led in contrary motion, e. g.  ." "If the bass progress upward a semitone, moving in major thirds with any part above, the fifth and third should be led in contrary motion either an octave apart, or to the unison; consequently to the second bass-note the third is doubled and the octave dropped, *e. g.*

 ."

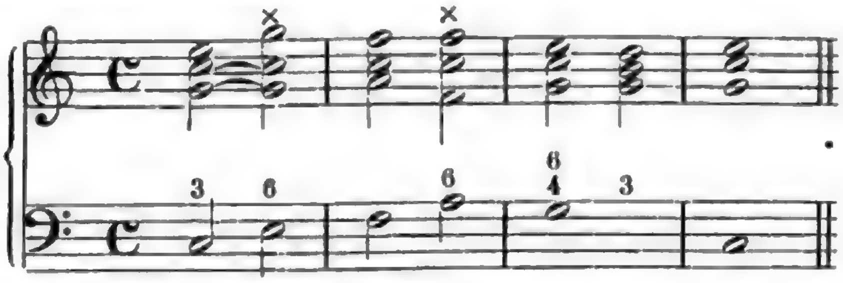
"If this progression be reversed the third should be doubled and the octave left out in the first chord:

 ."

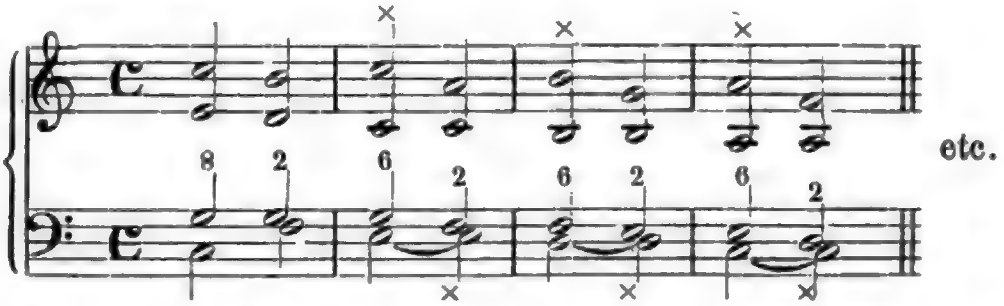
"Otherwise one of the parts is forced to take the unmelodic step of the augmented second  which should be avoided."

These examples clearly show that the desired end is attained more readily and with greater certainty through the theory of interconnection, which also renders mistakes far less probable.

Special attention should be called to the fact, that teachers of harmony have latterly shot over the mark in their pursuit of parallel fifths and octaves. The expressions "Quintenjägerei" and "Octavenjägerei" [chase after fifths and octaves] have for this reason become naturalized as German musical terms. Such pedantry finds it even blameable when octaves occur in two successive measures on the arsis, as in this example:



Octaves are found in the chords marked × between the soprano and tenor, and are sounded in both cases on the unaccented part of the measure, or arsis. Many a pedant might maintain that they should be avoided as parallel octaves; between them lies, however, a harmony differing from that of the first chord, the four-part progression being retained, so that this succession of chords is not to be regarded as faulty. To strike at the root of such a mistaken idea various examples of the employment of the chord of the sixth in the above manner have been brought forward. Even where chords, in which octaves occur, appear in successive measures on the thesis, the harmony cannot be called incorrect. This example



shows a sequence, in which octaves occur in several successive measures on the thesis between the soprano and alto, as marked by a × placed above. The × below indicates that in several successive measures fifths occur on the arsis between the two middle parts. The form of these progressions may be considered, on account of the sequences, as premeditated, but not as faulty. Where the composer considers it necessary to employ such sequences, it may be said that he does so designedly, this often appearing justifiable when certain tones of a melody are to be brought into prominence in the midst of the harmony, but not that he writes incorrectly. The motives for using such a form of expression should therefore be carefully weighed in each individual case.

CHAPTER XVI.

Thorough-bass Notation.

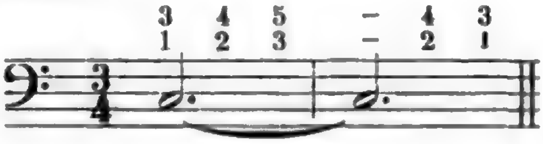
At the time of JOHANN SEBASTIAN BACH and his son, C. PH. E. BACH, thorough-bass notation was already a subject of study, and sometimes of controversy. The differences of opinion in regard to the proper method of figuring arose from a vacillation between the modern musical system and the old church-modes. The signatures of the various keys were also generally slighted by composers, a fact mentioned specially by C. PH. E. BACH. He says, that before his time *D*-minor was seldom distinguished by a *B*♭, or *C*-minor by an *A*♭, etc.; the minor keys had therefore one chromatic sign less than belonged to them, which is corroborated by old scores of the seventeenth and beginning of the eighteenth centuries. Even some contemporaries of BACH followed this custom, "perhaps from habit, perhaps through a love of antiquity, perhaps for other reasons", writes C. PH. E. BACH. "The composer", he continues, "often has the good intention of not confusing the performer by throwing in fresh signs at every turn, particularly in pieces where chromatics are freely used, or in recitatives, where great freedom in modulation is permitted, etc.; but contents himself with one and the same signature, or sets no chromatic signs whatever before the composition. In such cases these signs are likewise often left out in the figuring, a perfect acquaintance with all the keys being taken for granted".

It being the object of a manual of harmony to explain the chords and their employment, and also to give the student the ability to study the works of the masters, a fuller explanation of the thorough-bass notation met with in old scores is indispensable. When the pupil, by working out the exercises and through learning the various chords, has gained a sufficient understanding of the nature of harmonic combinations, there is no better discipline for fixing and widening his knowledge than the playing of figured basses taken from master-works.

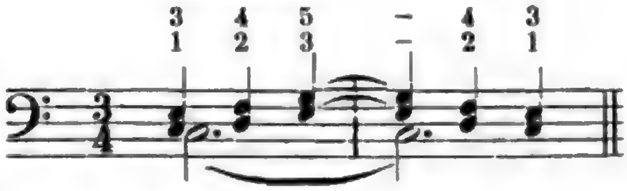
Those not sufficiently advanced in piano-playing may regulate and strengthen their musical faculty by writing out the exercises. At the end of the chapter basses taken from master-works are given for practice in this direction.

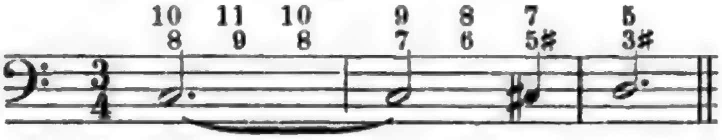
In the preceding exercises only such figures were given as were absolutely indispensable, in order that the pupil should not be tempted into a merely mechanical counting of the intervals from the bass. It now becomes necessary, for the purpose of insuring a comprehension of the harmonies in the works of the old masters, to learn the signification of other figures, till now unemployed. A compendious explanation of thorough-bass figuring will qualify the learner to overcome the difficulties in these works.

Firstly, it should be remarked that primes, tenths, elevenths and twelfths were formerly regarded as of like value with octaves, thirds, fourths and fifths, which opinion coïncides with the views of most modern musicians. The former were often designated, in old scores, by the figures 1 (prime), 10 (tenth), 11 (eleventh), and 12 (twelfth). These occurred, according to C. Ph. E. Bach, "most frequently in the gallant [free] style", which, cultivated specially by the French composers of the Bach period, formed a contrast to the strict "contrapuntal" or "metrical" style through a lighter, freer, and more graceful treatment of the themes. This figuring was generally used in three-part accompaniments, as shown by the following examples:

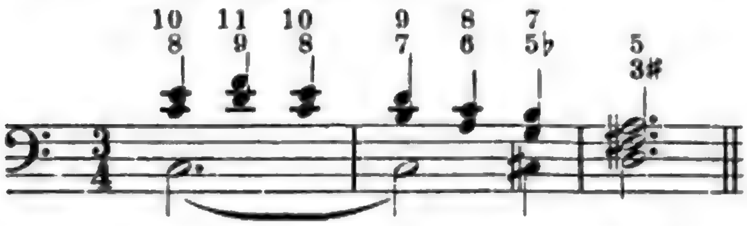


or, as given in full:

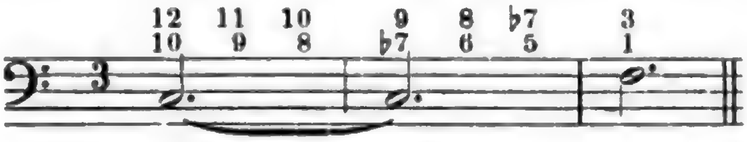




in full:



In the second measure a ♭ occurs beside the figure 5. This ♭ was formerly often employed to indicate the diminished or "false" fifth, and to duly distinguish it from the perfect fifth, as will be more fully explained further on.



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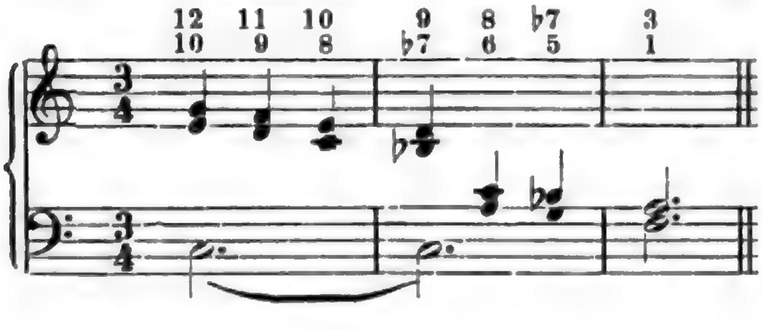

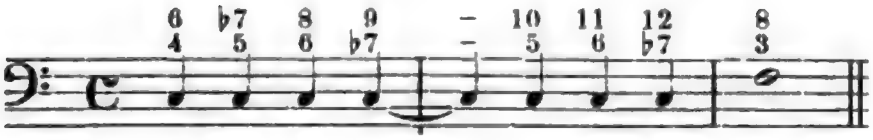

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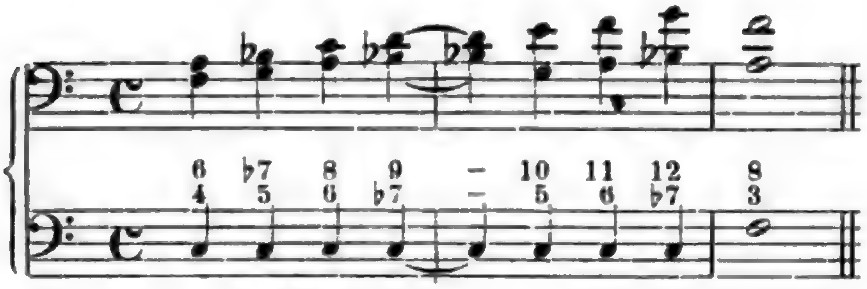

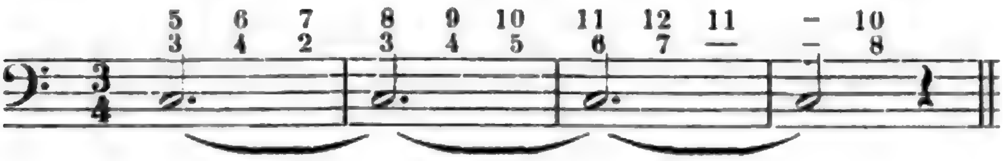

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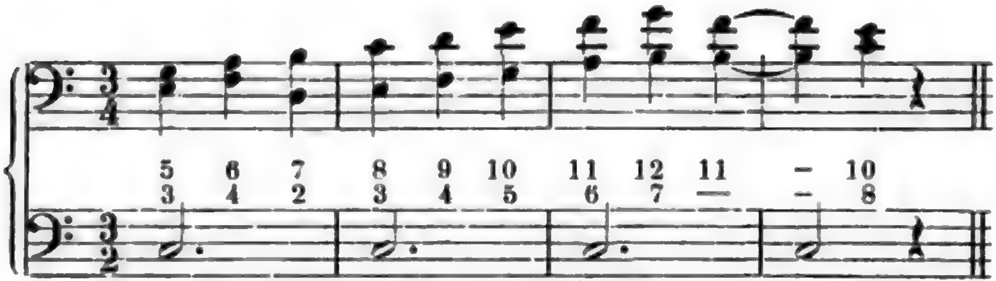
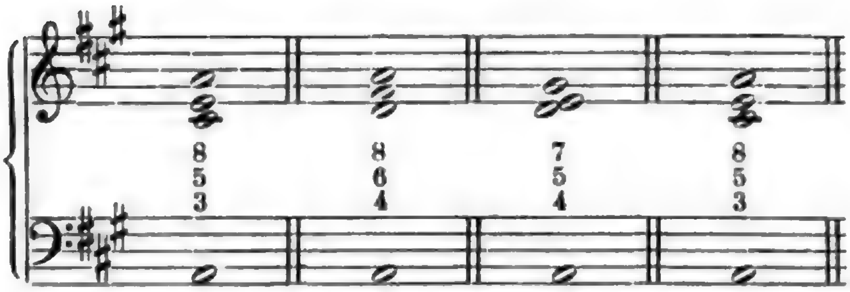


The figures 10, 11 and 12 are used by the older composers only when preceded or followed by 7, 8 and 9; otherwise 3 is written for 10, 4 for 11, and 5 for 12, *i. e.* the third for the tenth, the fourth for the eleventh, and the fifth for the twelfth.

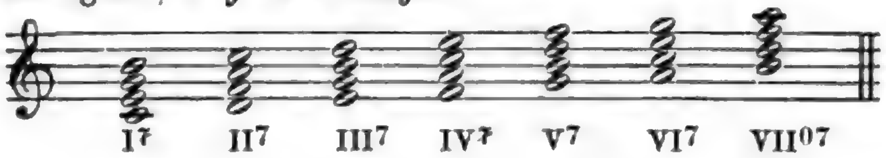
In the BACH period, as C. PH. E. BACH explicitly maintains, simple figures, without further addition, were employed to indicate the intervals as they occurred *within the key*, with the exception of the sign given above for the diminished fifth. For instance, in *G*-major, which has one ♯ as signature, the sixth of *a* would be no longer *f′* but *f′*♯, and a simple 6 would be written over the *a*, the ♯ being already given in the signature. On the other hand, if intervals occurred which were foreign to the key, special signs were employed.

A line drawn through the figure, or a ♯ beside it, denotes, that the interval is to be raised a chromatic semitone.

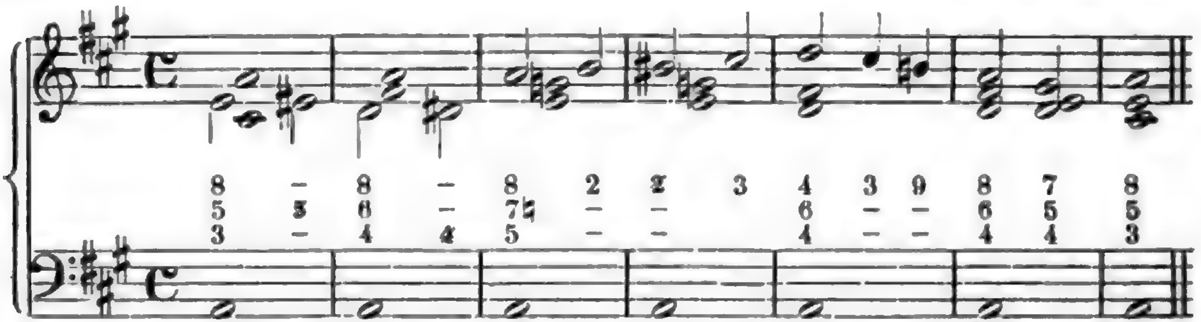
The manner of designating the intervals is determined by the key. The degrees are so precisely indicated by means of the signatures, that the mere figures suffice to mark the intervals, if the original key be adhered to. Taking, for example, the key of *A*-major, the intervals formed by the aid of the notes *f*♯, *c*♯ and *g*♯ are indicated by simple figures, *e. g.*



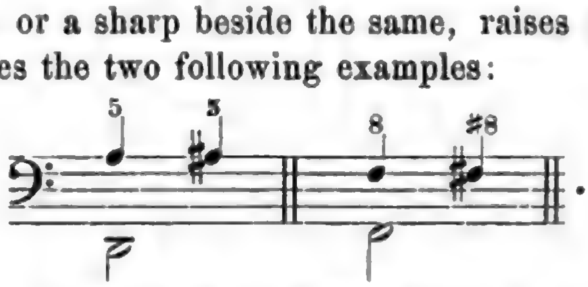
Attention must here be called to a mistake in Richter's Manual of Harmony which is calculated to perplex the student. In his presentation of the chords of the seventh in major, those chords having a major seventh, on the first and fourth degrees, are designated by a line drawn through the figure; *e. g.* in *C*-major:



This mode of notation is evidently wrong; because the stroke through the 7 is meaningless, the sevenths based on the first and fourth steps being naturally major intervals, and not artificially raised. When intervals are raised artificially they are marked by a line drawn through the figure or by a ♯ set beside the same. For instance, if we are writing in *A*-major, and employ chromatically raised notes which do not occur in that key, this must be indicated by a stroke through the figure, or by a chromatic sign, *e. g.*

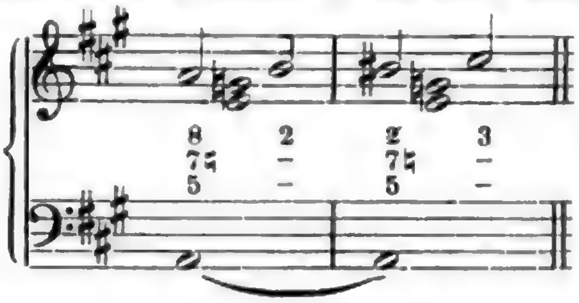


C. Ph. E. Bach remarks, regarding this mode of notation: "A stroke through the figure, or a sharp beside the same, raises the interval by a semitone", and gives the two following examples:



He then adds: "This stroke through the figure is in general use and everywhere understood by us Germans, and likewise by the Italians; the French alone deviate from this practice, thus causing confusion. Examine, for instance, Le Clair's figured basses, in which the *natural* major intervals, and also the *accidental* minor intervals, are marked alike, namely by a stroke".

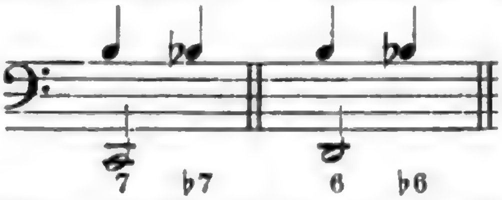
The "natural major" intervals are such major intervals as are found within the limits of the key; *e. g.* the sevenths on the first and fourth steps in *A*-major; the "accidental minor" intervals are those formed through being artificially lowered, and thus changed from major to minor intervals. In the example given above, with the organ-point on *A*, such an "accidental" minor interval occurs in the third and fourth measures:



where, instead of the major seventh *A*-*g*♯, the minor seventh *A*-*g* is found, as marked by 7♮.

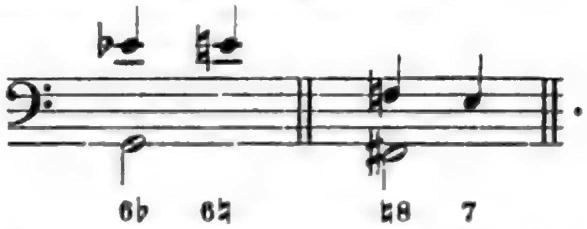
The natural (♮) restores any chromatically raised or lowered tone to the natural pitch [*i. e.* its pitch as shown on the staff without any chromatic sign]; it therefore lowers tones previously raised by sharps, and raises tones previously lowered by flats. The simple sharp (♯), as we already know, raises the note before which it stands by a chromatic semitone, and the simple flat (♭) lowers the note before which it stands by a chromatic semitone. The double sharp (×) raises a tone by two chromatic semitones (not, as often erroneously stated, by a whole tone); and the double flat (♭♭) lowers a tone by two chromatic semitones. In thorough-bass notation the raising or lowering of an interval by two chromatic semitones is indicated by writing the one or the other of these signs beside the figure. In the BACH period the sign ✕ was frequently employed instead of our ♯; the double sharp × was used as at present. In order to become familiar with the views of the musicians of that period, and to qualify oneself to understand and execute the figured basses in the scores, it is necessary to examine the disquisitions of C. PH. E. BACH more attentively. He explains the matter so clearly and concisely as to leave little room for error.

"A ♭ written through or set beside the figure lowers the interval by a semitone," whence it is evident that it makes no difference whether the chromatic sign be placed at the right or at the left of the figure.



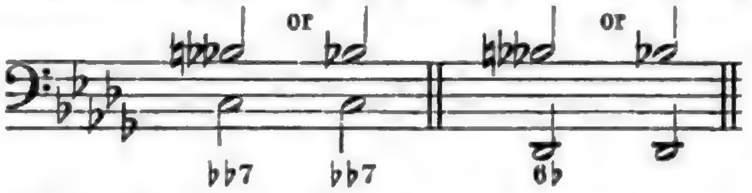
"A ♮ through or beside the figure restores the interval to its natural position."

"I hardly need to observe," he writes, "that this ♮ lowers tones in the sharp keys, and raises them in the flat keys."



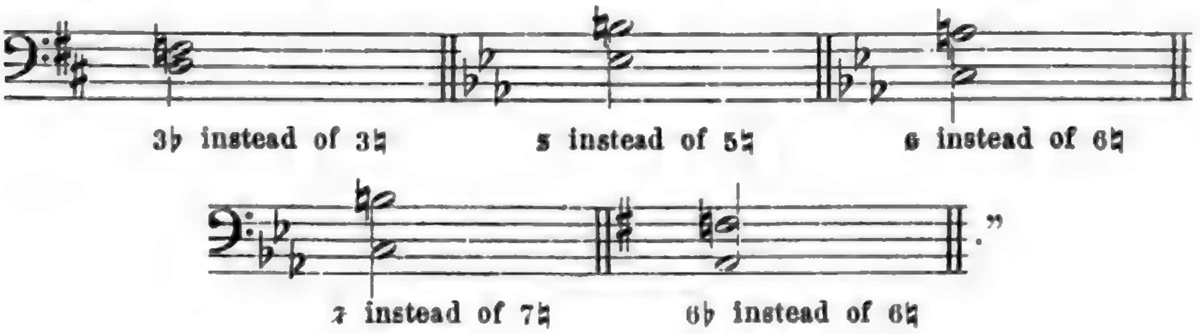
"Two strokes, two sharps (= our double sharp ×) or a simple cross through or beside the figure, raise the interval by a whole tone;" *i. e.* the higher tone of the interval is to be raised by two chromatic semitones. C. **Ph. E. Bach** adds, that the sign of two sharps was the least frequently used and the most indefinite.

Two ♭♭ or a great ♭ through or beside the figure lower the interval by two chromatic semitones (not by a whole tone, as **Bach** says); for instance:



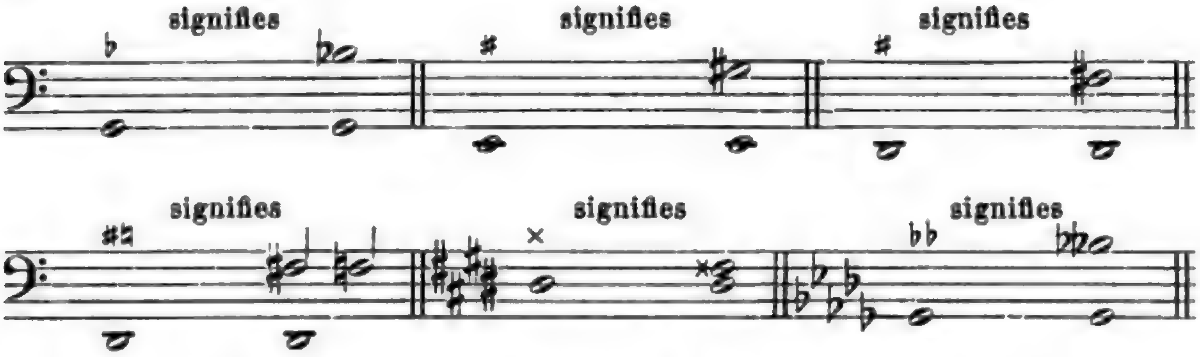
"The great ♭", writes **Bach**, "is as yet seldom employed, notwithstanding its convenience. The signs ♮♭ and ♮♯, which effect the return to a simple chromatic change after a double one, are not so generally employed as the correct style of notation would demand. As they nevertheless occasionally occur they are mentioned here with the rest, in order that they may not give cause for alarm."

It is also well to know that some composers of the **Bach** period wrote a ♭ instead of a ♮ when a tone was to be lowered, *e. g.* *f♯* to *f*; and drew a line through the figure for the purpose of raising the tone instead of using a ♮. The natural was formerly also called the "square *B*" (*Be-quadratum*). After this explanation it will be easy to understand what C. **Ph. E. Bach** says on this head: "One need not be surprised if some (composers) use flats and strokes through the figures instead of the *square B*. The various signification of the *square B*, here raising a tone and there lowering it, may account for this want of accuracy. *E. g.*



The diminished fifth, and the minor and diminished seventh, were often indicated by a ♭ placed beside the figure; for which reason C. Ph. E. Bach observes: "In the case of the diminished fifth, and of the minor and diminished seventh, we are rather accustomed to see them designated by a ♭".

"The third may be indicated, without using a 3, merely by a chromatic sign":

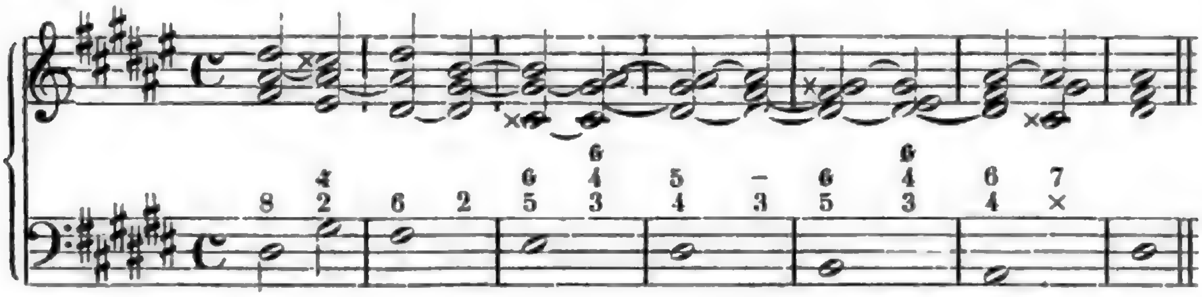


C. Ph. E. Bach usually writes only a ♭ where a tone is to be lowered by two chromatic semitones from the natural pitch, in case one flat is already present in the signature. This can, in point of fact, be viewed only with approval; the double sign ♭♭ is now, however, so generally employed, that it were useless to contend against the practice.

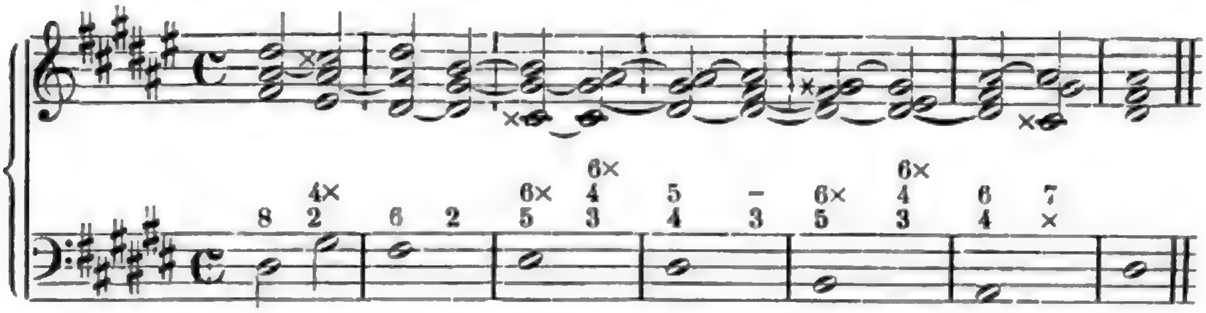
It is at present so customary to indicate the third simply by a ♯, ♭ or ♮, that it hardly seems necessary to call special attention to the fact Nevertheless, C. Ph. E. Bach does not pass over this circumstance in silence, but devotes an entire paragraph to it. He also remarks that it is better to write the chromatic signs and the strokes (which latter raise the tone by a chromatic semitone above its pitch in the key), *through* rather than beside the figures, in order to prevent mistakes and to avoid ambiguity where the figures stand near together.

We repeat, in order to ensure a comprehension of his remarks, that our natural (♮) was at that time often called a "*square B*", and our flat (♭) a "*round B*" (*Be-rotundum*), a usage of the middle ages from which the musicians of the eighteenth century could not entirely free themselves.

We would here again call attention to the strokes or lines drawn through the figures. These suffice, even in keys having numerous sharps, in cases where a double sharp would be required before the note itself. *E. g.*

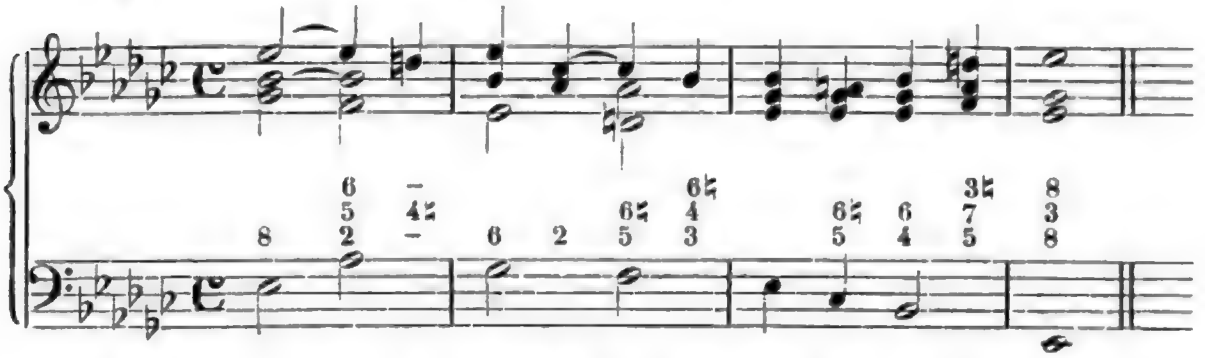


For this example the following figuring might also be employed:



The double sharp always signifies that the tone is to be raised by two chromatic semitones above its natural pitch, even where that degree in the key is already provided with a sharp in the signature. A line drawn through the *figure* suffices in such cases to indicate the interval, the upper note of which is marked by a double sharp (×).

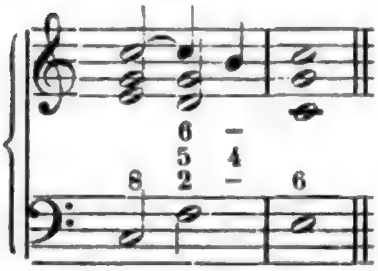
In the flat keys it is better to indicate the raising of a tone by a natural (♮) than by a stroke. *E. g.*

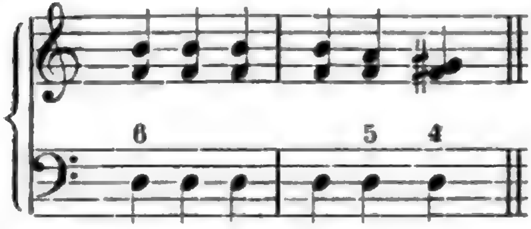


These signs should be carefully learned in order to prevent mistakes. Unquestioned accuracy can be attained only through perfect familiarity with the various keys. *G*♯-minor and *D*♯-minor, and also *B*♭-minor and *E*♭-minor, should be repeatedly reviewed, on account of the numerous chromatic signs in their signatures. Special attention must be paid to the figuring of the triads and chords of the seventh in their fundamental positions and inversions.

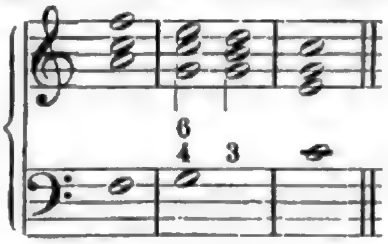
To ensure thoroughness the learner should write out and figure the triads and chords of the seventh in *B*-major, *F*♯-major, *D*♭-major, *G*♭-major, and the four minor keys given above, and also their inversions, so that these keys, in which mistakes are often made, may be thoroughly impressed upon the memory. **Hauptmann**'s axiom, that every tone has, in the key, a definite position and special signification, should always be borne in mind. If this truth be fully recognized, the above advice, that particular attention should be given to these difficult keys, will appear neither superfluous nor pedantic.

The student should notice particularly, that the chromatic signs preserve their influence until expressly annulled. In thorough-bass notation dashes are frequently used to show that the value of the figures is to be

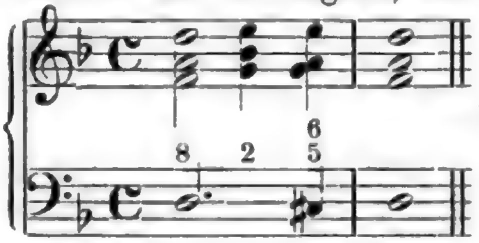
transmitted, *e. g.* 

. In the first measure the dashes after the figures $\begin{smallmatrix}6\\5\\2\end{smallmatrix}\ \begin{smallmatrix}-\\4\\-\end{smallmatrix}$ over the bass-note *f* signify that 2 and 6, the second and sixth, are the same in the second chord. Another custom was formerly prevalent which detracted from the clearness of the notation. Where several notes of like pitch occurred in succession in the *basso continuo* (*i. e.* fundamental part, or bass,) to each of which a harmony was to be struck in the accompaniment, a figure was written over the first bass-note alone when the same harmony was to be employed in every case, and the same harmony was sounded with each bass-note till a change was indicated by a different figure; the dashes not being written, as shown in the following example taken from C. Ph. E. Bach: 

. The sixth is here struck to the first four notes before the entrance of the fifth.

It is evident, from examples already given, that, where two successive chords are indicated by figures over one note, each of these chords has half the rhythmical value of the bass-note. *E. g.* 

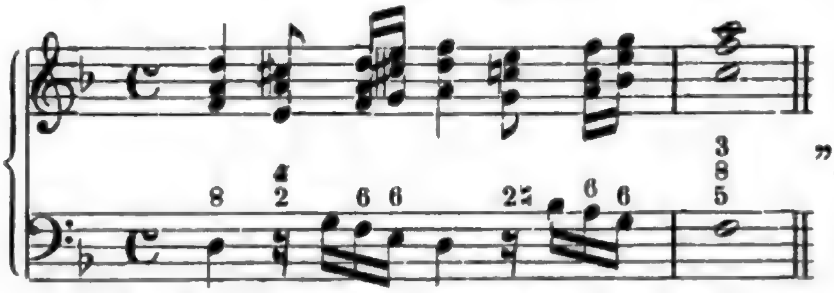
which shows the application of the above rule.

In the case of dotted notes the following rule should, according to C. Ph. E. Bach, be observed: "The harmony denoted by figures placed over a dot, which lengthens the note, should be struck when the rhythmical value of the dot begins; the figures referring to the dotted note, *e. g.* 

". The figure 2 therefore signifies, that on the third quarter of the measure, which is filled by the dot, the chord of the second is to enter. The figure 2 consequently belongs *harmonically* to the bass-note *d*, and *rhythmically* to the dot after the same.

A point to be carefully observed in notation is the use of rests, for which C. Ph. E. Bach gives this important rule:

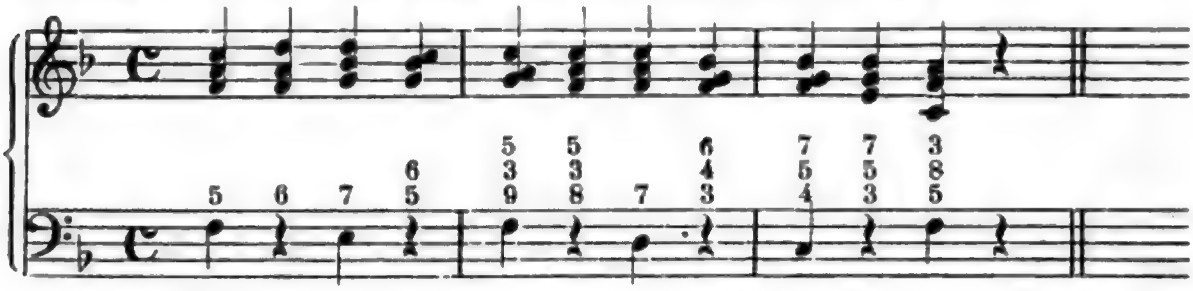
"The harmony indicated by figures over a *short* rest is struck to the rest, the figures belonging to the note *following*:



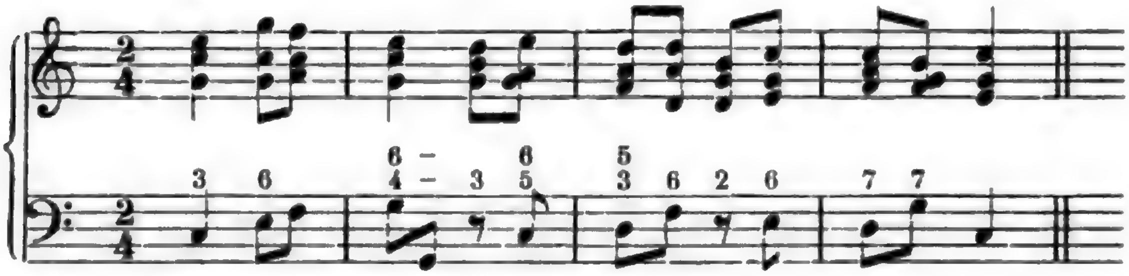
".

The figures over the sixteenth-rests show the application of the rule.

The harmony indicated by figures over a *long* rest is likewise struck to the rest, but the figures belong to the *preceding* note; *e. g.*



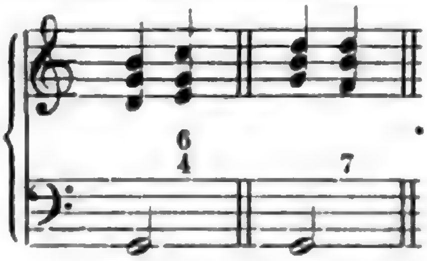
or:



which shows the correctness of the above rule.

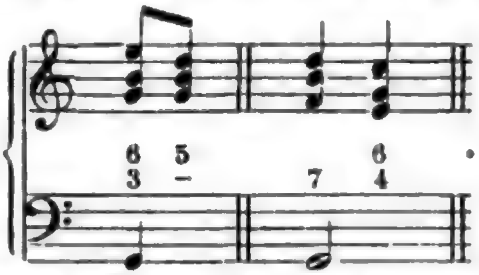
The musicians of the Bach period paid special attention to the rhythmical division of the notes, upon which the position of the figures was dependent. In this case also C. Ph. E. Bach gives precise, unequivocal directions.

"The figures representing the harmonies to be sounded are arranged, according to the value of the bass-note, as follows: If the bass-note be divisible into two equal parts, and have a figure, or several figures set perpendicularly, beside [*i. e.* not directly over, but to the right of] it, the harmony indicated by the figure or figures is to be struck to the second half of the bass-note, *e. g.*

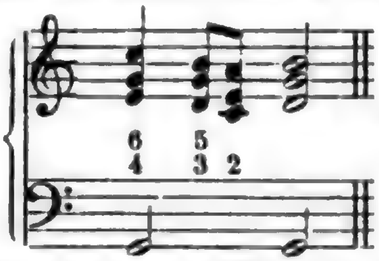


.

"In case a bass-note divisible into two equal parts has two figures beside each other, the rhythmical value of the bass-note is divided equally between these latter. *E. g.*

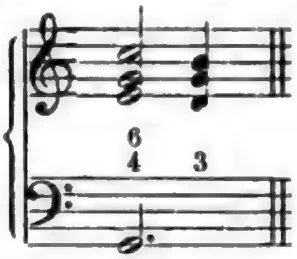


"Where three figures appear beside each other over such a bass-note, the first half of the bass-note belongs to the first figure, which stands directly above it, and the second half is divided equally between the other two figures. *E. g.*



."

"Where two figures are set beside each other over a bass-note having three equal parts, or, what amounts to the same thing, two unequal parts, the first or larger part (= two-thirds) belongs to the first figure, and the second, or smaller part (= one-third) to the second figure. *E. g.*



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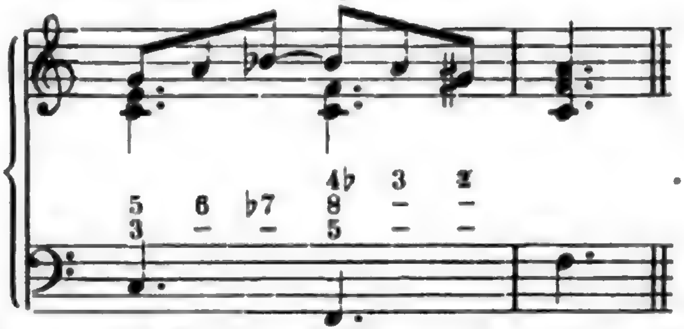
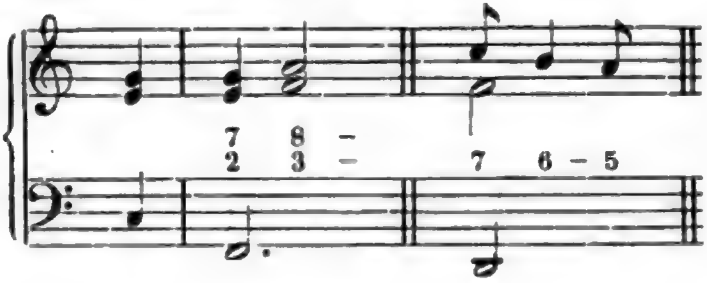
"If three figures be placed over such a bass-note, one-third of its rhythmical value falls to each figure. *E. g.*



."

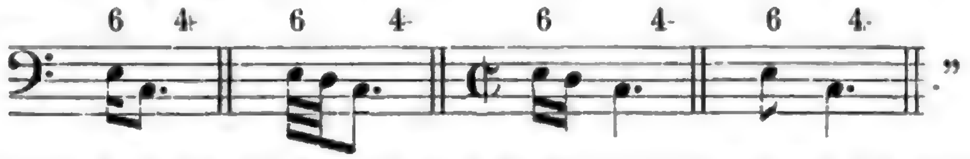
"The above manner of division is the most customary; any deviation must be expressly indicated; *e. g.*



."

Here "the little dash, which often indicates the prolongation of a figure, shows the division clearly."

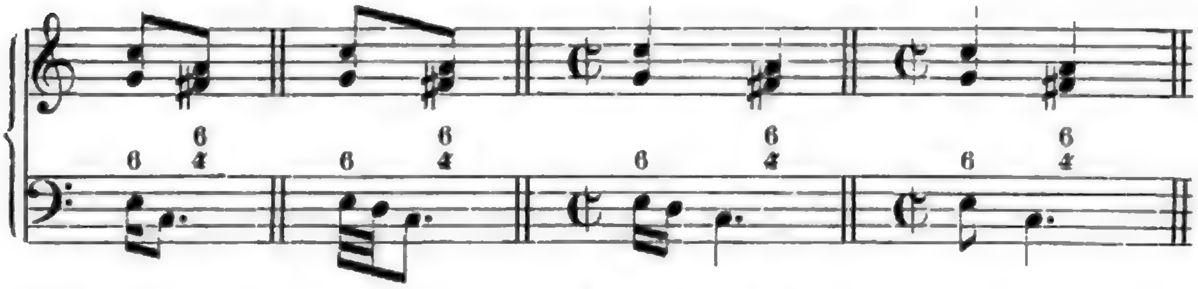
"In the following examples half the collective value of the bass-notes is allotted to each figure:



”

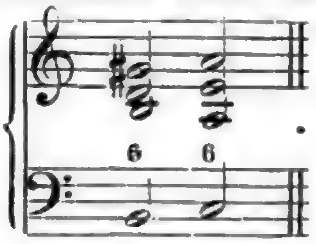
The sign 4+ is the same as 4̸, and in this case indicates the augmented fourth of *C*, which is *F♯*.

In three-part harmony these examples would appear thus:


6 $\genfrac{}{}{0pt}{}{6}{4}$
6 $\genfrac{}{}{0pt}{}{6}{4}$
6 $\genfrac{}{}{0pt}{}{6}{4}$
6 $\genfrac{}{}{0pt}{}{6}{4}$

The figuring $\genfrac{}{}{0pt}{}{8}{6}$ was at that period employed, in free piano-accompaniment, as a designation for three-part harmony. TELEMANN wrote a curve $\overset{\frown}{\genfrac{}{}{0pt}{}{8}{6}}$ for the purpose of indicating that three-part harmony should be employed; which custom is alluded to by C. PH. E. BACH in the following words: "A doubled figuring preceding and following this $(\genfrac{}{}{0pt}{}{8}{6})$ is usually a sign that the accompaniment should be in three parts; if *Telemann's curve* be written over these figures $(\overset{\frown}{\genfrac{}{}{0pt}{}{8}{6}})$ this fact is still more plainly shown."

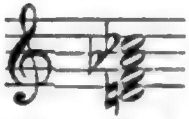
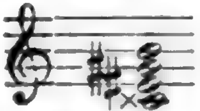
C. PH. E. BACH also recommends the employment of TELEMANN's curve if, in case the chord of the sixth derived from the diminished triad be used, the third of the triad should be doubled. *E. g.*



Here the first 6 might be written with TELEMANN's curve $\overset{\frown}{6}$.

A 5 being written at that period, "often for the sake of convenience", where $\genfrac{}{}{0pt}{}{6}{5}$ properly belonged in chords having a diminished fifth, TELEMANN also wrote a curve over the $\overset{\frown}{5}$, where he desired to indicate the diminished triad. In case it became necessary to add a chromatic sign to the 5, the curve was likewise employed to indicate that triad, *e. g.* $\overset{\frown}{5\flat}$. "By this means," says C. PH. E. BACH, "all confusion is avoided, and unpracticed players, who have not thoroughly grasped the science of modulation, are delivered out of serious difficulty."

Exercises to be worked out in Close Position.

Among these exercises may be found some with so-called *enharmonic changes*, which could have been employed only in the tempered system of tuning. These changes are effected by substituting for notes marked by sharps, others marked by flats, which latter are identical with the former *on keyed instruments*; *e. g.* writing *b♭* for *a♯*, or *e♭* for *d♯*, etc. One can thus transform an entire chord written in a flat key into one belonging to a sharp key, and *vice versa*; *e. g.* the chord  on the seventh degree in *E♭*-minor may, in the tempered system, *i. e.* on the piano, take this form: , which is the chord of the seventh on the seventh degree of *D♯*-minor.

The student is reminded that the chord of the fifth and sixth, with diminished fifth, may be indicated briefly by 5♭ instead of $^6_5$; and that in place of our ♮, and to indicate a diminished interval, a ♭ is often used. I have, however, employed the sign ♯ instead of 𝄪, which has the same signification, this change being also made in the printed scores. It should also be noticed that the chord of the second was, according to circumstances, indicated by 2, 4+, 4♮, $^4_2$ and $\substack{6\\4\\2}$; that the third may be indicated by a chromatic sign (♭, ♯, ♮) without a figure, the chord of the fourth and sixth by a 4 alone, and the chord of the third, fourth and sixth occasionally simply by a 6.

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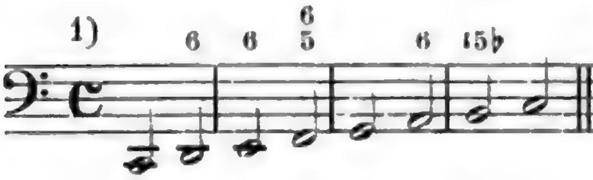

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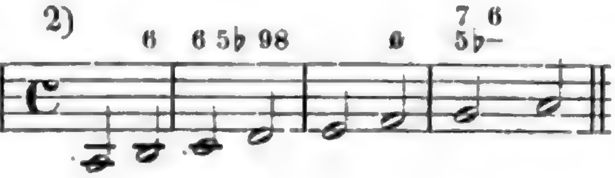

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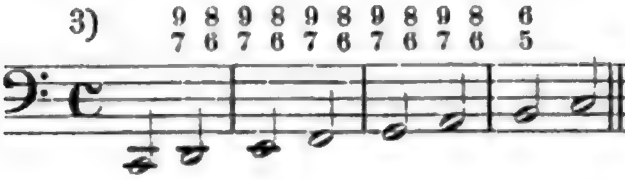

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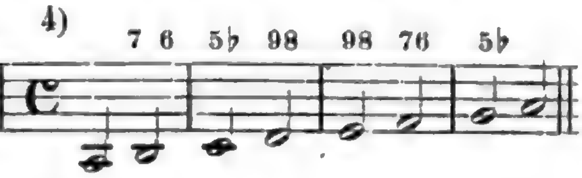

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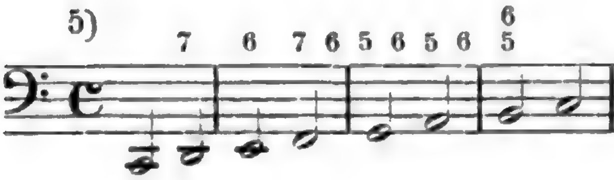

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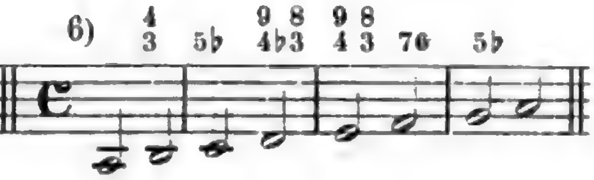

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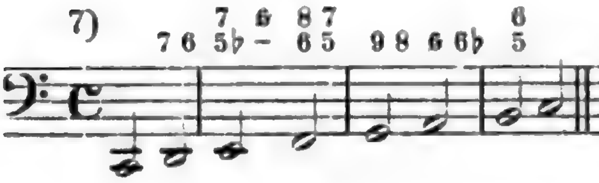

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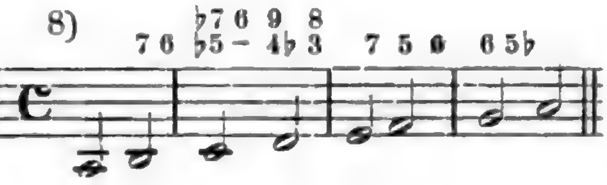
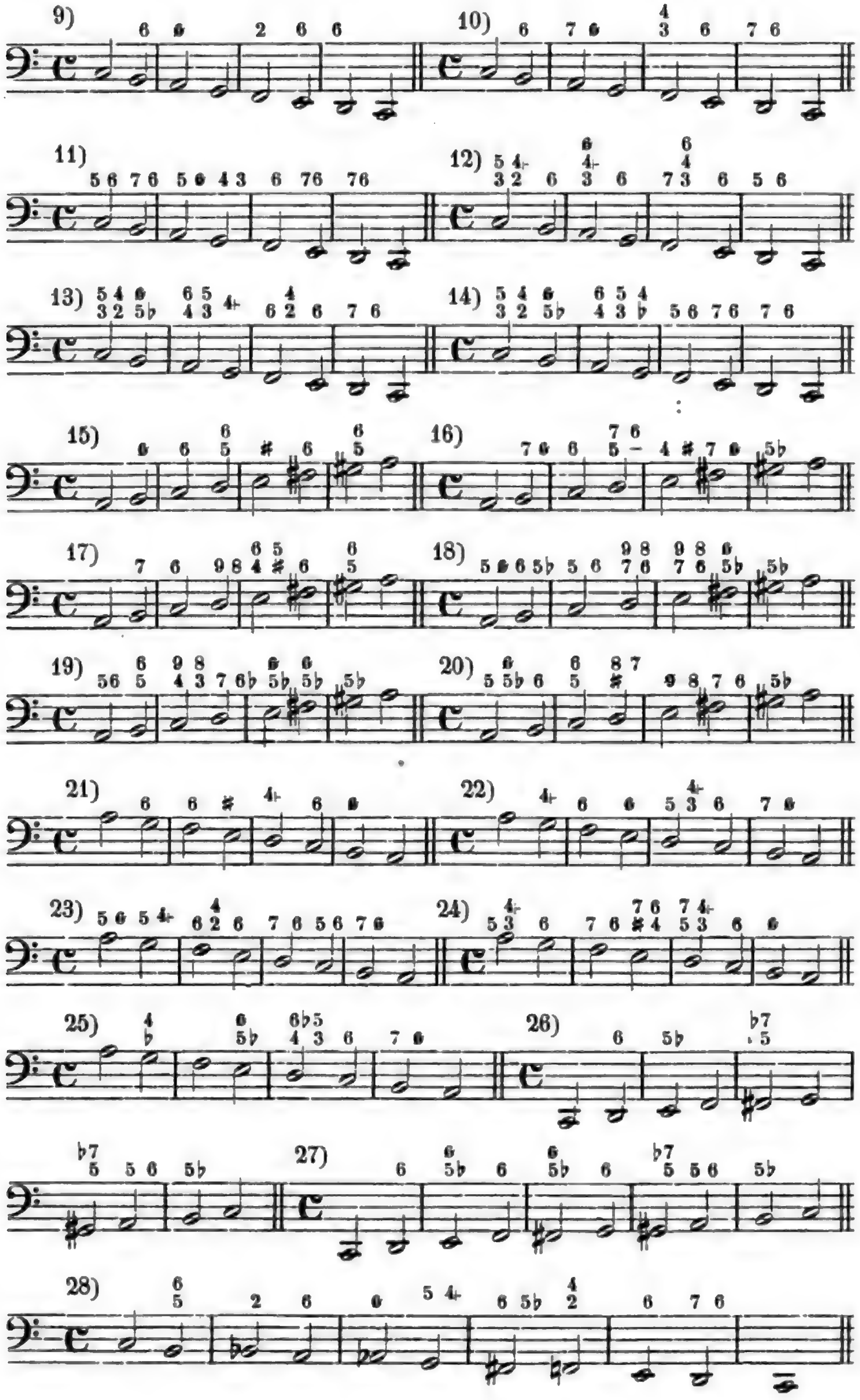
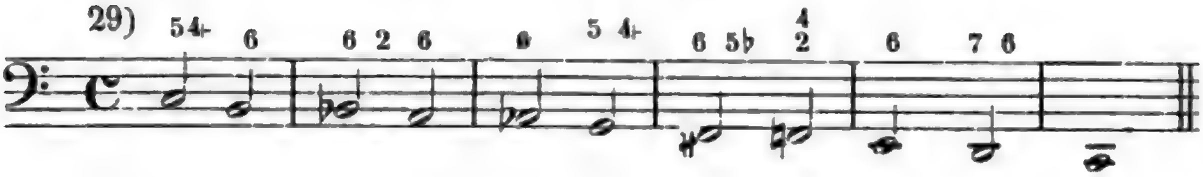



9) 6 6 2 6 6
10) 6 7 6 4 3 6 7 6
11) 5 6 7 6 5 6 4 3 6 7 6 7 6
12) 5 3 4 2 6 6 4 3 6 7 6 4 3 6 5 6
13) 5 3 4 2 6 5♭ 6 4 5 3 4+ 6 4 2 6 7 6
14) 5 3 4 2 6 5♭ 6 4 5 3 4 ♭ 5 6 7 6 7 6
15) 6 6 6 5 ♯ 6 6 5
16) 7 6 6 7 5 6 – 4 ♯ 7 6 5♭
17) 7 6 9 8 6 4 5 ♯ 6 6 5
18) 5 6 6 5♭ 5 6 9 7 8 6 9 7 8 6 6 5♭ 5♭
19) 5 6 6 5 9 4 8 3 7 6♭ 6 5♭ 6 5♭ 5♭
20) 6 5 5♭ 6 6 5 8 ♯ 7 9 8 7 6 5♭
21) 6 6 ♯ 4+ 6 6
22) 4+ 6 6 5 4+ 3 6 7 6
23) 5 6 5 4+ 6 4 2 6 7 6 5 6 7 6
24) 5 4+ 3 6 7 6 7 ♯ 6 4 7 5 4+ 3 6 6
25) 4 ♭ 6 5♭ 6 4 ♭5 3 6 7 6
26) 6 5♭ ♭7 5 ♭7 5 5 6 5♭
27) 6 6 5♭ 6 6 5♭ 6 ♭7 5 5 6 5♭
28) 6 5 2 6 6 5 4+ 6 5♭ 4 2 6 7 6

29)



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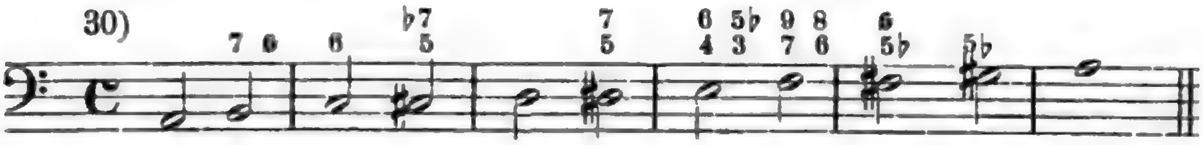

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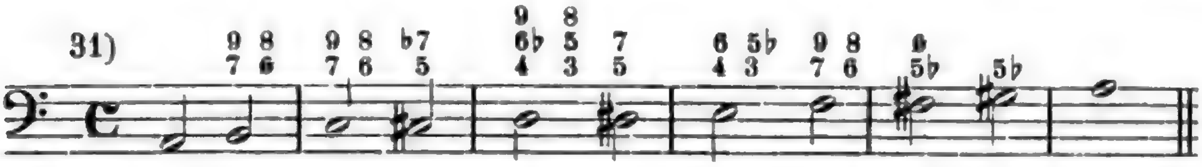

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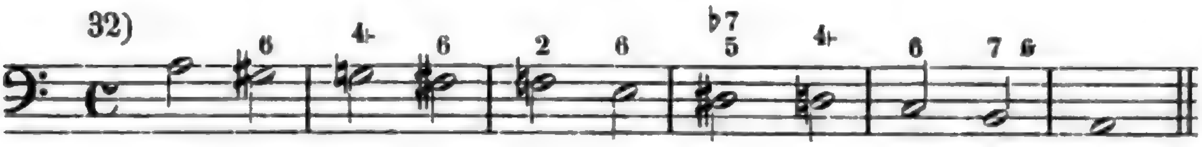

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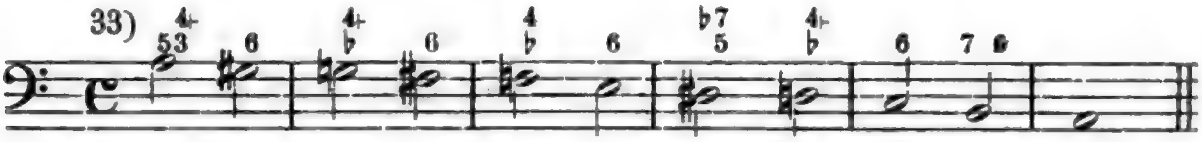

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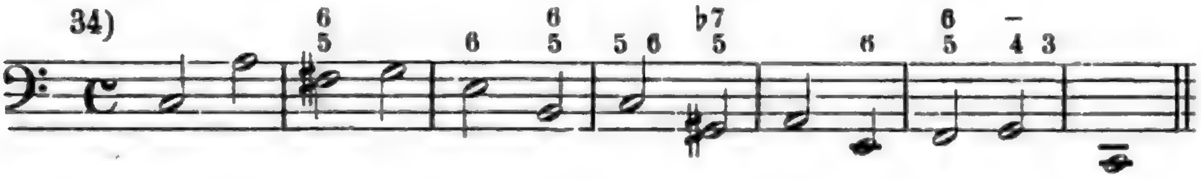

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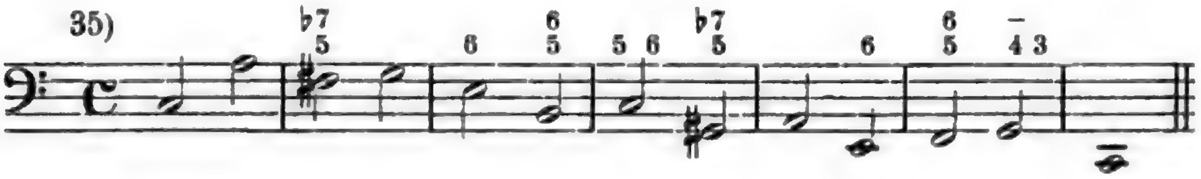

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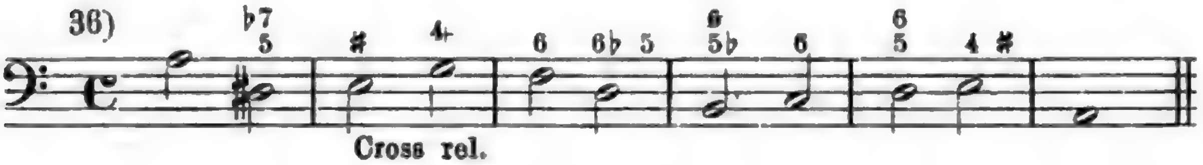
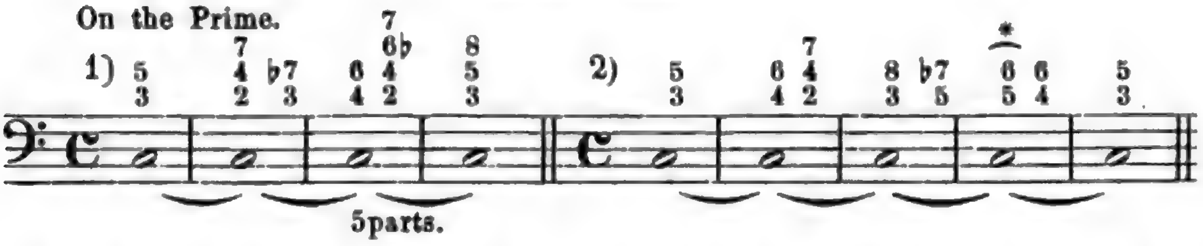


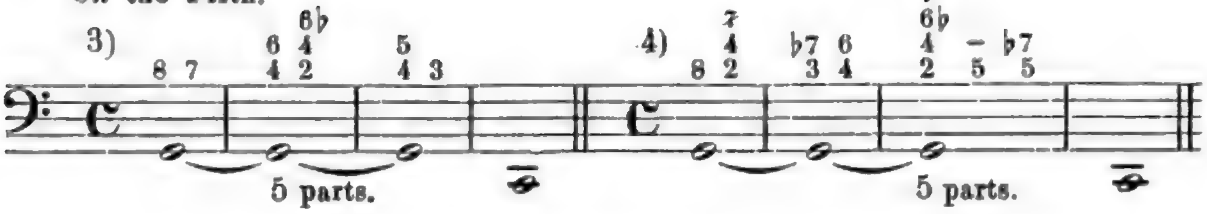
Organ-points.

On the Prime.

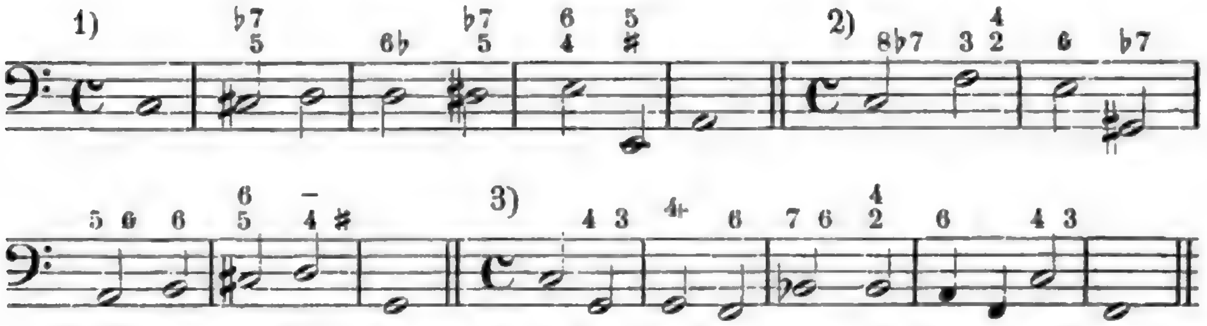


The chord marked * should be written with fifth and sixth without the third, the fundamental being doubled.

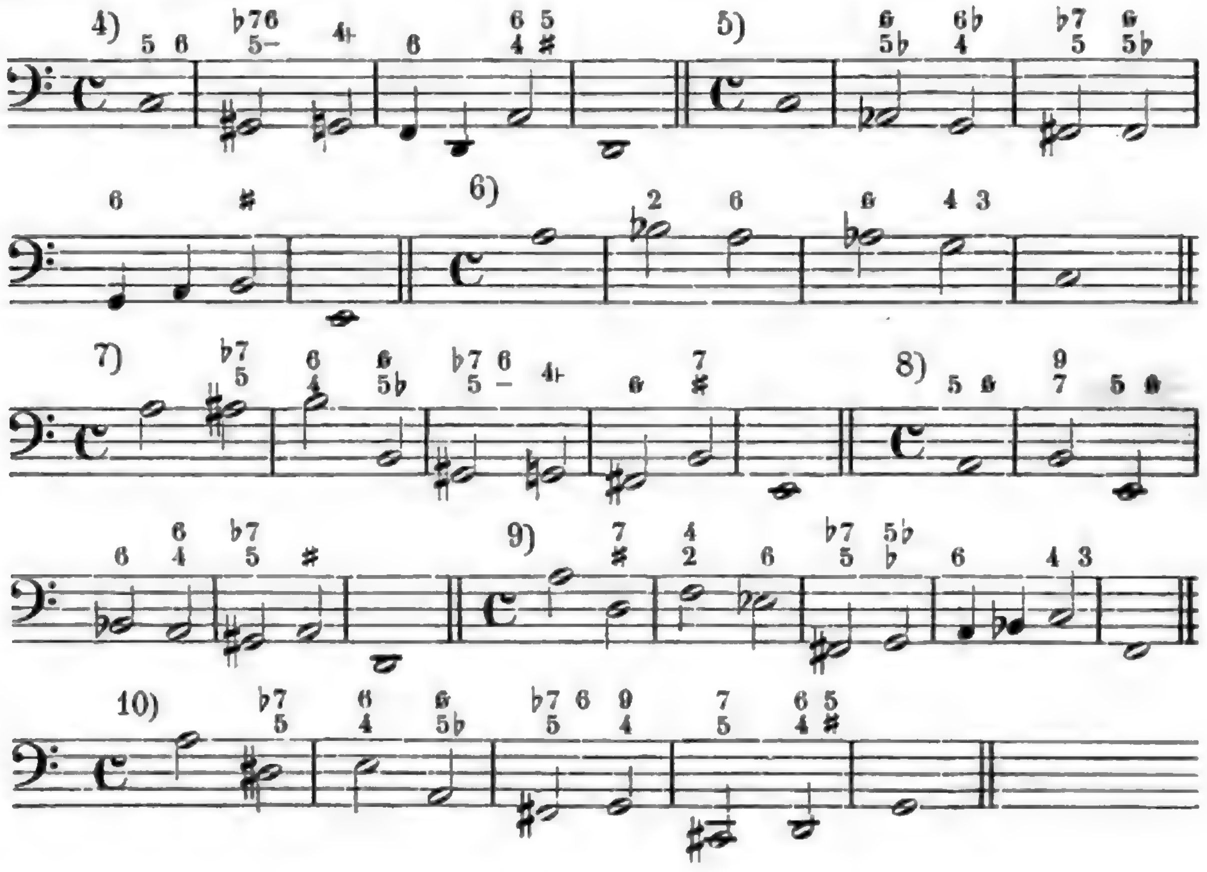
On the Fifth.



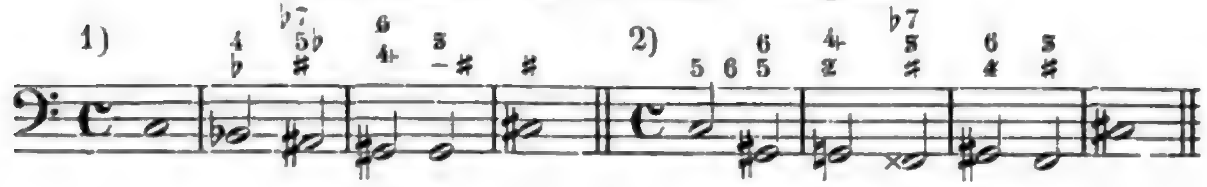
Modulations.

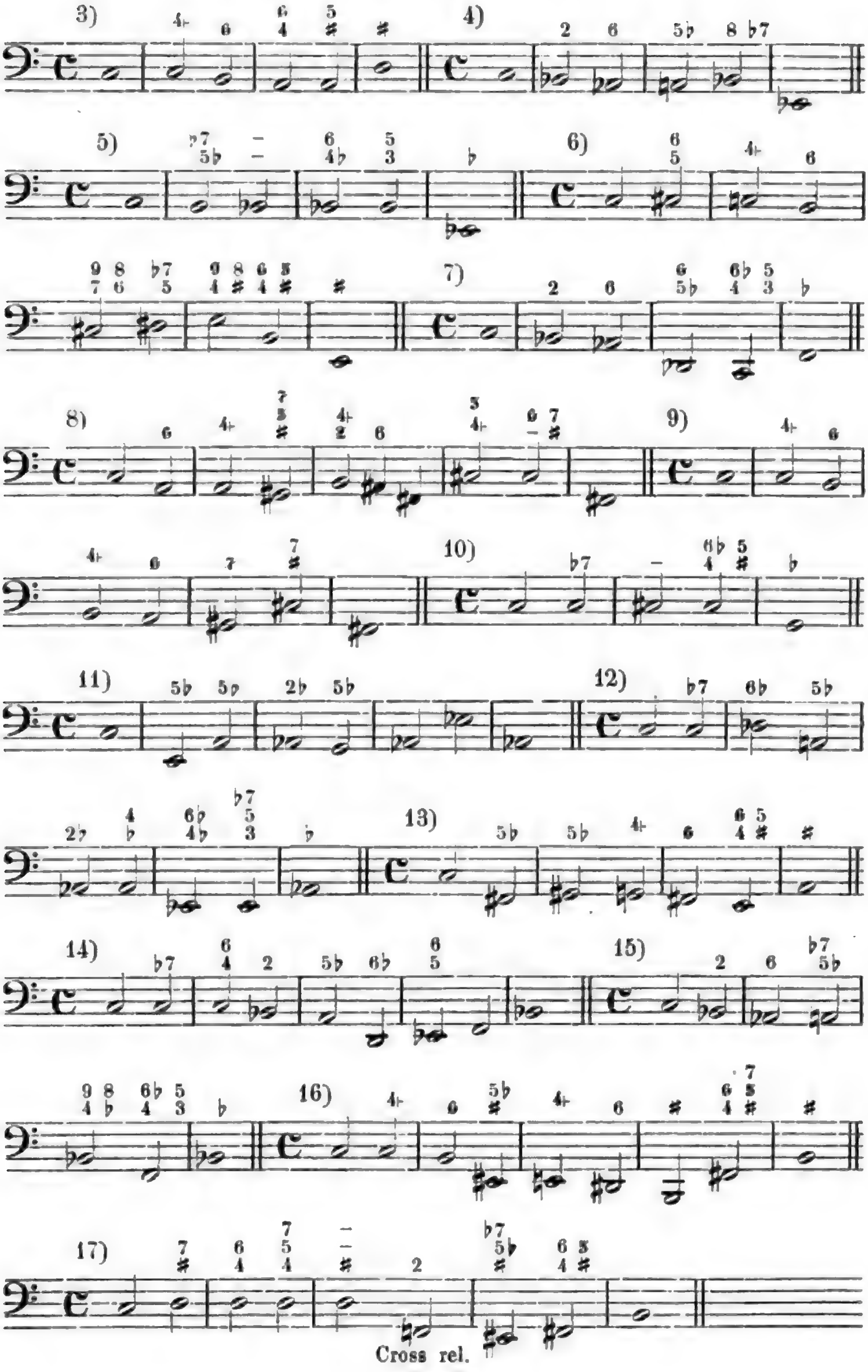


In this example the figure 4 in the first and fourth bars signifies the chord of the fourth and sixth $^{6}_{4}$, which chord is often thus indicated immediately before the 3, or before a chromatic sign (♯, ♭, ♮) taking the place of that figure.



Enharmonic and chromatic Examples.

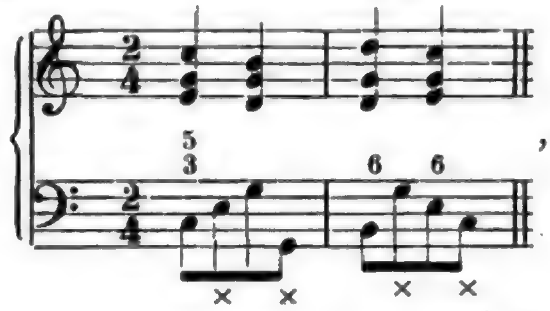


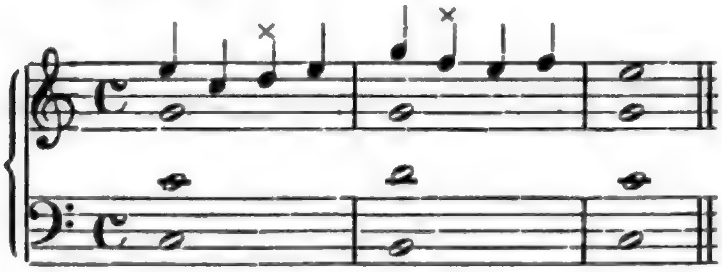

3)
4)
5)
6)
7)
8)
9)
10)
11)
12)
13)
14)
15)
16)
17)
Cross rel.

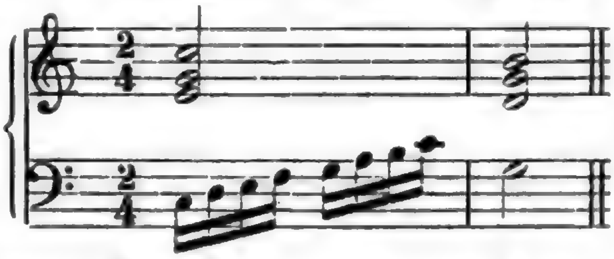
CHAPTER XVII.

Passing and Changing Notes.

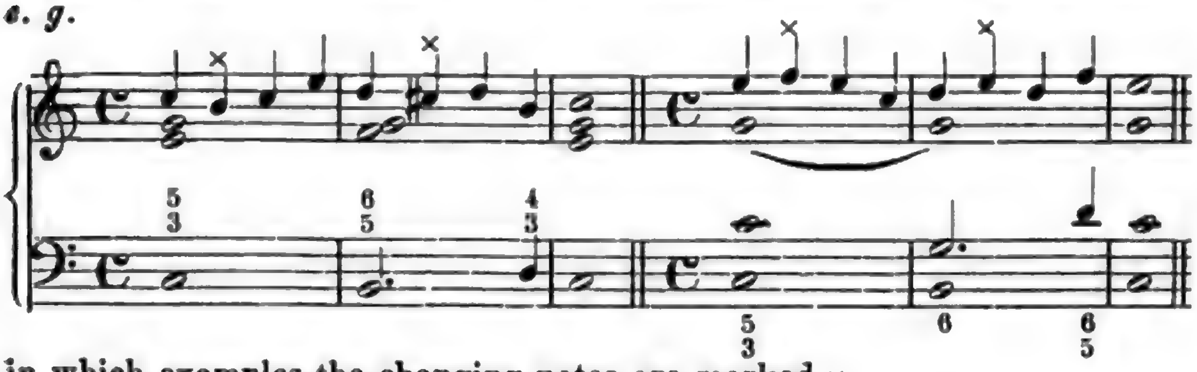
C. Ph. E. Bach rightly divides the passing notes into two classes; for these notes are either chord-notes, like those marked × in the following example: , or lead from one harmony to another, e. g. . The latter must therefore in strict harmony progress by steps, and never by skips. In figurate passages, especially in instrumental music, a number of passing notes may occur in succession, as shown in the following passage in sixteenth-notes: . In such cases the harmonic progression from the last unaccented note [of the passage] to the following harmony on the thesis is to be regarded as that connecting the preceding chord with the one next following.



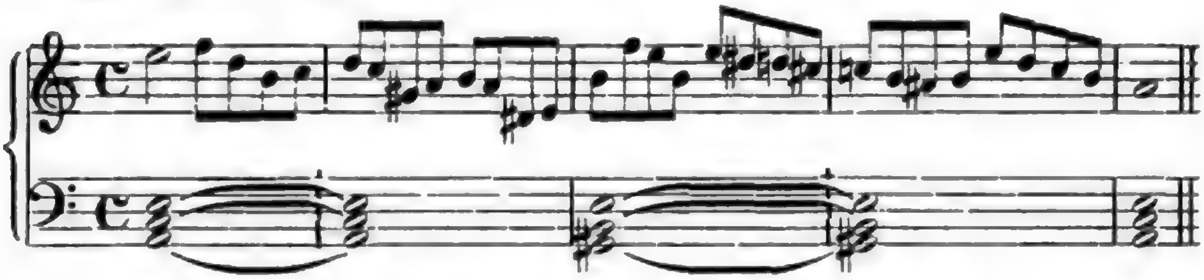




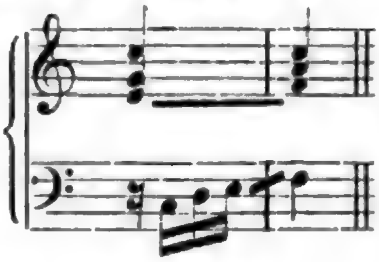
Changing notes, on the other hand, are such as appear in the unaccented part of the measure, and only a semitone or a whole tone above or below the next following note on the thesis belonging to the harmony, *e. g.* in which examples the changing notes are marked ×.

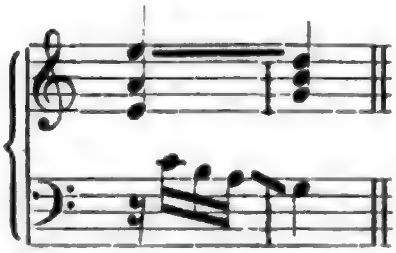


The passing and changing notes given above are all to a certain extent harmonically prepared; in free composition, however, such notes may, more especially in instrumental music, enter in steps or skips without harmonic preparation, in case a harmony be present which acts as basis, and, even if only of short duration, takes in some measure the place of an organ-point; *e. g.*

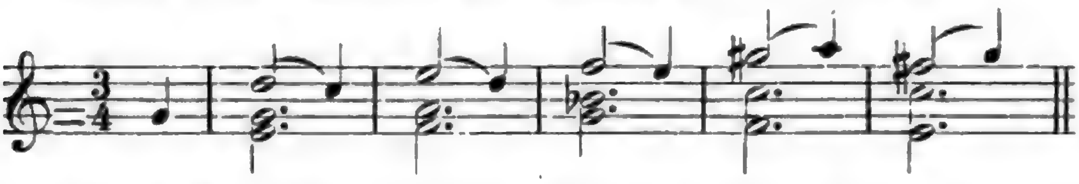


In *a cappella* style such liberties in figuration can, of course, not be taken. Wherever passing notes may occur, the harmonic progression must always be in accordance with the laws of harmony. Such a progression as the following: would be just as faulty as this: . The same laws must therefore be observed in the employment of passing notes as in progressions between simple chords.





It will now be clear, that in progressions like the following:



the chords on the thesis contain no changing notes, but on the contrary in every instance a dissonance which, entering freely [without preparation], finds its resolution on the arsis. In Richter's Manual of Harmony this matter of passing and changing notes is, in my opinion, explained in a manner at variance with the principles of the earlier masters. The same remark is also applicable in the case of the so-called passing chords. According to his theory all chords might be termed passing chords which appear on the arsis, and connect one harmony on the thesis with another on the next thesis.

It is undoubtedly simpler, and more conducive to thoroughness in the study of harmony, to take each chord individually into consideration in its relation to others; respecting the accent, the position of the chord may then be settled, according as it falls on the thesis or on the arsis, in conformity with rhythmical rules. Even in the harmony of an organ-point the chords cannot be briefly disposed of by calling them "passing chords"; each subsists by itself, and fills its peculiar rhythmical position in the composition.

CHAPTER XVIII.

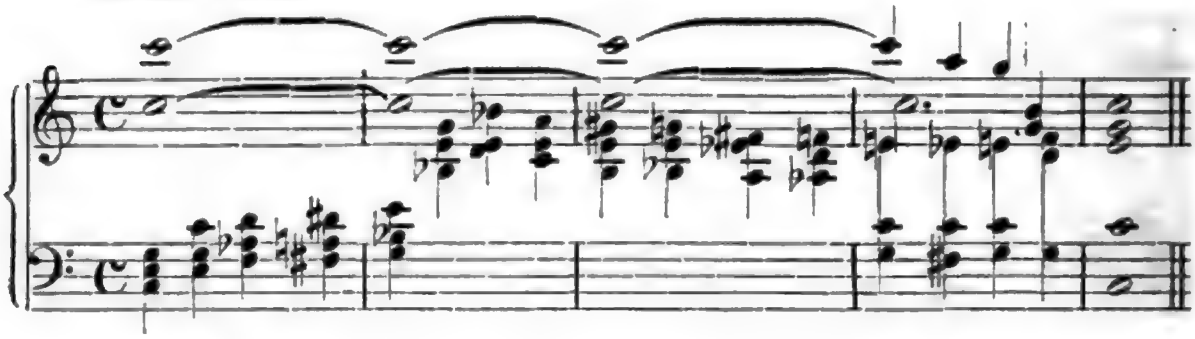
Organ-point.

The organ-point usually *enters* on the thesis. By its *close* on either the tonic or dominant the key is firmly established.

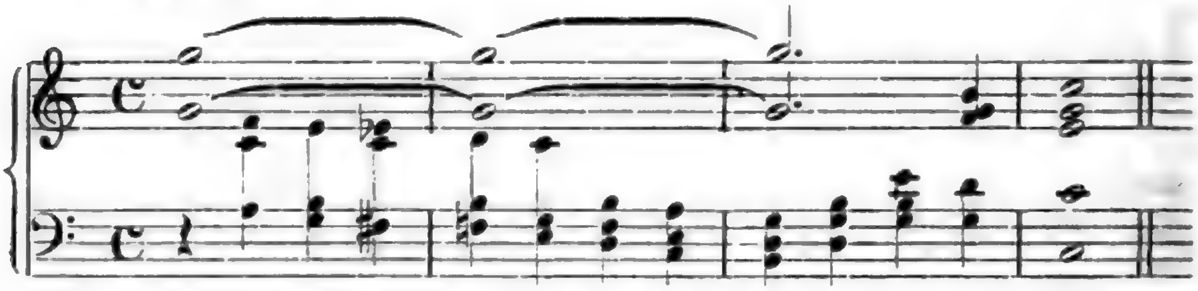
As remarked above, the organ-point generally lies in the bass, although sometimes found in the soprano, or in the middle parts; in later instrumental music it is even occasionally based on the third. These latter forms may, when the product of true inspiration, be extremely effective; the learner will nevertheless do well to be very cautious in employing the organ-point in the soprano or middle parts.

The following are examples of such organ-points:

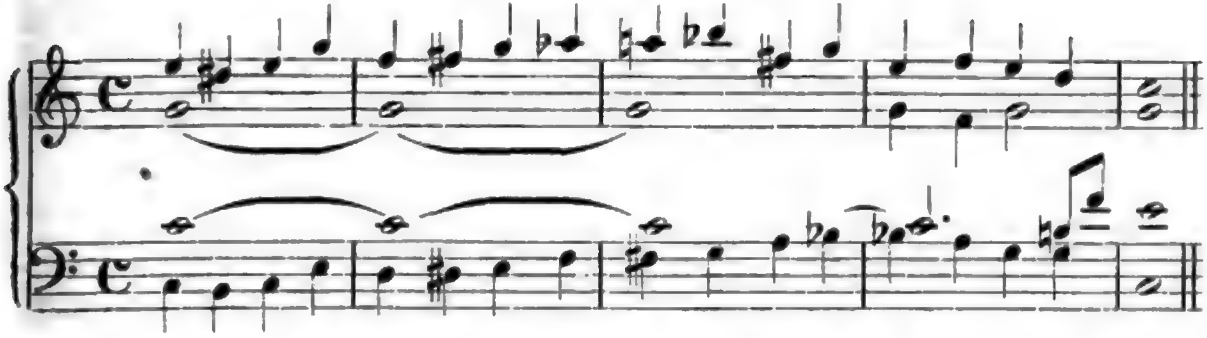
On the Tonic:



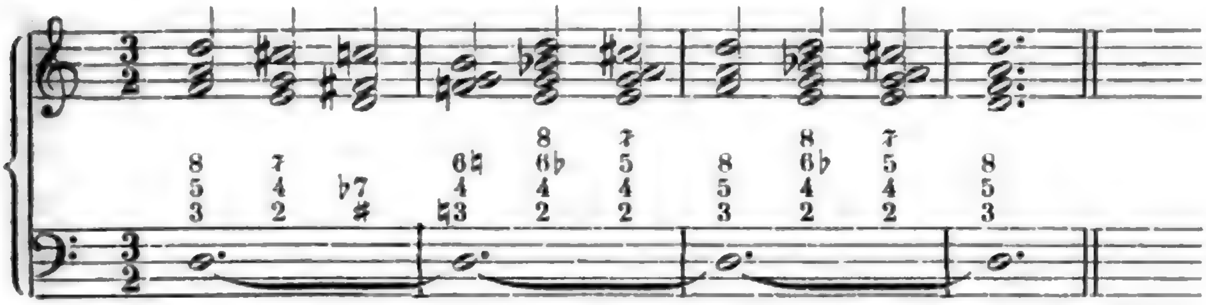
On the Fifth:



In the middle parts on Prime and Fifth:

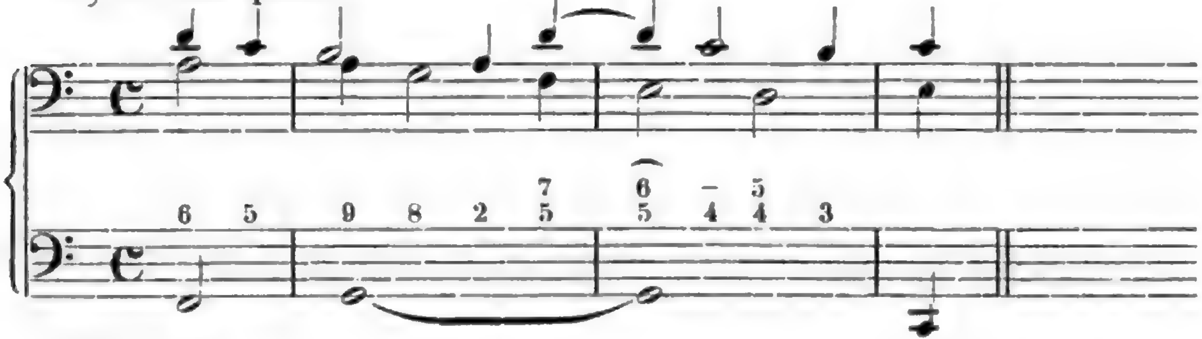


Organ-points of high pitch, with a succession of harmonies below, have come into favor during the modern development of instrumental music; in the **Bach** period the organ-point in the bass was generally employed both in vocal and instrumental compositions. By the French it was also usually employed in the latter manner, and was in most cases treated by them, and by the Germans as well, like a "*tasto solo*" in accompanying. The following example, marked "*point d'orgue*," is from **Rameau**:

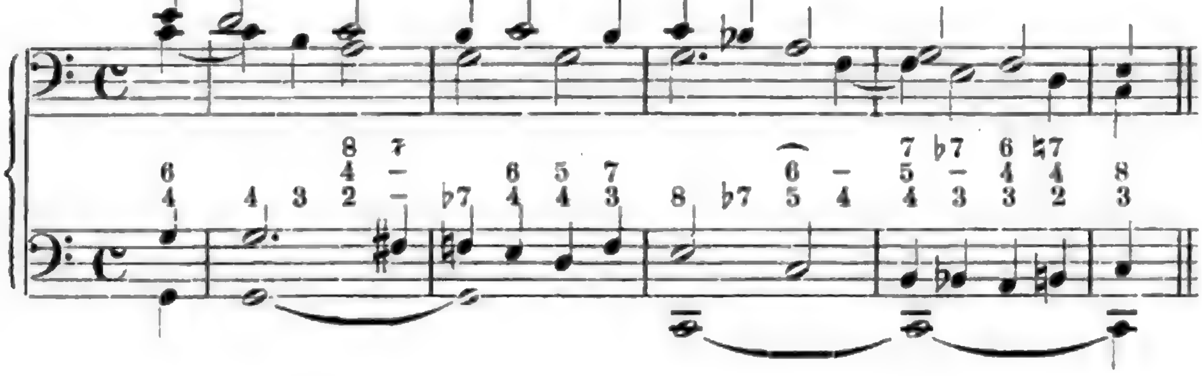


C. **Ph. E. Bach** gives examples of organ-points, which so frequently occur in his father's works, in three, four and five parts, as shown below:

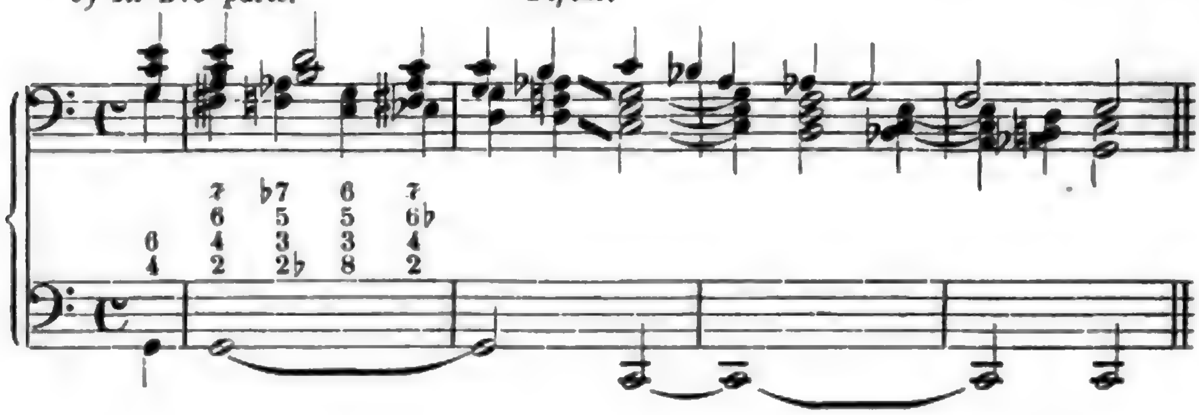
a) In three parts.



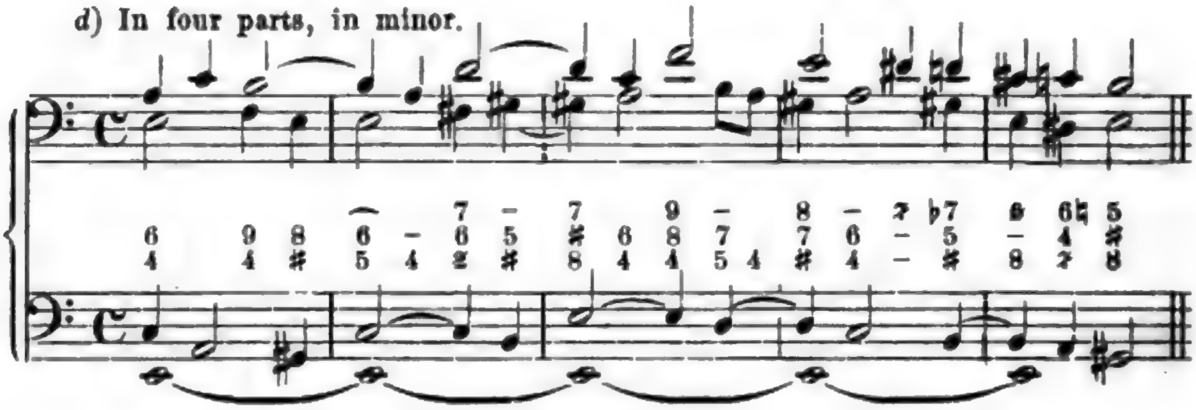
b) In four parts.



c) In five parts.



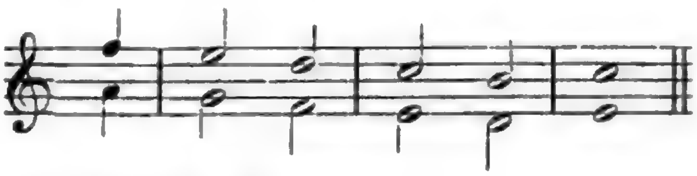
d) In four parts, in minor.



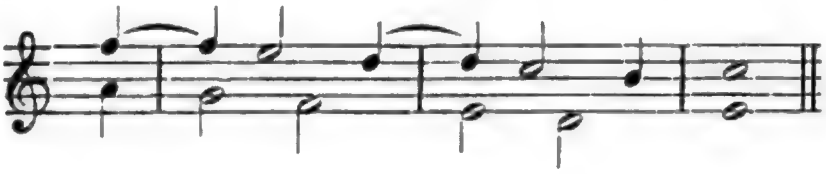
CHAPTER XIX.

Syncope, Retardation, Anticipation and After-striking.

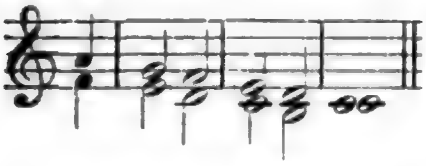
Anticipation is the advancing of one or more parts composing a harmony before the rest; which part or parts would, if all the parts progressed simultaneously, enter later. Through the employment of anticipation, and also of retardation, the syncopated form of harmony results. Syncope consists in binding an arsis to the following thesis; *e. g.* the simple succession of sixths:



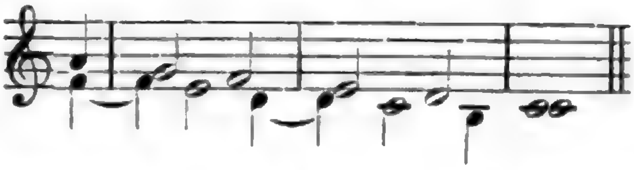
would appear in syncopated form thus:

,

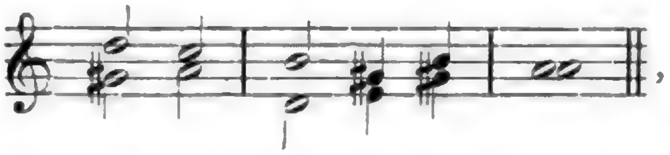
or in thirds:



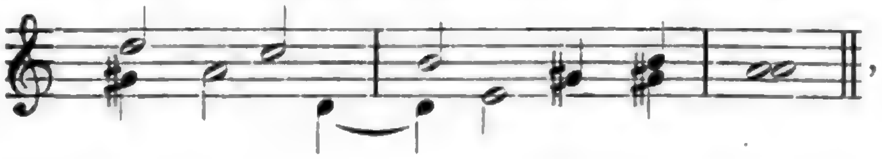
syncopated:



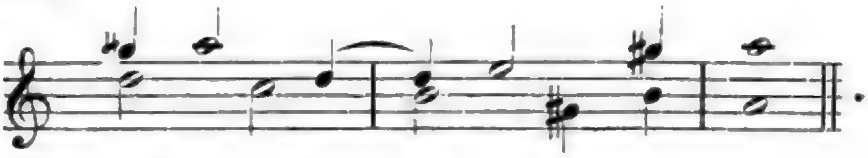
Here the syncope arises from retardation. If the following simple form be taken:



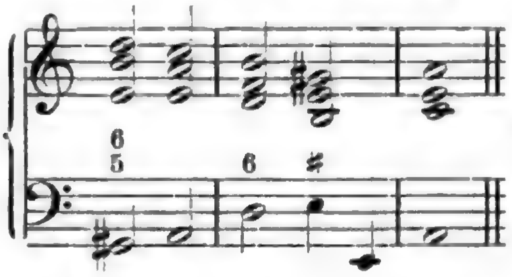
the harmony would, by employing anticipation, appear thus:



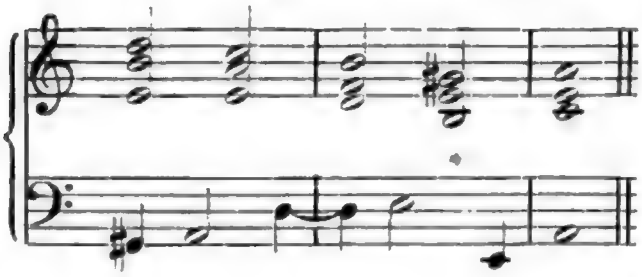
or as inverted:



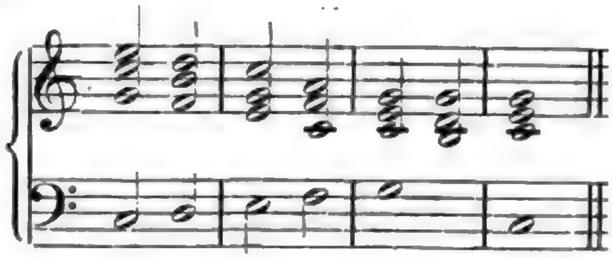
Or with chords:

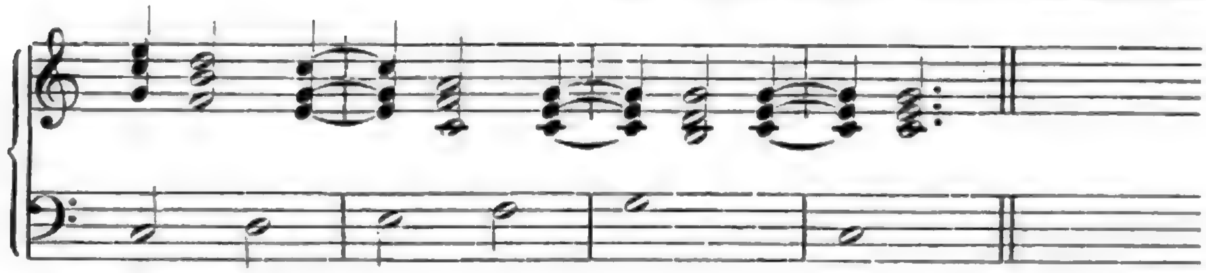


with anticipation of the bass:

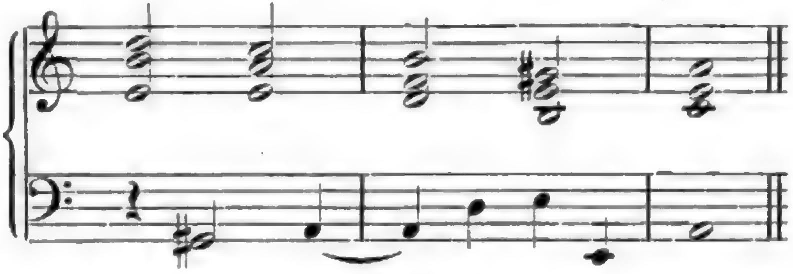


. Or with anticipation of the chords, the simple form being:

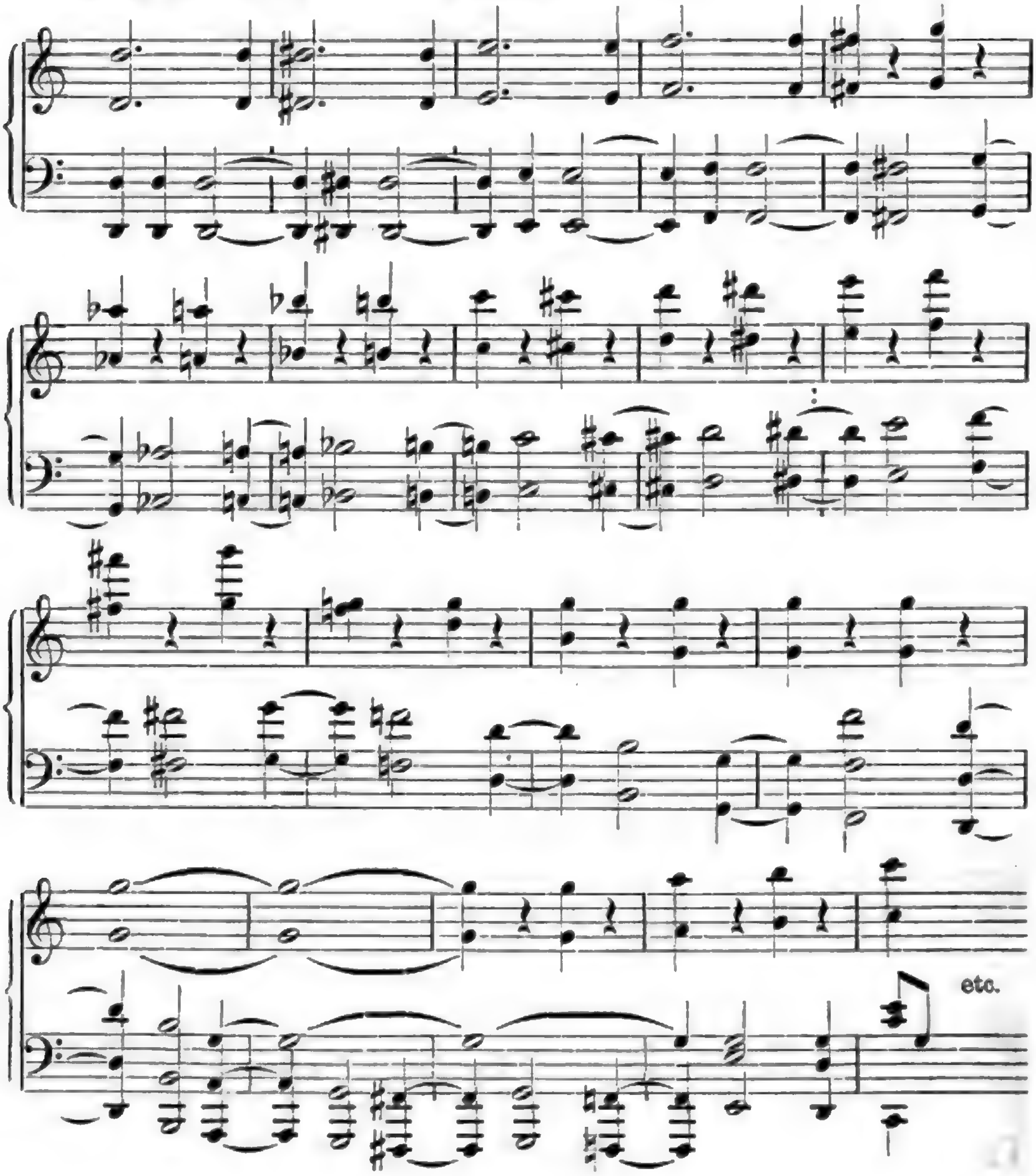




After-striking is precisely the reverse of anticipation in the bass; for instance, the above example in minor would appear, with after-struck bass, as follows:



The celebrated unison passage in BEETHOVEN's Leonore Overture No. 3 is often taken as a model example of after-striking, and may, although universally known, be allowed a place here.



CHAPTER XX.

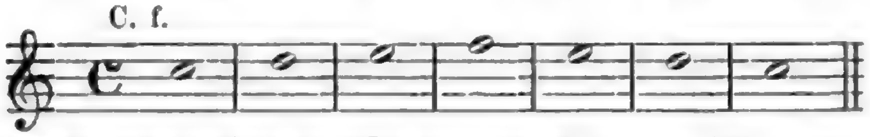
On Harmonizing a Melody, or Cantus Firmus. — Choral Composition.

Although the following remarks belong, in point of fact, to a work on counterpoint, a few hints as to the proper treatment of a choral-melody may be not unwelcome to the student.

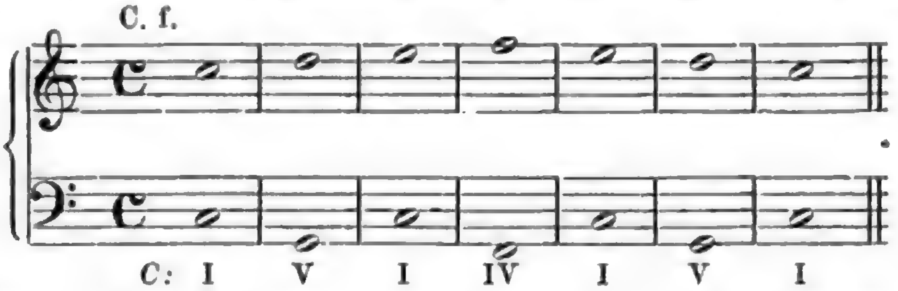
Scores for four-part composition are, at the present time, generally written only in the *G*-(violin) clef and *F*-(bass) clef. For those desiring to diligently study the works of Seb. Bach or G. F. Händel, facility in the employment of the other clefs is, however, indispensable. Where the tenor part is written in the violin-clef it should be remembered that the notes sound an octave lower than they are set.

If we at first use only the violin and bass-clefs, the comprehension of the harmonic treatment of a *cantus firmus* may be attended with fewer difficulties for the student.

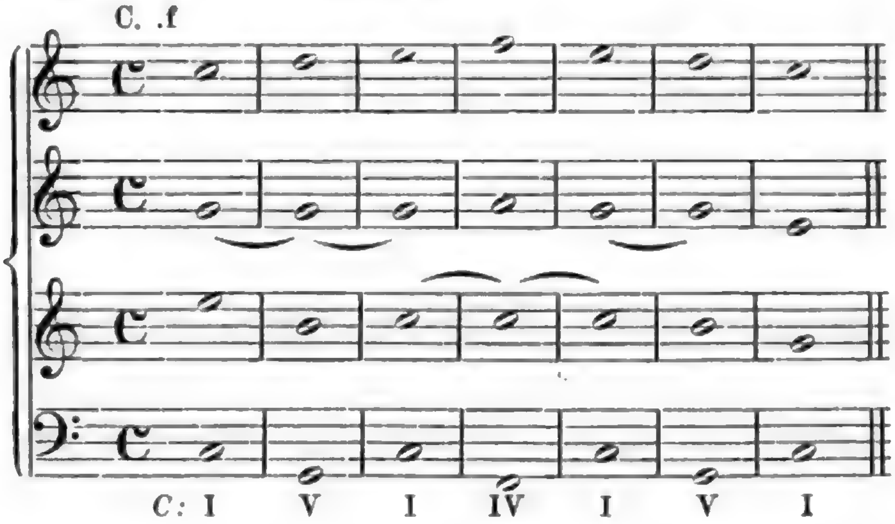
If a melody, or *cantus firmus*, marked *C. f.*, be given, *e. g.*



the bass may be set to this melody in various ways. By employing only triads in the fundamental position, the bass would take this form:

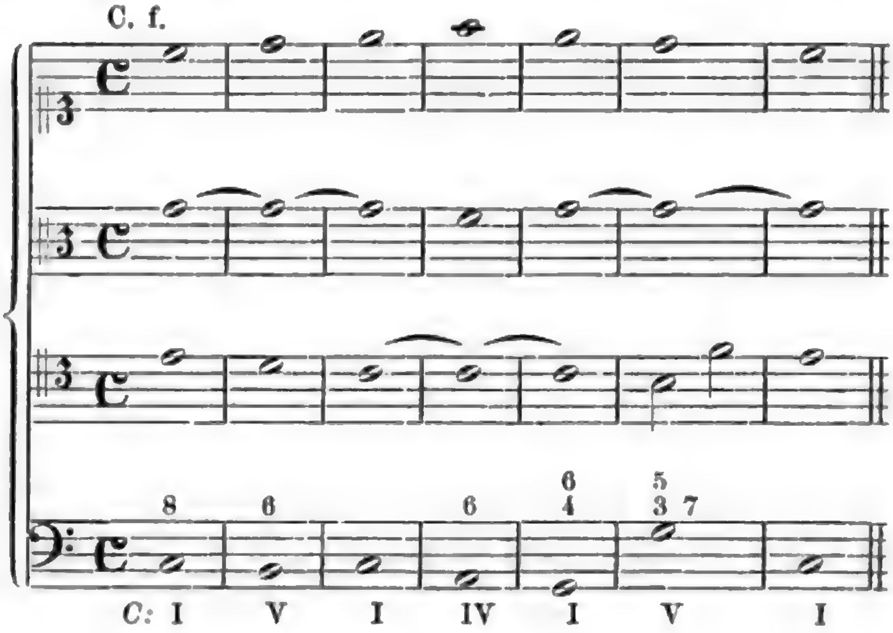


The addition of the middle parts offers no special difficulties to one practiced in four-part harmony, *e. g.*

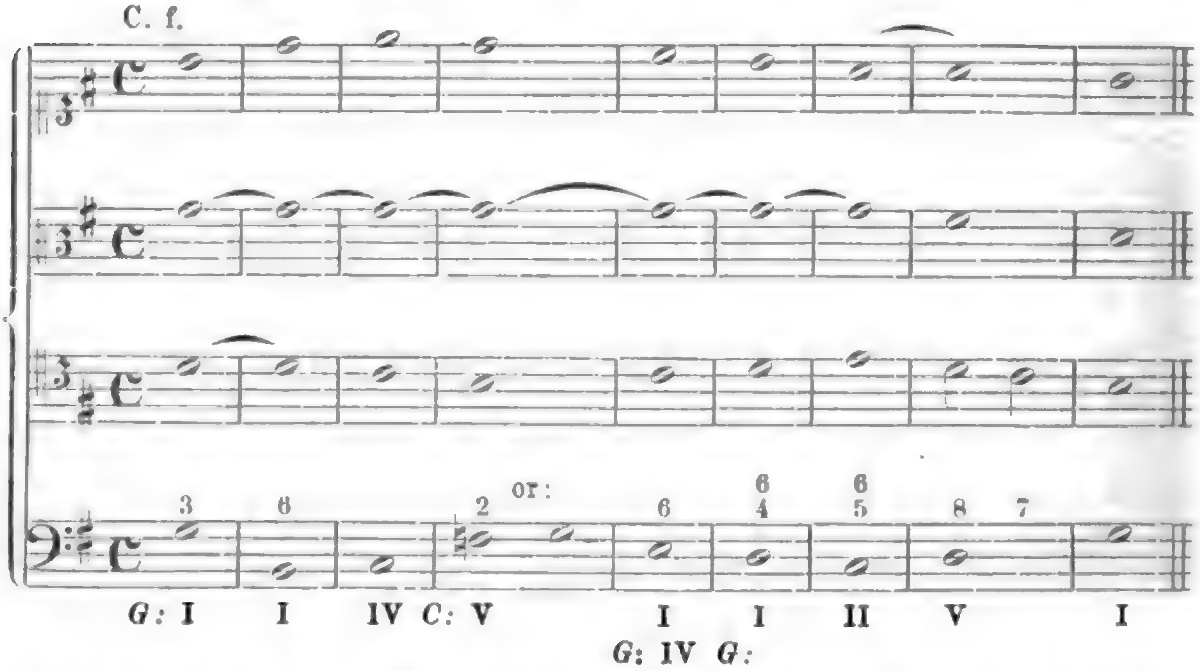


As however not only triads in the fundamental position, but also their inversions, and besides these the chords of the seventh and their inversions, may be employed, we need not hesitate to make use of this richer material. By adhering to a dignified diatonic style there need be no fear of obtaining too great harmonic variety. The works of **Bach** plainly show how infinite a variety of harmonic changes may be employed without in the least overloading the composition.

In simple style, but using the inversions of the chord in harmonizing, the example would appear (employing the soprano, alto and tenor-clefs, and marking the degrees in the key) as follows:

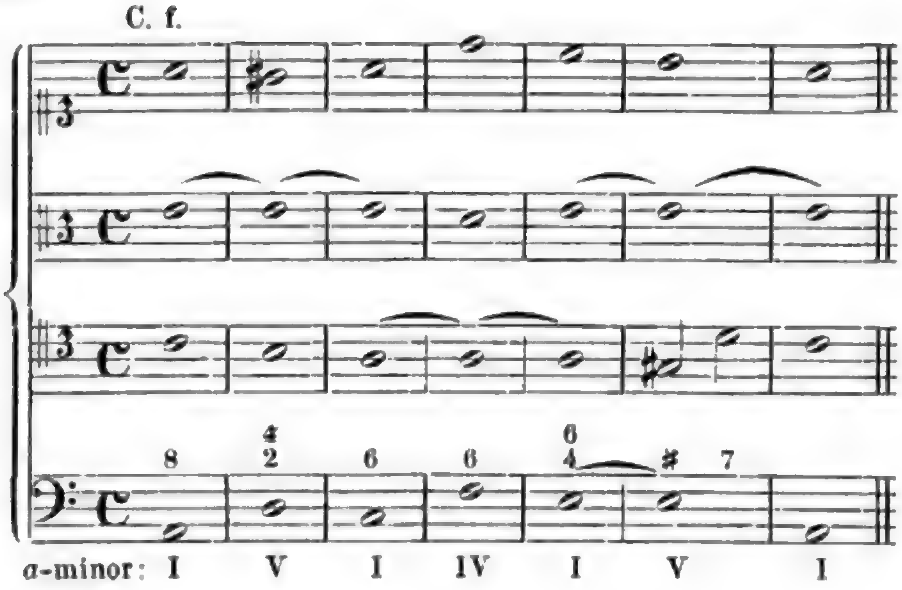


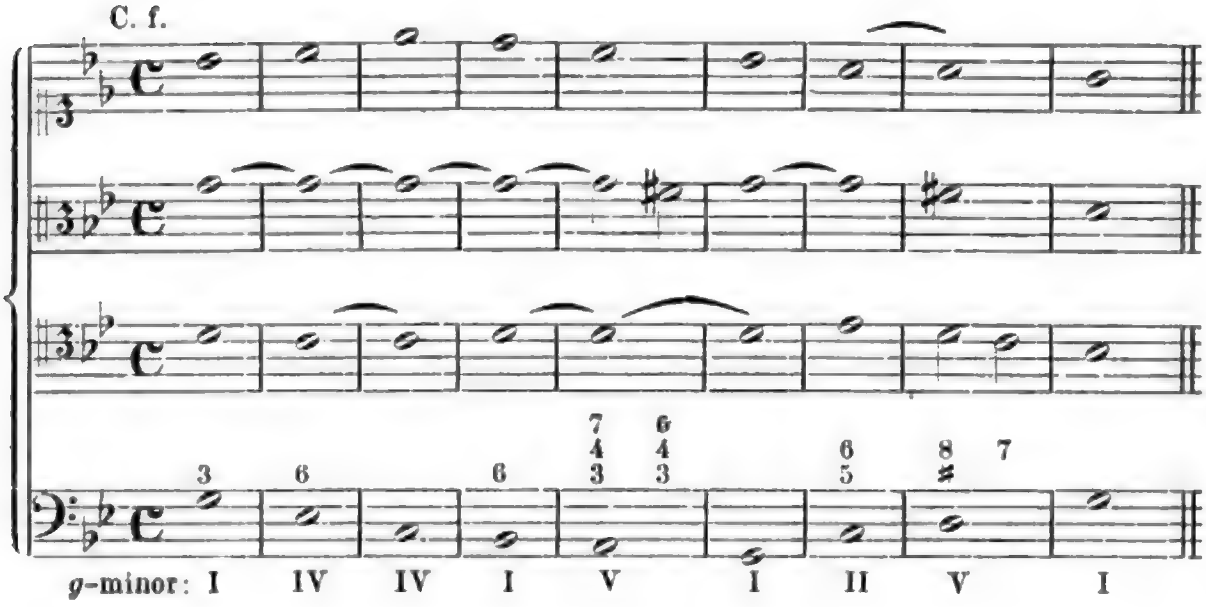
If the *cantus firmus* begin on the third:

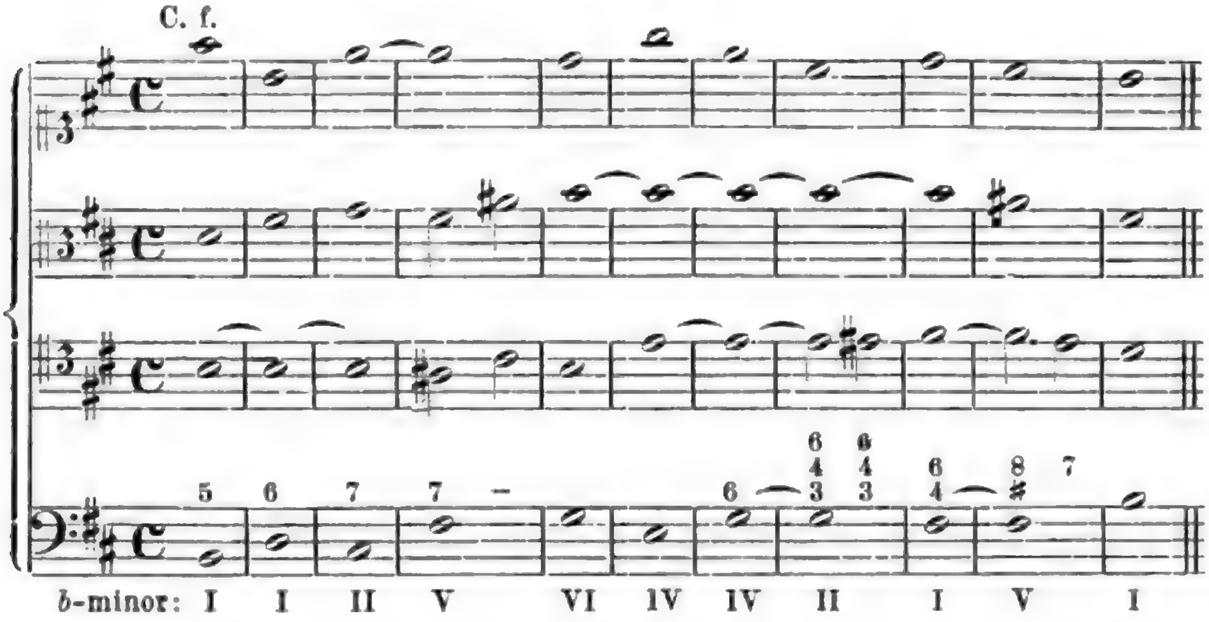


By taking *g* in the fourth measure instead of *f*, in the bass, modulation is avoided.

In minor:


C. f.
8 4/2 6 6 6/4 ♯ 7
a-minor: I V I IV I V I

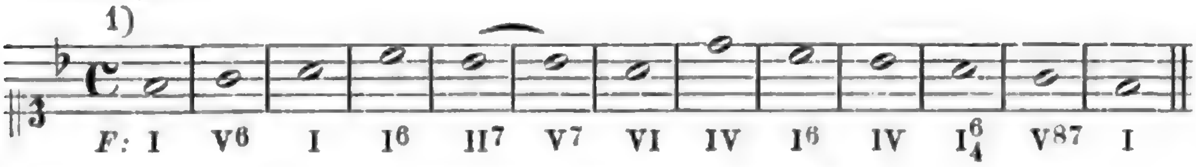

C. f.
3 6 6 7/4/3 6/4/3 6/5 8/♯ 7
g-minor: I IV IV I V I II V I


C. f.
5 6 7 7 – 6 6/4/3 6/4/3 6/4 8/♯ 7
b-minor: I I II V VI IV IV II I V I

The student, who has already had practice in writing the harmonies to figured basses, should now harmonize melodies set in the soprano, taking care to secure a melodious [not too monotonous] leading of the bass. The following melodies, each of which is to be set in the soprano as *cantus firmus;* may serve as a preliminary exercise to harmonizing the melodies of chorals.

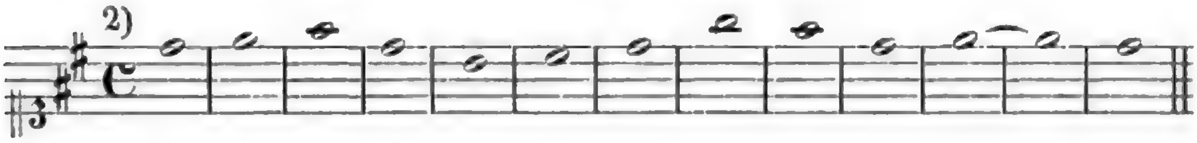
In major:

1)

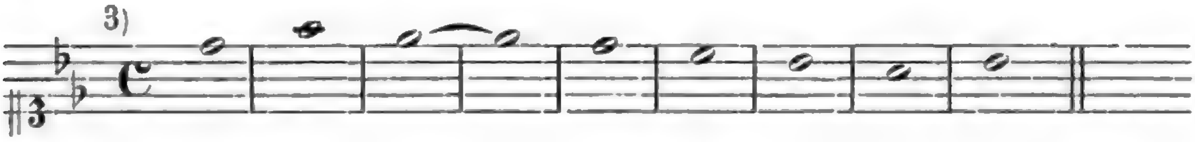


The rest should be harmonized without assistance.

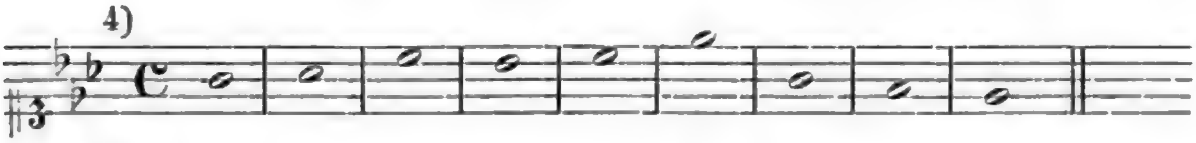
2)



3)



4)



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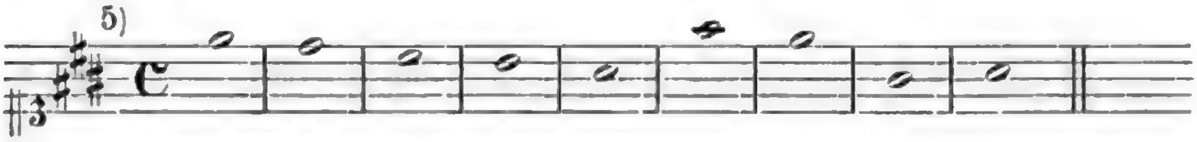

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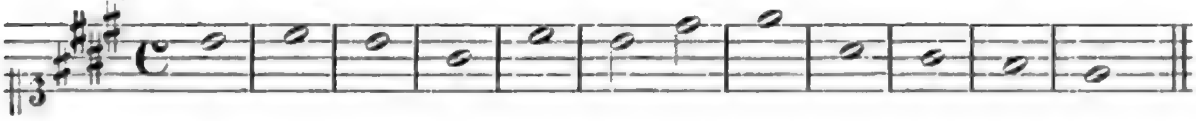

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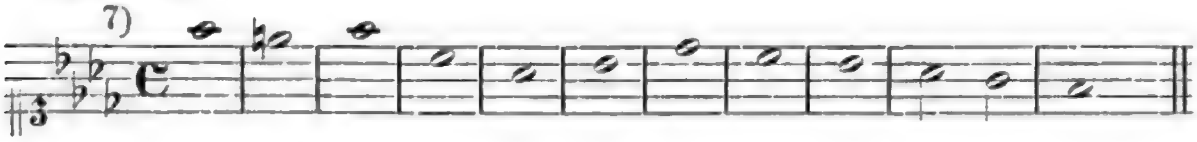

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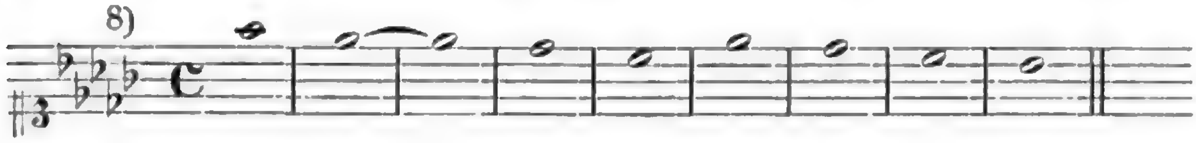

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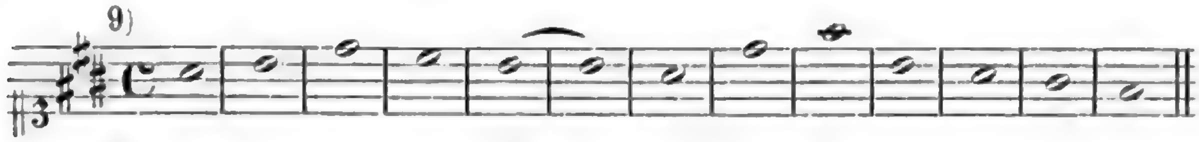

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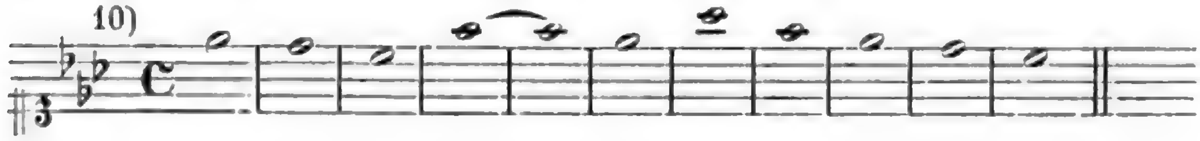


Harmonic exercises requiring the *cantus firmus* to be set in one of the middle parts are to be undertaken only after two-part contrapuntal composition has been studied. The exercises just given are intended merely to prepare the student quickly for harmonizing chorals. Intelligent pupils will be able to perform this even without the aid of these exercises; so many choral-books are to be obtained that it is scarcely necessary to treat of the simple harmonizing of chorals in a Manual of Harmony. The student who has had sufficient practice in harmonizing figured basses will do well to take up a good book of chorals, to analyse the harmonic structure of some of them, and then to harmonize the melodies independently.

The figuration of chorals does not come within the scope of this work, but belongs to Counterpoint.

Among the later choral-books to be recommended, that in use in the Evangelical Churches in Prussia, by CARL HEINRICH SÄMANN, (Leipzig, Breitkopf und Härtel) deserves special mention. Besides this voluminous and well-arranged work, the careful harmonizing of which is particularly noteworthy, that by ALFRED DÖRFFEL (Leipzig, Edition Peters) may, on account of the pure and simple style of its harmony, be warmly recommended to students and teachers alike. Organists will, of course, study the compositions of RINCK, FISCHER, SCHICHT, etc. It is the duty of every musician to study JOH. SEB. BACH'S 371 four-part choral-songs (Leipzig, Breitkopf und Härtel, fourth ed.). The routine acquired through the thorough-bass exercises, and an understanding of pure harmony, will enable the student to judge of the freedom and occasional harshness of BACH'S style correctly, and to fully appreciate the beauties in the melodic progression of the parts. The remarks made above on the subject of passing and changing notes will render an explanation of many such progressions easier.

We have reached the end. May this Manual of Harmony prove useful, more especially in preparing the student properly for the study of Counterpoint.

APPENDIX.

Note I. The range of musical tones, upward or downward, is limited only by the capacity of the ear for distinguishing the separate tones. The impressions made upon the sense of hearing are analysed by the science of acoustics, and are divided into two classes, noises and tones. Both are created by vibrations imparted to the atmosphere, and transmitted by the same to the ear. The most eminent scientist of the present day in the province of acoustics and physiological investigation, H. HELMHOLTZ, who by his work on "The science of the perception of tones" [Die Lehre von den Tonempfindungen, fourth ed. Braunschweig, Friedrich Vieweg und Sohn, 1877] has earned the thanks of every thinking musician, and has laid a secure foundation for the theory of music, explains perception by the ear as follows: "The irregular and changeable character of the impressions caused by noises leads us to the conclusion, that the vibrations of the air in such cases must also have an irregular and changeable movement; that musical tones, on the contrary, are caused by regular, constant vibrations of the air, which must, in their turn, be generated by precisely such regular vibrations of the body originating the tone, which vibrations are transmitted to the ear through the medium of the air."

The swinging of a pendulum is called a *periodic* movement, this movement being repeated at regular intervals. The movement imparted to the air by a tone-producing body is likewise periodic, but so exceedingly rapid as scarcely to be compared, in point of velocity, with the slow periodic movement of the pendulum, which can easily be distinguished by the eye. The periodic movements of tone-producing bodies are named vibrations, and "the equal divisions of time elapsing between repetitions of like movements are termed the periods of vibration."

As noises are distinguished from musical tones through a different manner of movement, the distinction between tones and noises is, according to HELMHOLTZ, to be defined as follows: "The sensation caused by a tone is produced by rapid periodic movements; that caused by a noise is produced by imperiodic movements."

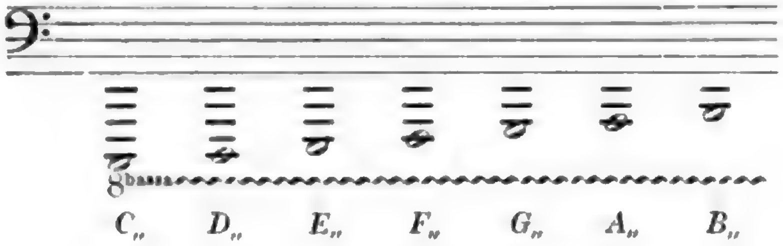
Musical tones may be classed according to intensity [loudness], pitch and quality. The intensity of a tone depends upon the amplitude of the vibrations of the tone-producing body [all other conditions being equal]. It is easily to be seen that the vibrations of strings are strongest at first, and that as they decrease in breadth the loudness of the tone also decreases. It is likewise observable, that on retreating from a tone-producing body in the open air the sound loses in strength. "But, as at a distance only the amplitude of vibration of the atmospherical atoms is changed [diminished] in the character of the air-waves, the *loudness* of the sound, but no other characteristic of the same, must be dependent upon the amplitude of vibration."

"The pitch of a tone depends upon the length of the period of vibration, or, which is the same thing, upon the number of vibrations made in a given time;" and the quality of the tone depends upon the form of vibration. Each orchestral instrument has its peculiar form of vibration.

The pupil should now memorize the designations of the musical tones [according to absolute pitch] in the various octaves.

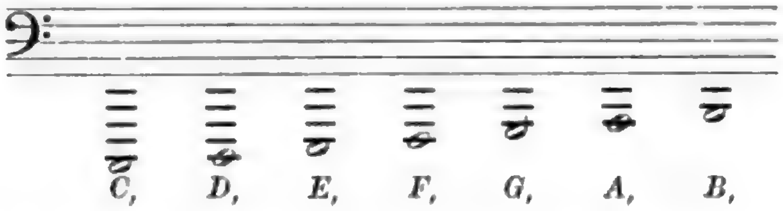
The lowest octave is:

1. The double contra-octave, also called, with reference to the organ, the 32-foot-octave, because its lowest tone, *C,,*, requires for its production an open pipe 32 feet in length. Its separate tones are designated, in letters: *C,, D,, E,, F,, G,, A,, B,,*; and on the staff as follows:

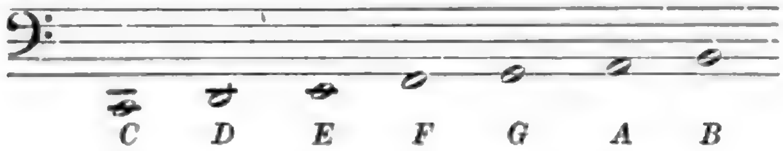


The next octave above the preceding is

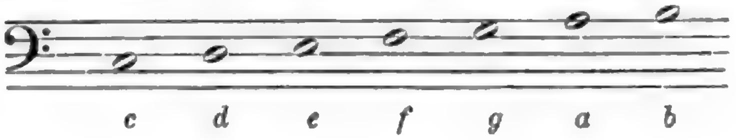
2. The contra-octave (16-foot-octave of the organ).



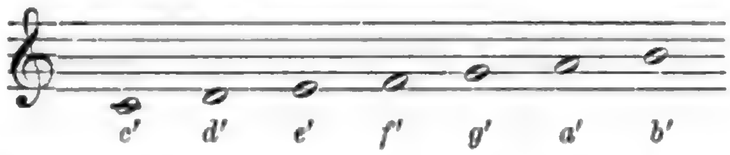
3. The *first* or *great* octave (8-foot-octave of the organ).



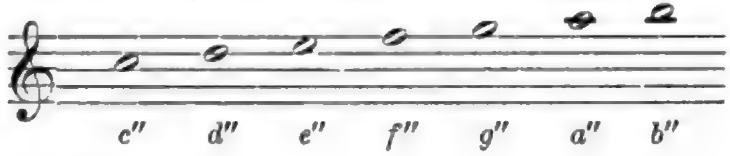
4. The second octave (4-foot-octave of the organ).



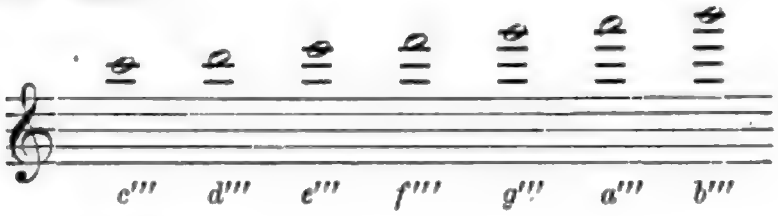
5. The third octave (2-foot-octave of the organ), the lowest tone of which, *c'*, is called "middle *C*."



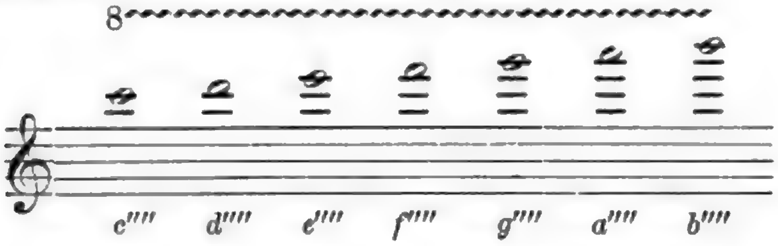
6. The fourth octave (1-foot-octave of the organ).



7. The fifth octave:



8. The sixth octave:



Note II. *Diatonic* and *Chromatic*. Both these terms are taken from the Greek. Diatonic is derived from διά *through* and τόνος *key* (*mode*), and therefore signifies that which goes through the whole key, *i. e.* whatever is to be found within the limits of any key. In modern music this expression denotes the normal relations subsisting between the tones found in the major and minor modes. The term chromatic, from the Greek word χρῶμα *color*, is used to designate all such relations between the tones as are created through an artificial raising or lowering of the same. In Oscar Paul's work on "Boetius and the Harmony of the Greeks" (Boetius und die Harmonik der Griechen, Leipzig, F. E. C. Leuckart, 1872) may be found a full description of the diatonic, chromatic, and enharmonic modes of the Greeks, and also of the *chromatics* (χρόαι) of Claudius Ptolemaeus.

Note III. "The greater the number of vibrations, or the shorter the period of vibration, the higher will be the pitch of the tone produced". Within the period of vibration the movement may take any form whatever, and, if the period of vibration of two tones be alike, they will still be of the same pitch. Experiments with acoustic instruments (Monochord, Sirene) have established the following proportions [for the vibration of the various tones of the scale as compared with the prime]: 1 : 1 unison, 1 : 2 octave, 2 : 3 fifth, 3 : 4 fourth, 4 : 5 major third, 5 : 6 minor third. From these proportions the period of vibration of each tone in the major scale, as compared with the tonic, may be deduced:

$$\begin{array}{ccccccccccccccc} C & : & D & : & E & : & F & : & G & : & A & : & B & : & C \\ 1 & : & \frac{9}{8} & : & \frac{5}{4} & : & \frac{4}{3} & : & \frac{3}{2} & : & \frac{5}{3} & : & \frac{15}{8} & : & 2 \end{array}$$

A comparison of the separate tones with one another gives the following proportions: $C : D = 8 : 9$, $D : E = 9 : 10$, $E : F = 15 : 16$, $F : G = 8 : 9$, $G : A = 9 : 10$, $A : B = 8 : 9$, $B : C = 15 : 16$.

For whole tones there are consequently two different proportions; the *greater* whole tone interval between the first and second, fourth and fifth, and sixth and seventh degrees; and the *lesser* whole tone interval between the second and third, and fifth and sixth degrees. The proportion for the diatonic semitone is in every case the same, namely 15 : 16.

The proportion for a chromatic semitone is found by subtracting a diatonic semitone from either the greater or the lesser whole tone interval. This gives in the former instance the proportion of 128 : 135, and in the latter the proportion of 24 : 25. *E. g.*: in *C* major we have the proportion $C : C\sharp = 128 : 135$; and $D : D\sharp = 24 : 25$. As the greater and lesser whole tone intervals differ from each other by 80 : 81, the greater and lesser chromatic semitones differ by the same proportion.

The proportions between the several tones in the minor scale are found in the same manner; for the augmented second between the sixth and seventh degrees we have the proportion 64 : 75; the proportions for the entire scale would appear as follows:

$$\begin{array}{ccccccccccccccc} A & : & B & : & c & : & d & : & e & : & f & : & g\sharp & : & a \\ 1 & : & \frac{9}{8} & : & \frac{16}{15} & : & \frac{9}{8} & : & \frac{10}{9} & : & \frac{16}{15} & : & \frac{75}{64} & : & \frac{16}{15} \end{array}$$

According to SCHEIBLER the tone a' makes 440 vibrations in a second of time; in the following table the number of vibrations made in a second by each tone in the entire range from C_1 to b'''' is given:

C_1	33	D_1	37,125	E_1	41,25	F_1	44	G_1	49,5	A_1	55	B_1	61,875
C	66	D	74.25	E	82,5	F	88	G	99	A	110	B	123,75
c	132	d	148,5	e	165	f	176	g	198	a	220	b	247,5
c'	264	d'	297	e'	330	f'	352	g'	396	a'	440	b'	495
c''	528	d''	594	e''	660	f''	704	g''	792	a''	880	b''	990
c'''	1056	d'''	1188	e'''	1320	f'''	1408	g'''	1584	a'''	1760	b'''	1980
c''''	2112	d''''	2376	e''''	2640	f''''	2816	g''''	3168	a''''	3520	b''''	3960

Note IV. *The pure and tempered systems of harmony.* On this subject two excellent articles by MORITZ HAUPTMANN, entitled "Klang" (musical tone) and "Temperatur" (temperament), in "CHRYSANDER'S Jahrbüchern für musikalische Wissenschaft," (Leipzig, Breitkopf und Härtel) [more easily obtainable in HAUPTMANN'S "Opuscula," Leipzig, F. E. C. Leuckart, 1874] contain all necessary information. The principal points are here presented to the student in condensed form, with the aid of the Pythagorean system of music. Fuller explanations may be found in the work of HELMHOLTZ (p. 433 etc.).

In the Pythagorean system we find only perfect fifths; from C upwards these would be:

$$C\text{-}G\text{-}D\text{-}A\text{-}E\text{-}B\text{-}F\sharp\text{-}C\sharp\text{-}G\sharp\text{-}D\sharp\text{-}A\sharp\text{-}E\sharp\text{-}B\sharp.$$

These fifths can be set together within the compass of an octave by progressing upward two fifths at a time, setting the fifth thus reached an octave lower, and then repeating the operation, beginning each time with the fifth last lowered. In figures this process would appear thus: $\frac{3}{2} \times \frac{3}{2} \times \frac{1}{2} = \frac{9}{8}$; and the final result thus:

C	D	E	$F\sharp$	$G\sharp$	$A\sharp$	$B\sharp$
1	$\frac{9}{8}$	$(\frac{9}{8})^2$	$(\frac{9}{8})^3$	$(\frac{9}{8})^4$	$(\frac{9}{8})^5$	$(\frac{9}{8})^6$

The series of perfect fifths downwards is:

$$C\text{-}F\text{-}B\flat\text{-}E\flat\text{-}A\flat\text{-}D\flat\text{-}G\flat\text{-}C\flat\text{-}F\flat\text{-}B\flat\flat\text{-}E\flat\flat\text{-}A\flat\flat\text{-}D\flat\flat.$$

These fifths may also be brought into juxtaposition within the compass of an octave, by progressing downward two fifths at a time, and setting the notes reached an octave higher, which gives the following result:

C	$B\flat$	$A\flat$	$G\flat$	$F\flat$	$E\flat\flat$	$D\flat\flat$
1	$\frac{8}{9}$	$(\frac{8}{9})^2$	$(\frac{8}{9})^3$	$(\frac{8}{9})^4$	$(\frac{8}{9})^5$	$(\frac{8}{9})^6$

For $D\flat\flat$ the proportion $(\frac{8}{9})^6 = \frac{262144}{531441}$, *i. e.* $\frac{1}{2} \times \frac{524288}{531441}$, or, as contracted $(\frac{8}{9})^6 = \frac{1}{2} \times \frac{73}{74}$ is obtained; and for $B\sharp$ the proportion $(\frac{9}{8})^6 = 2 \times \frac{74}{73}$.

The tone $B\sharp$ is therefore higher than the octave of C by the interval $\frac{74}{73}$ according to the calculation by perfect fifths; and the tone $D\flat\flat$ is lower by the same interval than the lower octave of C. The same proportion holds good in all the so-called enharmonic differences of the Pythagorean system, the pitch of the tones obtained from the ascending succession of fifths C-G-D etc. being by $\frac{74}{73}$ higher than that of those in the descending succession of fifths C-F-$B\flat$ etc.

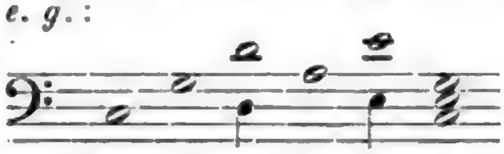
If we set $C = B\sharp = D\flat\flat$, and apportion the slight deviation of $\frac{74}{73}$ equally among the 12 fifths in either succession, the deviation for each fifth becomes so small as not to be noticeable in our keyed instruments tuned according to the tempered system. "The fifth in the system of equal temperament may be expressed very nearly by the figures $\frac{3}{2} \times \frac{885}{886}$. The employment of this fifth instead of the perfect fifth can in very few cases be regarded as objectionable," writes HELMHOLTZ. The fourths in the system of equal temperament are $\frac{4}{3} \times \frac{886}{885}$. Acoustic experiments have proved, that "the perfect consonances, the proportions of which are alike in the Pythagorean and modern systems of pure harmony, are not impaired to a serious extent by the equal temperament." The interval $\frac{885}{886}$ is very near the limit of perceptible differences in pitch, in the melodic progression of tones. According to WEBER's experiments the extreme limit is, with practiced violinists, $\frac{1000}{1001}$. HELMHOLTZ' experiments show that still smaller differences are made perceptible through the pulsations caused by the sounding together of two tones of unequal pitch.

The thirds and sixths of the system of equal temperament are nearer the true pitch than those of the Pythagorean system:

		Tempered.	Pythagorean.
Major Third	$\frac{5}{4}$	$\frac{5}{4} \times \frac{127}{126}$	$\frac{5}{4} \times \frac{81}{80}$
Minor Sixth	$\frac{8}{5}$	$\frac{8}{5} \times \frac{126}{127}$	$\frac{8}{5} \times \frac{80}{81}$
Minor Third	$\frac{6}{5}$	$\frac{6}{5} \times \frac{121}{122}$	$\frac{6}{5} \times \frac{80}{81}$
Major Sixth	$\frac{5}{3}$	$\frac{5}{3} \times \frac{122}{121}$	$\frac{5}{3} \times \frac{81}{80}$
Semitone	$\frac{16}{15}$	$\frac{18}{17}$ or $\frac{16}{15} \times \frac{147}{148}$	$\frac{21}{20}$ or $\frac{16}{15} \times \frac{80}{81}$

This proportion 80 : 81 is of great moment for a correct apprehension of the system of harmony. Precisely this proportion marks the difference between the tones obtained by the series of fifths, and those obtained through the addition of major thirds [compare Note IX]. HELMHOLTZ also considers this proportion of special importance, and mentions, with reference to the same, the analyses of HAUPTMANN as authoritative. He writes: "The only correct system of harmony is that which follows HAUPTMANN's method in marking the distinction between the tones obtained by fifths and those obtained by thirds."

HAUPTMANN's theory for tuning a pianoforte according to the system of equal temperament is, that a compromise should be effected between the *third*, and the fourth tone in the series of *fifths*, between which tones the actual proportion is 80 : 81. ["The method employed to avoid getting into the higher octaves is the following: to the second *fifth* in the series the lower octave is tuned, and then, from this lower tone, the third and fourth *fifths*; the lower octave of this last *fifth* being set as *third* of the tonic (starting) note, *e. g.*:



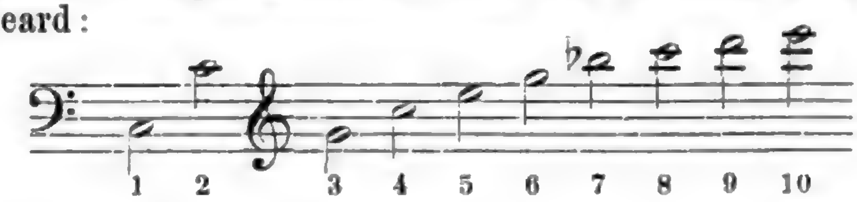
It is evident that all these *fifths* are to be held a trifle lower than the perfect *fifth*." PAUL, History of the pianoforte (Geschichte des Claviers, Leipzig, A. H. Payne, 1868, pp. 23—24.)]

The consonances in perfect harmony are ordered by HELMHOLTZ, in respect to euphony, as follows:

1. The most perfect consonances: octave, twelfth, double-octave.
2. The perfect consonances: perfect fifth and perfect fourth, the fourth being less agreeable than the fifth.
3. The mediate consonances: major sixth and major third.
4. Imperfect consonances: minor sixth and minor third.

Note V. The so-called harmonic overtones embrace, among others, those constituting the major triad. The ear perceives, by listening attentively (particularly when aided by an ear-trumpet or *resonator*) not only the lowest or primary tone, but also a number of higher tones, which are named "overtones".

Taking *c* for example as primary tone, the following tones may likewise be heard:



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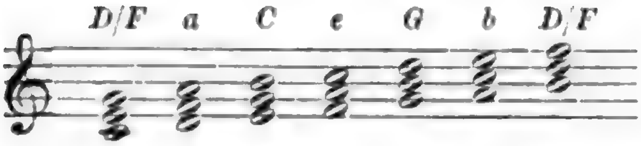
Each of these is to be regarded as a *partial tone*, as a component part of the compound tone which we call *c*.

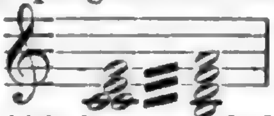
The quality of a tone is most agreeable when only such overtones as are consonant to the primary tone are distinctly audible. An uninterrupted succession of such consonant tones is found up to the sixth partial tone. The fourth, fifth and sixth partial tones form the major triad.

If a *resonator* having an individual tone, [*i. e.* vibrating when a particular tone is sounded], for instance *c''*, be employed, this tone is heard when the notes *c'*, *f*, *c*, *A*♭, *F* or *C* are sounded on a musical instrument. In each of these cases the *resonator* answers to the vibrations of one of the overtones peculiar to the tone sounded. It is therefore evident, that six consonant *undertones* are also present, and that the fourth, fifth and sixth of these collectively constitute the minor triad. RAMEAU, [b. 1683] had already taken the major and minor triads as a basis in his researches, and since that time an immense amount of preparatory experimental work had been done; to the Berlin scholar, HELMHOLTZ, the praise is due for having penetrated, sifted and arranged this mass of material, and for enriching the same by original investigations of the greatest value. His work, "Die Lehre von den Tonempfindungen", should be studied by every thoughtful musician. We should overstep the bounds of this manual were we to pursue the subject of acoustics further. Only such points as stand in close connection with harmony can be touched upon. The investigations concerning the tones of combination [Combinationstöne], the first species of which, named by HELMHOLTZ differential tones [Differenztöne], were discovered by SORGE and TARTINI in the last century, and the second species, called tones of summation, [Summationstöne], were discovered by HELMHOLTZ, are therefore to be sought for in the above-named work. It may nevertheless be noticed here, that the number of vibrations made by the differential tones in a given time is equal to the difference between the number of vibrations made by the [two given] primary tones; and that the number of vibrations made by the tones of summation is equal to the sum of the vibrations made by the primary tones.

Note VI. This idea of the opposition of the fifth to the tonic was a favorite theory of HAUPTMANN's. Perhaps the clearest explanation of the same is to be found in the first chapter of his "Science of Harmony" (Die Lehre von der Harmonik, Leipzig, Breitkopf und Härtel, 1868), a posthumous work edited by Dr., now Prof., OSCAR PAUL. For the student having a full command of the German language this work will also serve as an excellent introduction to HAUPTMANN's principal work, "Die Natur der Harmonik und der Metrik". *Translator's Note.*

Note VII. The proportion 80 : 81 exists between the fifth of the triad on the second degree in major, which I have named a diminished triad of minor character, and the perfect fifth of a major or minor triad. The key of *C*-major:



contains the triad *D/F-a* on the second degree. We obtain a perfect fifth by forming the triad *D-f♯-A*, taking *D* as basis; if the interval *D-a* be perfect fifth, it must therefore a be equal to *D-A*. In the ascending series of fifths $\overset{F\text{-}C\text{-}G\text{-}D\text{-}A}{1\;\frac{3}{2}\;\frac{9}{4}\;\frac{27}{8}\;\frac{81}{16}}$ the fourth fifth, *A*, appears in proportion to *F* as 81 : 16. The major third of *F*, or *a*, which tone is likewise the fifth of the triad *D/F-a*, stands in proportion to *F* as 5 : 4. $\frac{5}{4}$ set two octaves higher, beside *A*, gives $\frac{5}{4} \times 4 = \frac{20}{4} = \frac{80}{16}$. Consequently *a* is somewhat lower than *A*; this latter being, however, the perfect fifth of *D*, it follows that *a* is not the perfect fifth of *D*. The difference is, to be sure, a very slight one, $a : A :: 80 : 81$, but nevertheless perceptible. Because of this difference HAUPTMANN maintains that a progression like the following, with the addition of a dissonant tone:  is allowable, in spite of the parallel fifths *C-G* and *D-a*, if it be conceded as a general rule that a diminished fifth may follow a perfect one. This fifth of the triad on the second degree in major is, however, scarcely sufficiently diminished for such a progression; which the musician, at least, must regard as inharmonious, even if [theoretically] permissible. It should also be remarked, that the interval *D/F* has not the proportion 5 : 6 of a minor third; for *D*, as the second fifth above *C*, has the proportion of $\frac{9}{4}$, or, when set two octaves lower, $\frac{9}{4} \times \frac{1}{4} = \frac{9}{16}$; *F*, as the fifth below *C*, has the proportion $\frac{2}{3}$; consequently the proportion between *D* and *F* is 27 : 32. This proportion of $\frac{32}{27}$ is known as the Pythagorean minor third. It is found in ARCHYTAS' system of tetrachords in the chromatic mode, in which the proportions were: $\frac{32}{27} \times \frac{243}{224} \times \frac{28}{27} = \frac{4}{3}$.

5 : 1

Note VIII. The terms *plagal* and *authentic* were already often employed in the Middle Ages. They were also used by the Greeks in much the same sense as in the church-modes; αὐθεντικός = authentic; πλάγιος = plagal, as the opposite of authentic, derived from τὸ πλᾶγος or τὸ πλάγος, the side, an old Doric word, from which the Latin *plaga* is also derived. By these terms the manner of dividing the octave into two parts, a fifth and a fourth was designated. If the fifth lay below the fourth, this mode of division was called authentic; if the fourth lay below the fifth, the division was called plagal. *E. g.*:

$\overbrace{c\ d\ e\ f\ g}\overbrace{\ a\ b\ c'}$ with the division *c-g-c'*

was an authentic mode; and

$\overbrace{c\ d\ e\ f}\overbrace{\ g\ a\ b\ c'}$ with the division *c-f-c'*

was a plagal mode.

The so-called church-modes are explained at length in OSCAR PAUL'S "Absolute Harmony of the Greeks" [Absolute Harmonik der Griechen, Leipzig, Alfred Dörffel, 1866] according to SETHUS CALVISIUS. It will suffice here to give merely the modes themselves, with the observation, that in the Middle Ages the word *modus* [mode] was used for *key*. They are here presented according to the arrangement of the eminent theoretician **Zarlino** (16th century). Upon these modes are based the compositions of **Palestrina**, **Orlandus Lassus**, and **Giovanni Gabrieli**; the scores of these old masters are incomprehensible to one unacquainted with the church-modes.

A distinction was made between the *regular* and the *transposed* systems. The regular system had no *b♭*, being formed with the aid of the so-called *Be-quadratum*, equivalent to our *b*. On the other hand, the transposed system had not the *Be-quadratum*, but the *Be-rotundum*, equivalent to our *b♭*. These terms, "square *B*" and "round *B*", were still employed by C. Ph. E. Bach. The modes may now be exhibited as follows:

A. Regular System.

Authentic Modes.

Ionic Mode: *c d e f g a b c'*
Doric Mode: *d e f g a b c' d'*
Phrygian Mode: *e f g a b c' d' e'*
Lydian Mode: *f g a b c' d' e' f'*
Mixolydian Mode: *g a b c' d' e' f' g'*
Aeolian Mode: *a b c' d' e' f' g' a'*

Plagal Modes.

Hypoionic Mode: *G A B c d e f g*
Hypodoric Mode: *A B c d e f g a*
Hypophrygian Mode: *B c d e f g a b*
Hypolydian Mode: *c d e f g a b c'*
Hypomixolydian Mode: *d e f g a b c' d'*
Hypoaeolian Mode: *e f g a b c' d' e'*

B. Transposed System.

Authentic Modes.

Ionic Mode: *f g a b♭ c' d' e' f'*
Doric Mode: *g a b♭ c' d' e' f' g'*
Phrygian Mode: *a b♭ c' d' e' f' g' a'*
Lydian Mode: *b♭ c' d' e' f' g' a' b'♭*
Mixolydian Mode: *c' d' e' f' g' a' b'♭ c''*
Aeolian Mode: *d' e' f' g' a' b'♭ c'' d''*

Plagal Modes.

Hypoionic Mode: *c d e f g a b♭ c'*
Hypodoric Mode: *d e f g a b♭ c' d'*
Hypophrygian Mode: *e f g a b♭ c' d' e'*
Hypolydian Mode: *f g a b♭ c' d' e' f'*
Hypomixolydian Mode: *g a b♭ c' d' e' f' g'*
Hypoaeolian Mode: *a b♭ c' d' e' f' g' a'*

Breitkopf und Härtel's edition of PALESTRINA'S works offers the student, by means of practical examples, the greatest advantages for studying the manner in which these modes were employed. It should be remarked, that in the 16th century the *pitch of intonation* of the various compositions, which were written in either the regular or the transposed system, was left to the discretion of the leader of the choir. At the same time attention should be called to an extremely meritorious and valuable work: "Old German Song-book. National Songs of the Germans, both words and melodies, from the twelfth down to the seventeenth century." [Altdeutsches Liederbuch. Volkslieder der Deutschen nach Wort und Weise aus dem 12. bis zum 17. Jahrhundert. Gesammelt und erläutert von FRANZ M. BÖHME. Leipzig, Breitkopf und Härtel, 1877]. From this work also the student may learn the manner of employing the church-modes; at the same time he will recognize, that this system of music was well adapted for the expression of deep and fervent emotion.

Note IX. From previous explanatory remarks the student will have gained a correct general idea of the systems of equal temperament and of pure harmony. For more extended computations, requiring a combining of the major and minor modes, it becomes necessary to employ HELMHOLTZ'S system of alphabetical notation, and to collate A. v. OETTINGEN'S work: "The System of Harmony in its Dual Development" [Das Harmoniesystem in dualer Entwickelung. Dorpat und Leipzig, 1866]. For the practical study of harmony HAUPTMANN'S formula, which exhibits the minor mode as entirely independent of, and not as derived from, the major, is better adapted. Either method may be employed, according to the object in view, as soon as the mathematical relations of the tones are firmly fixed in the memory. In order to clearly exhibit the distinction between the major and minor modes in the practical course of harmony, I have chosen HAUPTMANN'S formula. After thoroughly mastering this system, it will be easy for the student to learn that of HELMHOLTZ for use in the mathematical analysis of problems in acoustics.

A treatise by M. W. DROBISCH, written with the penetration and scientific thoroughness peculiar to that author, was published in 1877 as an "Abdruck aus den Berichten der math.-phys. Classe der Königl. Sächs. Gesellschaft der Wissenschaften". It is entitled: "On pure harmony and equal temperament" [Ueber reine Stimmung und Temperatur der Töne]. The labors of this eminent and honored scholar have yielded, in the department of musical science no less than in others, results, of which the accuracy is unquestionable. Below is quoted as much as appears necessary for attaining the end proposed by this manual.

M. W. Drobisch denotes the perfect fifth by Q, and the major third by T. $Q = \frac{3}{2}$, and $T = \frac{5}{4}$. By employing these two different values for determining the tones contained within the compass of the octave C—c, two series of tones result, as given below.

I.	II.
$C = 1 \qquad = 1$	
$D\flat = \frac{2^3}{Q^5} = \frac{2^8}{3^5} = 1{,}05350$	$\underline{D\flat} = \frac{2^5 T}{Q^9} = \frac{2^{12}.5}{3^9} = 1{,}04049$
$C\sharp = \frac{Q^7}{2^4} = \frac{3^7}{2^{11}} = 1{,}06787$	$\underline{C\sharp} = \frac{Q^3 T}{2^2} = \frac{3^3.5}{2^7} = 1{,}05469$
$D = \frac{Q^2}{2} = \frac{3^2}{2^3} = 1{,}125$	$\underline{D} = \frac{2\,T}{Q^2} = \frac{2.5}{3^2} = 1{,}11111$
$E\flat = \frac{2^2}{Q^3} = \frac{2^5}{3^3} = 1{,}18519$	$\underline{E\flat} = \frac{2^4\,T}{Q^7} = \frac{2^9.5}{3^7} = 1{,}17055$
$D\sharp = \frac{Q^9}{2^5} = \frac{3^9}{2^{14}} = 1{,}20136$	$\underline{D\sharp} = \frac{Q^5 T}{2^3} = \frac{3^5.5}{2^{10}} = 1{,}18652$
$F\flat = \frac{2^5}{Q^8} = \frac{2^{13}}{3^8} = 1{,}24859$	$\underline{F\flat} = \frac{2^7\,T}{Q^{12}} = \frac{2^{17}.5}{3^{12}} = 1{,}23317$
$E = \frac{Q^4}{2^2} = \frac{3^4}{2^6} = 1{,}26562$	$\underline{E} = T = \frac{5}{2^2} = 1{,}25$
$F = \frac{2}{Q} = \frac{2^2}{3} = 1{,}33333$	$\underline{F} = \frac{2^3\,T}{Q^5} = \frac{2^6.5}{3^5} = 1{,}31687$
$E\sharp = \frac{Q^{11}}{2^6} = \frac{3^{11}}{2^{17}} = 1{,}35152$	$\underline{E\sharp} = \frac{Q^7\,T}{2^4} = \frac{3^7.5}{2^{13}} = 1{,}33484$
$G\flat = \frac{2^4}{Q^6} = \frac{2^{10}}{3^6} = 1{,}40466$	$\underline{G\flat} = \frac{2^6\,T}{Q^{10}} = \frac{2^{14}.5}{3^{10}} = 1{,}38732$
$F\sharp = \frac{Q^6}{2^3} = \frac{3^6}{2^9} = 1{,}42383$	$\underline{F\sharp} = \frac{Q^2\,T}{2} = \frac{3^2.5}{2^5} = 1{,}40625$
$G = Q = \frac{3}{2} = 1{,}5$	$\underline{G} = \frac{2^2\,T}{Q^3} = \frac{2^3.5}{3^3} = 1{,}48148$
$A\flat = \frac{2^3}{Q^4} = \frac{2^7}{3^4} = 1{,}58025$	$\underline{A\flat} = \frac{2^5\,T}{Q^8} = \frac{2^{11}.5}{3^8} = 1{,}56074$
$G\sharp = \frac{Q^8}{2^4} = \frac{3^8}{2^{12}} = 1{,}60181$	$\underline{G\sharp} = \frac{Q^4\,T}{2^2} = \frac{3^4.5}{2^8} = 1{,}58203$
$A = \frac{Q^3}{2} = \frac{3^3}{2^4} = 1{,}6875$	$\underline{A} = \frac{2\,T}{Q} = \frac{5}{3} = 1{,}66667$
$B\flat = \frac{2^2}{Q^2} = \frac{2^4}{3^2} = 1{,}77778$	$\underline{B\flat} = \frac{2^4\,T}{Q^6} = \frac{2^8.5}{3^6} = 1{,}75583$
$A\sharp = \frac{Q^{10}}{2^5} = \frac{3^{10}}{2^{15}} = 1{,}80203$	$\underline{A\sharp} = \frac{Q^6\,T}{2^3} = \frac{3^6.5}{2^{11}} = 1{,}77979$
$c\flat = \frac{2^5}{Q^7} = \frac{2^{12}}{3^7} = 1{,}87288$	$\underline{c\flat} = \frac{2^7\,T}{Q^{11}} = \frac{2^{16}.5}{3^{11}} = 1{,}84976$
$B = \frac{Q^5}{2^2} = \frac{3^5}{2^7} = 1{,}89844$	$\underline{B} = QT = \frac{3.5}{2^3} = 1{,}875$
$c = 2 = \qquad = 2$	$\underline{c} = \frac{2^3\,T}{Q^{14}} = \frac{2^5.5}{3^4} = 1{,}97531$
$B\sharp = \frac{Q^{12}}{2^6} = \frac{3^{12}}{2^{18}} = 2{,}02729$	$\underline{B\sharp} = \frac{Q^8\,T}{2^4} = \frac{3^8.5}{2^{14}} = 2{,}00226$

Taking both these series into consideration M. W. Drobisch gives the exact formula $C, D, \underline{E}, F, G, \underline{A}, \underline{B}, c$ for the scale of C-major, in which $\underline{E} = T$, $\underline{A} = \frac{2T}{Q}$, and $\underline{B} = Q\ T$.

"As, in these two series, the tones having like names differ from each other by a syntonic comma $\left(\frac{81}{80} = \frac{Q^4}{2^2 T}\right)$, it is impossible, so long as Q represents the perfect fifth $\frac{3}{2}$, and T the major third $\frac{5}{4}$, ... to attribute to these like-named tones common periods of vibration, and thus to reduce the two series to one. It might, however, be possible to find values for Q and T which would differ imperceptibly from the true values, and through which the equation $\frac{Q^4}{2^2 T} = 1$ could be obtained, by which means the difference between the tones having like names would dissappear, the values of the tones in the two series becoming equal. If we call these nearly perfect fifths and thirds *tempered*, there are three several ways in which the equalization of the two series may be brought about. The fifth may remain perfect, and only the third be tempered; or the third may remain perfect, and the fifth be tempered; or both fifth and third may be tempered".

These various cases were carefully investigated by M. W. Drobisch, and the results of the different methods of temperament arranged in convenient tables. This highly interesting exposition is of no mean value to the musician, as it leads him to recognize what degree of delicacy human perception can reach; for although these subtile distinctions could not, in earlier times, be expressed with such mathematical precision, correct perception led to the practical result of constructing keyed instruments, whose mechanism permitted the employment of a more or less complete system of temperament.

The system of equal temperament is undoubtedly indispensable in modern music; Hauptmann calls it a necessary evil, but still an evil which, in consideration of the comparatively slight deviations from pure harmony, is quite bearable. We can *think* only in accordance with the system of pure harmony, at the foundation of which are our modes. Only when based on these latter can the values of the consonances and dissonances be established, which Helmholtz has most ingeniously accomplished through his observations of the pulsations caused by two tones of unequal pitch vibrating simultaneously.

In the model-keys of C-major and C-minor we find the following series of proportions:

$$b - D \;/\; F - a - C - e - G - b - D \;/\; F - a$$

$$\tfrac{6}{5} \quad \tfrac{32}{27} \quad \tfrac{5}{4} \quad \tfrac{6}{5} \quad \tfrac{5}{4} \quad \tfrac{6}{5} \quad \tfrac{5}{4} \quad \tfrac{6}{5} \quad \tfrac{32}{27} \quad \tfrac{5}{4}$$

$$b - D \;/\; F - a\flat - C - e\flat - G - b - D \;/\; F - a\flat$$

$$\tfrac{6}{5} \quad \tfrac{32}{27} \quad \tfrac{6}{5} \quad \tfrac{5}{4} \quad \tfrac{6}{5} \quad \tfrac{5}{4} \quad \tfrac{5}{4} \quad \tfrac{6}{5} \quad \tfrac{32}{27} \quad \tfrac{6}{5}$$

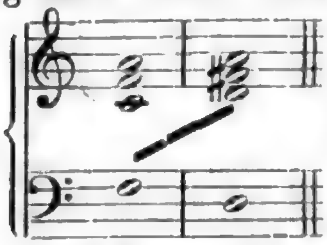
In these two modes three varieties of thirds, and as many varieties of sixths, may be found.

1. The natural major third ($\frac{5}{4}$) and its inversion, the minor sixth ($\frac{8}{5}$), both consonant intervals.

2. The natural minor third ($\frac{6}{5}$) and its inversion, the major sixth ($\frac{5}{3}$), both consonant.

3. The Pythagorean minor third ($\frac{32}{27}$) between *D* and *F*, *i. e.* between the dominant fifth and the sub-dominant fundamental. This third is dissonant, although differing from the minor third only by the syntonic comma $\frac{81}{80}$. In practical harmony it is treated as a consonance. For determining the degree of euphony [of any interval] the tones of combination are best employed.

The Pythagorean major third would be formed by setting the fourth *fifth*, reckoning from the fundamental of the sub-dominant, beside this latter. Taking, for instance, the series of fifths *F-a-C-e-G-b-D-*$f\sharp$*-A*, *F-A* would be the Pythagorean major third $\frac{81}{64}$, consisting of two greater whole tones ($= \frac{9}{8} \times \frac{9}{8}$). In a progression with cross-relation:



the $a = \frac{5}{4}$ of *F* passes into the $A = \frac{3}{2}$ of *D* or $= \frac{81}{64}$ of *F*.

The inversion of the Pythagorean minor third D'/F' ($\frac{32}{27}$) is the Pythagorean major sixth (F'-D'' ($\frac{27}{16}$), which interval is greater by a syntonic comma than the natural major sixth ($\frac{5}{3}$).

Further, four varieties of fifths are to be noticed.

1. The perfect fifth C''-G'' ($\frac{3}{2}$), consisting of a natural major third ($\frac{5}{4}$) and a natural minor third ($\frac{6}{5}$). Its inversion is the perfect fourth G'-C'' ($\frac{4}{3}$). Both intervals are consonant.

2. The diminished fifth in the diminished triad of minor character, *e. g.* D''-a'', named by HELMHOLTZ the imperfect fifth ($\frac{40}{27}$), is narrower by the syntonic comma $\frac{81}{80}$ than the perfect fifth; its inversion, which HELMHOLTZ calls the imperfect fourth, is a'-$D'' = \frac{27}{20}$.

3. The diminished fifth, named by the old theoreticians the "false" fifth: b'-$F'' = \frac{64}{45}$, is composed of a natural minor third ($\frac{6}{5}$) and a Pythagorean minor third ($\frac{32}{27}$). It is dissonant, as is likewise its inversion, the tritone F'-$b' = \frac{45}{32}$ (also called "augmented" and "false" fourth), an interval which embraces three whole tones, $\frac{9}{8} \times \frac{10}{9} \times \frac{9}{8} = \frac{810}{576} = \frac{45}{32}$.

4. The augmented fifth, as it occurs in the harmonic minor mode between the third and seventh degrees, embraces two major thirds. In *C*-minor $e\flat''$-$b'' = \frac{25}{16}$ is the augmented fifth, whose inversion is the diminished fourth b'-$e\flat'' = \frac{32}{25}$; both intervals are dissonant.

The sevenths and seconds are as follows:

1. The diminished seventh in minor b'-$a\flat''$, formed by combining b'-D'', D''/F'', and F''-$a\flat''$, has the proportion $\frac{128}{75}$, and is a harsher dissonance than its inversion, the augmented second $a\flat$-$b = \frac{75}{64}$.

2. The lesser minor seventh G'-F'', or b'-a'', or D''-$C''' = \frac{16}{9}$. According to HELMHOLTZ it is not so harsh a dissonance as the diminished

seventh, an opinion at variance with the established practice. It is the inversion of the greater whole tone C''-$D'' = \frac{9}{8}$, which the above-named scientist regards as the harsher dissonance.

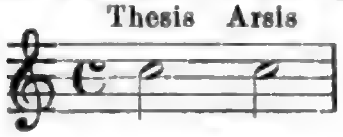
3. The greater minor seventh, for example: e'-D'', or a'-$G'' = \frac{9}{5}$, is greater by a syntonic comma than the lesser minor seventh, and is sharply dissonant, like its inversion, the lesser whole tone D'-$e' = \frac{10}{9}$; this latter being a sharper dissonance than the greater whole tone.

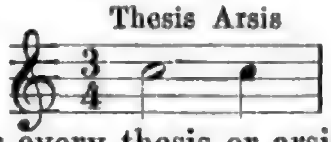
4. The major seventh, for example: F'-e'', or C''-$b'' = \frac{15}{8}$, is, according to **Helmholtz**, dissonant to about the same degree as the lesser whole tone. Its inversion, the diatonic second b'-$C' = \frac{16}{15}$, is the sharpest dissonance found in the scale.

Finally, the augmented sixth of the "extended" mode: $f\sharp'$-$a\flat'$-C''-$e\flat''$-G''-b''-D'''-$f\sharp'''$, or $a\flat'$-$f\sharp'' = \frac{225}{128}$, must be mentioned as a dissonant interval. Its inversion is the diminished third $f'\sharp$-$a\flat' = \frac{256}{225}$, likewise dissonant. "Altered" intervals or chords are therefore non-existent. It should also be observed, that every chord containing a dissonant interval is dissonant.

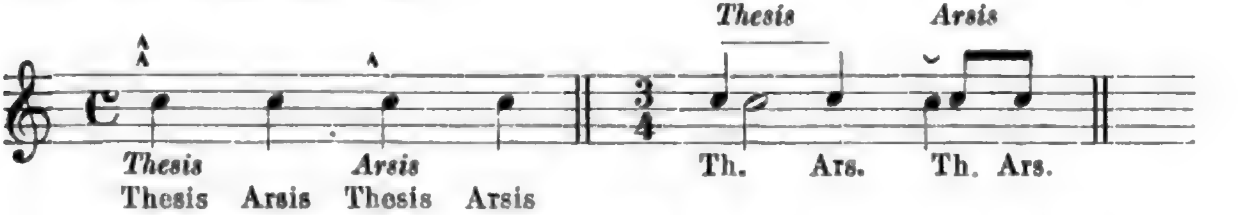
Note X. On the chord of the fourth and sixth and its treatment a thorough and instructive treatise has been written by the deserving theoretician, **Wilhelm Rischbieter**. It is entitled: "Three theoretical dissertations on Modulation, the Chord of the fourth and sixth, and the Organ-point" [Drei theoretische Abhandlungen über Modulation, Quartsextaccord und Orgelpunkt, Dresden, F. Ries, 1879].

Note XI. For the better comprehension of the terms *thesis* and *arsis*, the downward beat and the upward beat, which are derived from the Greek, and originally signified the treading with and lifting of the foot in the processional songs, we should remark, that every measure is divisible into thesis and arsis. The Grecian method of division for the measures of two or three equal parts is the best.

In the measure of two parts the thesis is of like duration with the arsis: 

. In the measure of three parts the thesis has twice the length of the arsis: 

. Within the measure subdivisions can be made; for every thesis or arsis may be, in turn, divided into thesis and arsis; *e. g.*



At the foundation of this measurement lie two species of metrical feet, the spondee (‒ ‒) and the trochee (‒ ⏑); through their aid an easy explanation may be found for any form of the two-part or three-part measure. The best rhythmical and metrical analyses are those of **Rudolph**

Westphal, in his work: "The Elements of Musical Rhythm" [Elemente des musikalischen Rhythmus, Jena, Costenoble, 1872]. The examples taken from the works of **Gluck**, **Mozart**, and other masters, confirm the opinion, that the Grecian mode of rhythmical analysis is also the simplest and clearest for modern music.

Note XII. The opinions of C. **Ph. E. Bach** find corroboration in **Johann David Heinchen's** "Manual of Thorough-bass" [Lehre vom General-Bass]. Both editions, of 1711 and 1728, agree, in the views expressed on the treatment of Recitatives, with the exposition of C. **Ph. E. Bach**. In the earlier edition the subject is handled more briefly, but none the less forcibly, than in the later.

"The recitative-style", he says, "holds the same place in comparison with *lawful* music, as free prose compared with versified and metrical poetry".

With reference to the employment of chords, where no figures are given over the bass, § 4 runs thus:

"Two things are to be taken into consideration:

1. We must know what style of figuration and what chords are usually employed in recitatives.
2. We must be able to judge, from the vocal part, what manner of accompaniment is suitable in a given case".

(§ 5) "Respecting the first we find, by employing the *artem combinatoriam*, which is so helpful in all sciences, that besides the triad no more than eight varieties of chords may be used by a composer in the recitative-style; these being:

$$\begin{array}{c|c|c|c|c|c|c|c|} \cancel{7} & \flat 7 & 7\cdot & 6 & 6 & 6\cdot & 5\flat & 4+ \\ 4 & 5\flat & & 4 & 4 & & & \\ 2 & & & 2 & & & & \end{array}$$

It follows, that, to any recitative whatsoever, the accompanist must employ either the triad, or one of the chords just mentioned; excepting that at times such other resolutions as 4 3, 9 8 etc. may properly be used, which however seldom happens; these latter resolutions would in any case have been learned from the foregoing".

Here follows an exposition of the recitative-style by means of practical examples. He then writes, in § 27: "The manner of handling recitatives varies greatly according to the instrument employed in accompanying. In sacred ['church'] recitatives, where we have to do with the prolonged and droning tone of the organ-pipes, the matter is very simple; the notes being generally struck and held down without further ceremony till another chord follows, which is treated in like manner. (§ 28) Should, however, the hands be lifted immediately after striking a fresh chord, thus producing a pause in the accompaniment, this occurs, according to circumstances, for the better hearing of the singer or of the other accompanying instruments. Further, the hands may be lifted when the bass has the same tone and chord during three or four or more measures, and the ear might become wearied through the continued monotonous humming of the organ-pipes. All of which is left to the judgment and pleasure of the accompanist. (§ 29) For playing on the *clavicembalo*, and accompanying theatre-recitatives, more skill is required. One must learn above all things to play a good, but short, arpeggio, which is the foundation of a good theatre-accompaniment, and can be learned far better by practice than by theorizing. (§ 30) Therefore one must learn to play it with the left hand, or the right, or both together, correctly and at the proper time. For, when the singer is, so to speak, in the midst of his recital, it has a very ill effect if the accompanist amuse himself by continually playing arpeggios, as if he desired to outdo the singer. On the other hand it is not well for the accompanist to stop short at every trifling pause, or when the singer takes breath, as if it had been mutually agreed upon beforehand." § 31 and 32 explain, that the most must be learned through practice. From p. 228 to p. 260 the subject is practically exemplified by the aid of two cantatas.

Note XIII. The views of C. Ph. E. Bach respecting the accompaniment are also confirmed by later authors. Johann Samuel Petri touches on this subject in his "Introduction to Practical Music" [Anleitung zur praktischen Musik, Leipzig, Breitkopf, 1782]. He recommends however that the pedal-bass be played *staccato*, so that the singer may not be overpowered; observing, on p. 311: "I have still to remark, that when an organ contains a very soft stopped flute-register, the chords, in recitatives without other accompanying instruments, may be quietly held in the left hand, and the fresh bass-notes executed *staccato* in the pedal. The singer is thus well supported, and not overpowered; the tenor chords are likewise more suitable for accompanying tenor or bass recitatives than the higher, shriller chords. It is a different matter in the case of the aria, where the octaves cover and reinforce each other *all' ottava* when the first violin and flute play the melody together with the tenor; for in the recitative the organist strikes simple chords without melody, and it would be absurd to play these high".

In another passage, speaking of supporting the singer by means of the accompaniment, he regards it as desirable that the accompanist should play the separate notes of the vocal part, which in recitatives were generally chord-notes, on the organ; *i. e.* he should play broken chords. This was, of course, no *arpeggio* in the present acceptation of the term, in which sense it was also employed by C. Ph. E. Bach for the pianoforte-accompaniment, but a [rhythmically measured] playing of the separate notes of the chord, probably in recitatives which were to be sung slowly. This manner of playing in broken chords was at that time designated as an "*arpeggio*", as also confirmed by J. D. Heinchen (or Heinichen).

After this explanation it will be easy to understand the remarks of the authors of the 18th century on the *arpeggio*, and to bring the same into agreement with the observation of C. Ph. E. Bach, that "on the organ the *arpeggio* (*i. e.* the pianoforte-*arpeggio*, in which the chord as a whole retains its position in the measure) is not to be employed". What Petri says, on p. 171, has reference to this playing of the notes of a chord separately. "The organist should never play the melody together with the singer, but must keep to his chords; *except in recitatives*, where he may, if the singer be unpracticed, and hesitates, or sings false, occasionally help him back by playing broken chords, or if necessary even a few tones of his melody". In connection with the above remark stand the rules which he gives just before it: "in recitatives he should not simply hold the chords down, but should play broken chords, or the *arpeggio*, and avoid all trills, which ought to be left to the singers and to the other instruments. In recitatives he should, after striking the chord, hold the bass alone, at times in the pedal, at others in the manual, according to the strength of the singer. At the same time he cannot regulate the accompaniment merely in accordance with the measure, but rather with the singer, who cannot possibly bind himself to the exact time if he is to give the proper form to his delivery".

Note XIV. In using the edition of the Bachgesellschaft the excellent prefaces, by that eminent student of BACH'S compositions, Dr. W. RUST, should not be overlooked.

CHRYSANDER'S instructive biography of HÄNDEL is also well worthy of notice in connection with the study of HÄNDEL'S works.

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INDEX.

A,, A, A a a' a'' a''' a'''' 153—154, absolute pitch acc. to number of vibrations 155.

Absolute pitch, *v.* Pitch.

A cappella style (= pure vocal composition without instr. accom.) 141.

Accidental interval 127.

Acoustics, science of, as applied to musical tones 152 *et seq.*

After-striking 146.

Alphabetical notation, *v.* Notation.

Altered chords 103, 107, 165.

Alto-clef 111.

Amplitude of vibration 153.

Anticipation 145.

Arabic numerals, used to indicate degrees of scale 1—3.

ARCHYTAS 159.

Arpeggio in accomp. recitatives 167—168.

Arsis 83, 165.

Augmented chords 100, acc. to HAUPTMANN 103; *second* unmelodic 3, treatment of in minor 32—34, 57—58, 68, 101; *intervals* 6—9, non-melodic 24, 31; *triad* 30, progression 67, free entrance not allowable 67, preparation 67.

Authentic close in major 23, in minor 37; *modes* 160.

B,, B, B b b' b'' b''' b'''' 153—154, absolute pitch acc. to number of vibrations 155.

BACH, CARL PH. E., "Essay on the true method of playing the piano" 119, recommends contrary motion in playing from figured bass 119, on covered octaves 120, on dimin. fifth following perfect, and *vice versa* 120, rules foreshadowing those for "interconnection of the chords" 120, on peculiarities in writing signatures at and before his time 123, on use of simple figures 125, ♯ beside figure or line through the same 126, on the flat (♭) 127, natural (♮) 127, 128, double-sharp (×) and its equivalents 127, 128, double-flat (♭♭) or great flat (♭) 127, 128, double chromatic signs (♮♯ and ♮♭) 128, on use of flats or lines through figure instead of natural 128, best manner of writing chromatic signs in thorough-bass notation 129, figuring when a chord is to be repeated 130—131, with dotted notes 131, with rests 132, rhythmical value of figures 132—134, remarks on TELEMANN'S curve 134, examples of organ-points 137—138, the *arpeggio* in accomp. recitatives 167—168.

BACH, JOH. SEB., 371 four-part choral-songs 151.

BEETHOVEN, passage from Leonore Overture No. 3, 146.

Be-quadratum 128—129, 160.

Be-rotundum 129, 160.

BOEHME, FRANZ M., "Old German Song-book" 161.

BOETIUS and the Harmony of the Greeks 154.

C,, C, C c c' c'' c''' c'''' 153—154, absolute pitch acc. to number of vibrations 155.

C-clef 111.

Cadence, *v.* Close.

CALVISIUS, SETHUS 159.

Cantus firmus 44, directions for harmonizing 147.

Changing notes 140—141, false 141.

Choral composition 147.

Chorals, books of, by SÄMANN and DÖRFFEL 151.

Chords, *interconection of* 12, applied to progressions between triads 15.
Chromatic, definition of term 154, progressions, 82—83, signs (♯, 𝄪, ×, ♭, 𝄫, ♭, ♮,) 127—128; also compare Thorough-bass notation.
CHRYSANDER, biography of HÄNDEL 168.
Church-modes 159—161.
Clefs, view of 111.
Close (or *cadence*) authentic 23, plagal 23, perfect 23, 52, in minor 37—38, deceptive 38, 56, 78, most complete *c.* with triads 41, *c.* with dominant chord of the seventh 52, 54.
Close position 39.
Combination, tones of 158.
Combined mode 62, 70—71, in modulation 81—82.
Comma, syntonic 163.
Common chord, *v.* Triad.
Consonances 6—7, perfect 6, 9, 147, nature of 48, 53, in perfect harmony as ordered by HELMHOLTZ 157, 163—165.
Contra-octave 153.
Contrary motion 18, 46, between indirectly connected chords 17—19, 54, between chords connected through the third or fifth 22—24, for avoiding parallel fifths 119.
Covered fifths 33, 54, 119; *octaves* 44, 46, 54.
Cross-relation 10, 27.

D,, D, D d d' d'' d''' d'''' 153—154, absolute pitch acc. to number of vibrations 155.
Dash or *stroke* through a figure 48, 126—130, 134 (4+) after a figure 130—134.
Deceptive close 38, 56, 78.
Degrees of scale, indicated by Arabic numerals 1—2, by Roman numerals 45.
Diatonic, defined 154, composition 32.
Differential tones 158.
Diminished chords of the *seventh* 60—61, need no preparation 62; *intervals* 8—9, indicated by the nought (°) 105; *triads* in major 16—17, in minor 31, on 2nd degree in major non-interchangeable with that on 4th degree of relative minor 79, mathematical proof 159.
Dissonances, diatonic 10, chromatic 10, character of as apposed to consonances 48, 53, various degrees of sharpness in, acc. to HELMHOLTZ 163—165.
Dissonant triads, on 2nd and 3rd degrees in minor 30—31, on 7th degree in major 17, 48, in minor 48.
DÖRFFEL, "Book of Chorals" 151.
Dominant chord of the 7th 51, preparation 52—53, resolution 53—59, in closes 52, 54, its inversions 74 and their progressions 74—75; *triad*, in major 12, in minor 28.
Dotted notes, rhythmical value of figures over 131.
Double-contra-octave 153.
Double-flat (𝄫) 128, or great flat (♭) 128, replaced by a ♭ 128.
Double-sharp (×) 36, 127, through or beside figure 128, its equivalents 128, invariable signification in thorough-bass notation 130, replaced by line through figure 129.
DROBISCH, M. W., "On pure harmony and equal temperament" 161—163.

E,, E, E e e' e'' e''' e'''' 153—154, absolute pitch acc. to number of vibrations 155.
Elevenths = fourths 124.
Enharmonic step 8, 9, *differences* of Pythagorean system 156, *changes* 135, *progressions*, examples of 138—139.
Equal temperament, *v.* Temperament.
Exceptions in progression based upon rules of strict composition 63.
Extended modes 103, 105—7.

F,, F, F f f' f'' f''' f'''' 153—154, absolute pitch acc. to number of vibrations 155.
F-clef (= bass-clef) 111.
False fifth (= dim. fifth) 120, indicated by a ♭ 124, mathematical value of 164; *fourth* (= augm. fourth) 164.
Fifth, mathemat. value of 154, 164, connection through the 13, covered fifths 33, 54, parallel 25, 101, chase after 122, dimin. following perfect, and *vice versa* 120, 159; natural progression of dimin. fifth, 121, dimin. fifth indicated by a ♭ 124, succession of *f's* on thesis or arsis not necessarily incorrect 122.
Fifth and sixth, (= third, fifth and sixth) chord of 73, indicated by 5♭ 134, augm. chord of 100—101, 103—104.
Fifth octave 154.
Figures, *Figuring*, *v.* Thorough-bass notation.
First or *great octave* 153.

Flat (♭) defined 2, 127, indicates dimin. fifth 124, through or beside figure 127, replaces ♮ 128, formerly called "*Be-rotundum*" (= round B) 129, 160, double-flat (♭♭) 127—128, great flat (♭) 128.

Four-part score, as now and formerly written 147.

Fourth, mathemat. value of 154, 164, parallel *f's* accomp. by the third 43, 54—55, 69, 76.

Fourth and sixth, chords of 40—45, progressions in connection with 43—44, successions of 55, modulatory power of 45, 55, Rischbieter's treatise on 165.

Fourth octave 154.

Fundamental harmony 38—39, *position* of triad 38, *triads* in major 13, in minor 29—30.

G,, G, G g g' g'' g''' g'''' 153—154, absolute pitch acc. to number of vibrations 155.

G-clef (= violin-clef) 111.

Gabrieli, Giovanni 160.

Gallant style 124.

Great flat (♭) 128; *octave* (= first octave) 153.

Händel, Chrysander's biogr. of 168.

Harmonic minor scale 3, superseded by melodic 31.

Harmonic overtones 158.

Hauptmann, Moritz, system of alphabetical notation in major 12—20, in minor 28—36, on indirect connection of triads 17, on derivation of melodic preparation of chord of the fourth and sixth 42, on successions of those chords 44, definition of chord of the seventh 66, of chord of the ninth 88, on augmented chords 103, on "Klang" and "Temperatur" 155, collection of short articles entitled "Opuscula" 155, on equal temperature of pianoforte 157, "Science of Harmony" 158, principal work, "The Nature of Harmonic and Metrical Science" 158 opinion on equal temperament 163.

Helmholtz, H., "Science of the Perception of Tones" 152, 158, on the fifth of the tempered system 156, arrangement of consonances in pure harmony 157, mathemat. values of consonances and dissonances 163—165, system of alphabetical notation 29, 161.

Heinchen, Joh. David, "Manual of Thorough-bass" 166.

Imperfect consonances 157, *fifth* 164.

Indirect connection 18.

Inharmonic relation, *v.* Cross-relation.

Interconnection of the *chords* 12.

Interpolated notes 18, 19, 24, 44, 46, 54 etc.

Intervals, defined 6, consonant 6, perfect 6, inversion of 7—9, limits in formation of 7, augmented 7—8, diminished 9, mathematical relations of 154—159, 161—165.

Inversion of intervals 7—9, of triads 38, of chords of the seventh 73.

Key-note, *v.* Tonic.

Keys, *major* and *minor*, *v.* Major and Minor.

Lassus, Orlandus 160.

Leading-note 48.

Letters, as symbols of tones, *v.* Notation, alphabetical.

Ligature, correct use of 56, signifies preparation 62.

Line or *Dash* drawn through a figure 48, 125—130, replaces × 129.

Major Keys, view of 19—20; *thirds*, successions of 64; *triad* 12, character of contrasted with minor 28—30.

Mediant (= Third) 12.

Melodic minor scale 3, 86.

Middle-C 154.

Minor Keys, view of 34—36; *character* as contrasted with major 28—30; *triad* 29.

Mode, defined 12; major 12, minor 28, difference in character of major and minor 28—30; combined (major and minor) *m.* 62, 70—71; plagal and authentic 159—160; church-modes 159—160, regular and transposed systems of 160; extended modes 103—107.

Modulation 108, with chord of the fourth and sixth 45, 55, with chords of the seventh, diatonic 78—81, chromatic 81—82, in pure and tempered harmony 79, augm. triad in 81, combined keys in 80—81, Rischbieter's treatise on 165.

Monochord 154.

Natural (♮) defined 2, 127, through or beside figure 128, replaced by flat 128, by line through figure 128, formerly called "*Be-quadratum*" (square B) 128—129, 160.

Ninth, chord of the 86—93, defined by HAUPTMANN 88.
Noises distinguished from musical tones 152.
Notation, *alphabetical*, HAUPTMANN'S system 12, 28; HELMHOLTZ'S system 29, 161.
Nought (°) in Thorough-bass notation 105.
Numerals, Roman and Arabic, *v.* Roman and Arabic.

Octaves, covered 44, 46, 54, parallel 25, 119, chase after parallel fifths and *o's* 122, successions of *o's* on either thesis or arsis not necessarily incorrect 122; arranged according to absolute pitch 153—154, named with reference to length of organpipes 153—154.
OETTINGEN, A. v., "System of Harmony in its Dual Development" 161.
Open position 39.
Organ, as employed in recitatives 166—168, octaves in named acc. to length of pipes 153—154.
Organ-point 88—89, 142—144, harmony taking the place of 141, RISCHBIETER'S treatise on 165.
Overlapping or *Overreaching system*, *v.* Extended modes.
Overtones 157.

PALESTRINA 160.
Parallel fifths and *octaves* 25, 101, 109, chase after 122; *fourths* accomp. by the third 43—44; 54—55, 69, 76; *motion* 18.
Partial tones 158.
Parts, how numbered 14.
Passing chords 141—142; *notes* 140, chromatic 103; *sevenths* 103, free entrance of 83.
PAUL, OSCAR, "Absolute Harmony of the Greeks" 159, "BOETIUS and the Harmony of the Greeks" 154, "History of the Pianoforte" 157, editor of HAUPTMANN'S "Science of Harmony" 158.
Perfect close in major 23, 52, in minor 38; *consonances* 6—7, former classification 12, alike in Pythagorean and modern systems 156.
Period of vibration 152, used to establish compar. values of intervals 154—5.
Periodic movement 152.
PETRI, JOH. SAMUEL "Introduction to Practical Music" 167—168.
Pitch, natural 127; absolute 153, dependent upon period of vibration 153; limit of perceptible differences in 156.
Plagal, defined 159; *close* in major 23, in minor 37; *modes* 160—161.
Position, *close* 39; *fundamental* 38; *open* 39.
Preparation, of chord of the fourth and sixth, harmonic 40, melodic 42; of dominant chord of the seventh 52; of subordinate chords of the seventh 60, 62.
Prime, augmented on impossibility 7, regarded as the same as the octave 124.
Principal chord of the seventh 52.
PTOLEMAEUS, CLAUDIUS 154.
Pythagorean system 156, minor third 159, 164, major third 164.

RAMEAU 158.
Range of musical tones 152.
Recitatives, accompaniment of 166—168.
Regular system of modes 160.
Relative major Key 68, modulation to 72, 77; *minor Key* 60.
Resolution, defined 53; progression to a dissonance no *r.* 63, 65, 69.
Resonator 158.
Rests, signification of figures over 132.
Retardation 144—145.
RICHTER, E. FR., Manual of Harmony, v, 100, 126.
RISCHBIETER, W., "On Modulation, the Chord of the Fourth and Sixth, and the Organ-point" 165.
Roman numerals, used to indicate degrees of scale 45, 51, with the nought (°) 51, in modulation 108, 110.
Round B 129, 160.
RUST, DR. W., Prefaces to edition of Bachgesellschaft 168.

SÄMANN, "Book of Chorals" 151.
Scales, major, minor and chromatic 1—6.
Score, *four-part*, as now and formerly written 147—150.
Second, general definition 1, progressions 81—82, mathemat. value of 155, 164; *augmented* unmelodic 3, treatment in minor 32—34, 58, 68, 101.
Second (= second, fourth and sixth) chord of 73, various modes of figuring 134; *augmented chord* of the 102.
Second octave 153.
Semitone, diatonic and chromatic 1—2, mathemat. values of 155, 165.

Sequences 25, with chords of the seventh 71—72.
Seventh, resolution 53, 81—82, mathemat. value 155, 164, earlier employment 61, preparation 62, free entrance 62, 68, upward resolution 63, 70, relation to fundamental 81, passing *s's* 83, 103.
Seventh, chords of the 51, sequences of 71—72, in modulation 78—83, passing 83; *dominant chord* of the 51, its preparation 52—53, its resolution 53—59, in closes 52, 54, *subordinate chords* of the in major 60—65, their preparation 59—62, in minor 66, 73, their progressions 67—73. *Inversions* 73.
Sharp (♯) defined 1, 127, beside a figure 36, 48, 125—126, above over bass-note 36, 129, replaced by stroke or line through figure 48, 128—129.
Sirene 154.
Sixth, mathemat. value of 155, 164, chord of 39, augm. chord of 100, 101.
Sixth and fourth, *v.* fourth and sixth.
Sixth octave 154.
Skip, defined 1, not allowed in chromatic progressions in strict harmony 82.
Soprano-clef 111.
SORGE 158.
Spondee 165.
Square B 128—129, 160.
Step, defined 1.
Sub-dominant, triad of the, in major 13, in minor 29.
Subordinate chords of the *seventh* 60—65. preparation 60, 62; *triads*, in major 15, in minor 29—31.
Summation, tones of 158.
Suspensions 93—98, false 98.
Syncope 144.
Syntonic comma 163.

TARTINI 158.
TELEMANN, his use of curve $\left(\overset{\frown}{8}\atop 6\right)$ over figure 134.
Temperament, equal 11, 16—17, enharmonic changes effected through the medium of 135, compared with system of pure harmony 155—157, as presented by DROBISCH 161—163, indispensable in modern music 163, HAUPTMANN's practical system of 157.
Tempered system, *v.* Temperament.
Tenor-clef 111; *part*, in violin-clef 147.
Tenths, = thirds 124.
Tetrad 51.
Thesis 83, 165.
Third, indicated by a 3 pp. 14—16, 124, by chromatic sign alone 129, connection through the 15, doubled 34, mathemat. value of 155, 164, succession of major thirds 64.
Third and fourth, (= third, fourth and sixth) chord of 74, augm. chord of 100, 102.
Third and sixth (= sixth) chord of 39.
Third, fifth and sixth (= fifth and sixth) chord of 74.
Third octave 154.
Thorough-bass notation, defined 47—48, fundamental principle of 40, 74, in scores of BACH period 123—139. *Chromatic signs* in: (1) the ♯ beside figure 36, 48, 125, alone over bass-note 36, 129, replaced by line through figure 48, 128—130; (2) the × 36, 127—128, its equivalents 128, invariable signification of 129, replaced by line through figure 130; (3) the ✕ instead of ♯ in old scores 127; (4) the ♭ 2, 127, through or beside figure 128, indicates dimin. fifth 124, takes the place of 𝄫 129, of ♮ 128, formerly called "*Be-rotundum*" (=round B) 129, 160; (5) 𝄫 127 or great flat (♭) 128, replaced by ♭ 129; (6) the (♭) (great flat) 128; (7) the ♮ 2, 127, through or beside figure 128, replaced by ♭ 128, formerly called "*Be quadratum*" (= square B) 128—129, 160; (8) double chrom. signs (♮♯) (♮♭) 128; (9) *line* or *stroke* through figure 48, 126, 128, 129. The *nought* (°) 105. TELEMANN'S *curve* $\left(\overset{\frown}{8}\atop 6\right)$ over figure 134.
Three-part harmony, as formerly figured 124, 134.
Tones, musical, as distinguished from noises 152, classification of 153.
Tonic (= Key-note) 12, triad of the in major 13, in minor 29.
Transposed system of modes 160—161.
Triad, major 12, minor 15, 29, three fundamental *t's* in major 13, in minor 29, subordinate *t's* in major 15, in minor 29—31, dimin. *t's* in major 16—17, 159, in minor 31, view of all

triads in major 17, in minor 30, augm. triad in minor 30, 67. *Inversions* 38—48. *Dissonant t's, v.* Dissonant.
Tritone unmelodic 24, 64, 69.
Trochee 165.
Twelfths = fifths 124.

Undertones 158.

Vibrations, theory of 152.
Violin-clef (= *G*-clef) 111, as employed in writing tenor-part 147.

Westphal, Rudolph, "Elements of Musical Rhythm" 166.
Whole tone, greater and lesser, mathemat. values of 155, 164.

Zarlino 160.

Errata.

Page 17, line 7 from below, for *form* read *from*.
» 44, » 3 from above, for *A* read *a*.
» 76, » 8 from below, supply comma between *can* and *however*.
» 78, » 9 from above, for *suited* read *adapted*.
» 79, » 8 from above, for *subtle* read *subtile*.

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